5 March 2019  Problem Set 7  These problems are due in class on Tuesday, 12 March. These 5 problems are based on lectures through March 7.

Please put your name and section number on the first page of your solutions. The graded homework will be returned in your sections.

1. **Radiation Pressure**

Suppose you are designing a spaceship powered by solar radiation pressure.

(a) With the spaceship a distance \( r \) from the Sun, find the gravity force on the spaceship of mass \( M \).

(b) Determine the intensity of the sunlight at the spacecraft a distance \( r \) from the Sun. Assume the light power emitted by the Sun is \( P \).

(c) Your spaceship has a reflecting sail of area \( A \). You orient the sail so it reflects light directly back towards the Sun. Find the resulting radiation force on your spaceship at distance \( r \).

(d) Find the minimum area \( A \) needed to power the spaceship out of the solar system, assuming the spaceship mass \( M \) is 1000 kg. You will need to look up data about the Sun.

2. **Slow Glass**

Bob Shaw has written science fiction stories about “slow glass”: a glass whose index of refraction is so big that it takes, say, a year for light to travel the thickness of a standard pane. He envisions you position the glass overlooking a picturesque scene for a year, then move the glass to your home. For a year you would then enjoy the scene at home. Consider slow glass such that it takes visible light a year to travel a distance of 1 cm through it.

(a) Determine the index of refraction of this glass.

(b) Suppose you took a square meter of this glass with 1 cm thickness and exposed its face “24/7” over a year to the full light of the sun. How much energy is stored in the glass? The sun radiates at 1300 W/m\(^2\). Assume no reflection in this problem.

(c) Estimate the maximum electric field strength in the glass.

3. **Slow Glass Part II** Now consider reflection for the slow glass in problem 2.

(a) Determine the reflection coefficient.

(b) Based on your answer in (a), explain why useful slow glass would be very hard to realize.

4. **Pulse of radiation moving in the \( x \) direction.** Consider the fields

\[
E(r,t) = \hat{i}F_1(x-ct) + \hat{j}F_2(x-ct) + \hat{k}F_3(x-ct), \quad cB(r,t) = \hat{i}G_1(x-ct) + \hat{j}G_2(x-ct) + \hat{k}G_3(x-ct),
\]

where the functions \( F_i, G_i \) approach 0 in the limits \( x \to \pm \infty \). These fields satisfy the wave equation and correspond to a pulse of radiation moving in the \( +x \) direction. The Maxwell equations place sever restrictions on the components \( F_i, G_i \).

(a) Show that the Maxwell equations require that \( F_1 = G_1 = 0, G_3 = F_2 \) and \( G_2 = -F_3 \).

(b) Suppose \( F_2(\xi) = G_3(\xi) = E_0 \exp(-\xi^2/a^2) \) and other components are 0. Make a sketch that shows a snapshot of the fields at a time \( t \).

5. **Cosmic microwave background radiation** Most of the electromagnetic energy in the Universe is in the cosmic microwave background radiation, a remnant of the Big Bang. This radiation was
discovered by A. Penzias and R. Wilson in 1965, using observations with a radio telescope. The radiation is electromagnetic waves with a wavelength around 1.1 mm. The time-averaged energy density is $4.0 \times 10^{-14} \text{ J/m}^3$.

(a) Determine the maximum value of the electric field strength of the cosmic microwave background radiation.

(b) How far from a 1000 W transmitter would you have to go to have the same field strength? Assume the power from the transmitter is isotropic.