In class example

\[ B = \frac{\mu_0}{4\pi} \left( \frac{3m \cdot r^2 \cdot \hat{m}}{r^3} \right) \]

**Very Long cylinder** \( M = \text{constant along axis} \)

**B outside, B inside?**

Take \( B/2 \)

outside compute \( \mathbf{B} = \frac{\hat{m}}{2} \mathbf{M} \text{ Volume} \)

\[ \mathbf{B} = \frac{\mu_0}{4\pi} \left( \frac{3m \cdot \hat{m}}{r^3} \right) \]

inside \( \mathbf{B} = 0 \)

\[ \frac{1}{2} \text{ Last time obtained } \mathbf{B} \text{ due to magnetization} \]

\[ \mathbf{A} (\mathbf{r}) = \mu_0 \int_{V} d^3r' \mathbf{J}_b (r') \frac{1}{r - r'} + \mu_0 \int_{S} \mathbf{J}_o (\mathbf{r}) \left( \mathbf{r} - r' \right) \]

\[ \frac{1}{r} \]

\[ \mathbf{J}_b (r') = \nabla \times \mathbf{H} \quad \mathbf{K}_b = \mathbf{M} \times \mathbf{H} \text{ on surface} \]

These formulae gives power to compute \( \mathbf{A} \) therefore \( \mathbf{B} \).

There are many sources of \( \mathbf{B} \).

free currents & bound currents
We need a combined approach that includes both kinds of currents:
\[ \mathbf{J} = \mathbf{J}_f + \mathbf{J}_b \rightarrow \text{Materials with magnetic properties - may dipole} \]

Think currents where we turn on south pole:
\[ \mathbf{J} \rightarrow \text{Current flows} \]
\[ \mathbf{J} = \mathbf{J}_f + \mathbf{J}_b \]

In computing \( \mathbf{A} \) we need to include both kinds of currents.

In computing \( \mathbf{B} = \mathbf{\nabla} \times \mathbf{A} \) need to include both kinds of currents.

So \( \mathbf{A} = \frac{\mu_0}{4\pi} \int \mathbf{d}^3 r' \left( \mathbf{J}_f (r') + \mathbf{\nabla} \times \mathbf{M} (r') \right) \frac{1}{r-r'} \)

when taking \( \mathbf{\nabla} \times \mathbf{A} = \mathbf{B} \) we find

\[ \mathbf{B} = \mathbf{\nabla} \times \mathbf{A} \]

Want Part Diff Eq for \( \mathbf{B} \)

\[ \mathbf{\nabla} \times (\mathbf{B} - \mu_0 \mathbf{M}) = \frac{\mu_0}{c^2} \mathbf{J}_f \]
\[ \nabla \times (\mathbf{B} - \mu_0 \mathbf{M}) = \mu_0 \frac{\partial \mathbf{J}_f}{\partial t} \]
\[ \mathbf{H} = \mathbf{B} - \mathbf{M} \]
\[ \nabla \times \mathbf{H} = \mathbf{J}_f \]

we control \( \mathbf{J}_f \)

Name: Griffiths

\( \mathbf{B} = \text{mag. field} \)
\( \mathbf{H} = \text{auxiliary field of} \mathbf{H} \)

other books:
\( \mathbf{B} = \text{mag. ind. flux dens.} \)
\( \mathbf{H} = \text{mag. field} \)

\( \mathbf{B} \) is the fundamental field that

exerts force on moving charged particles.

Flux calculated from \( \mathbf{B} \)

SI Units:
\( [\mathbf{B}] = \text{tesla} \quad T = \frac{N}{(\text{Am})} \)
\( [\mathbf{H}] = \frac{N}{\text{Am} \cdot \mu_0} = \frac{N}{\text{Am}} \quad \text{m A/m} \)

In free space, \( \mathbf{B} = \mu_0 \mathbf{H} \)

Diagnaloic or paramagnetic \( \mathbf{B} = \mu \mathbf{H} \)

\( \uparrow \text{permability} \)

Ferromagnetic \( \mathbf{B} \) is nonlinear function of \( \mathbf{H} \)
Relation between \( B \) and \( H \): 
\[
\vec{H} = \vec{B} - M
\]

In general, this:
\( M \) is caused by free currents;
\( \vec{H} \) is caused by \( \vec{H} \) in some way that depends on the material.

In param. or diamag (not ferromagnetic):
Simplifying:
\[
\vec{M} = \chi_m \vec{H}
\]
Linear media

\( \chi_m \) magnetic susceptibility
for PnD, \( \chi_m \approx 1 \times 10^{-5} \)

\[
\vec{B} = \mu_0 (\vec{H} + \vec{M}) = \mu_0 (1 + \chi_m) \vec{H}
\]

\[
M = \mu_0 (1 + \chi_m) \text{ Permeability}
\]

\[
\vec{B} = \mu \vec{H}
\]

Works for diamagnetism or paramag.

Lect demo 5G30.

\[
\vec{B} = \mu \vec{H}
\]

Remember
\[
\vec{E} = \nabla \times (M \cdot \vec{B})
\]

Manganese C1 & att
But both are different

\[ \nabla \cdot \mathbf{B} = 0 \quad \nabla \times \mathbf{H} = -\nabla \cdot \mathbf{M} \text{ which is not always 0} \]

Both satisfy \( \nabla \cdot \mathbf{B} = 0 \)

Want to solve problems with \( \mathbf{M} \) to understand if it operates between \( \mathbf{B} \) & \( \mathbf{H} \)

Nerd Boundary conditions on \( \mathbf{B} \) & \( \mathbf{H} \)

From \( \nabla \cdot \mathbf{B} = 0 \) always true (so far)

\[
\text{Surf} \left( \frac{1}{\mu_0} \right) \mathbf{B}_\text{in} - \mathbf{B}_\text{in} = 0
\]

For tangential components use \( \mathbf{H} \)

When we have \( \nabla \times \mathbf{H} \) use Stokes thm

\[ \oint_c (\mathbf{H}_\text{in} - \mathbf{H}_\text{out}) = \lim_{\omega \to 0} \frac{1}{\mu_0} \iint_S \mathbf{J} \cdot d\mathbf{a} \]

For \( \mathbf{J} \neq 0 \), must have surface current \( \mathbf{J} \) to area

\[ \mathbf{H}_\text{ill} - \mathbf{H}_\text{in} = \mathbf{k} \times \mathbf{n} \]

Lecture Demo 5 6 30. Para magnetism & Diag magnetism

56.20.55  Remesley basis.
Computing $\mathbf{B}$ & $H$ for a uniform magnetized sphere

Outside

\[ \mathbf{M} = \mu_0 \mathbf{H}_{\text{inc.}} \]

Outside - nothing

determine $\mathbf{B}$, $\mathbf{H}$ outside

Physics is not a spectator sport

\[ \mathbf{B} = \mu_0 \left( \frac{3 \mathbf{m} \cdot \hat{r} \hat{r} - \mathbf{m}}{r^3} \right) \]

\[ \mathbf{m} = \frac{4\pi R^3 M_0}{3} \]

d) compute $\mathbf{B}$ at $\theta = 0, 90^\circ, \pi$

\[ \hat{r} = \hat{h} \]

\[ \mathbf{B}(0^\circ) = \frac{\mu_0}{4\pi} - \frac{2m}{r^3} \]

\[ (\theta = 90^\circ, \mathbf{B}(90^\circ) = -\frac{\mu_0}{4\pi} \hat{h} \]

\[ (\theta = \pi, \hat{m} = -\mathbf{m}, \hat{r} = -\hat{h}) \]

\[ \mathbf{B}(\pi) = \mathbf{B}(0) \]

Inside

\[ \nabla \times \mathbf{H} = 0, \nabla \cdot \mathbf{H} = 0 \]

\[ \nabla \cdot \mathbf{H} = -\nabla \cdot \mathbf{M} \]

Know Div. with no $\nabla \times$? Sound familiar? Think electrostatics.
Magnetic

Solve as in electrostatics

\[ \mathbf{H} = -\nabla \Phi \] magnetic scalar potential

The \[ \nabla \cdot \mathbf{H} = -\nabla^2 \Phi = -\nabla \cdot \mathbf{M} \]

Poisson's eq for \( \Phi \)

\(-\nabla \cdot \mathbf{M} \) is source of \( \mathbf{H} \)

Now we need to work out divergence of \( \mathbf{M} \)

Look at \( \mathbf{M} \) as a function of \( r \)

\[ \mathbf{M} = M_0 \Theta(R-r) \mathbf{\hat{r}} \]

\( \nabla \cdot \mathbf{M} = \frac{\partial M}{\partial r} = \frac{\partial}{\partial r} \left( M_0 \Theta(R-r) \right) = M_0 \Theta'(R-r) \frac{\partial r}{\partial r} \left( \frac{x}{r^2} \right) \]

Need some math concepts

Theta function \( \Theta(x) = 1 \) at \( x > 0 \)

\[ = 0 \quad x < 0 \]

\[ \mathbf{M} = M_0 \Theta(R-r) \mathbf{\hat{r}} \]

\[ = M_0 \Theta(R-r) \mathbf{\hat{r}} \]

\[ \nabla \cdot \mathbf{M} = \frac{\partial M}{\partial r} = \frac{\partial}{\partial r} \left( M_0 \Theta(R-r) \right) = M_0 \Theta'(R-r) \frac{\partial r}{\partial r} \left( \frac{x}{r^2} \right) \]

\[ z = r \cos \theta, \quad r = \sqrt{x^2 + y^2 + z^2} \]

\[ \frac{\partial r}{\partial x} = \frac{x}{r^2}, \quad \frac{\partial r}{\partial y} = \frac{y}{r^2}, \quad \frac{\partial r}{\partial z} = \frac{z}{r^2} \]

\[ \frac{\partial r}{\partial \theta} = \frac{r \sin \theta}{r} = \sin \theta \]

\[ \frac{\partial r}{\partial \phi} = \frac{r \cos \theta}{r} = \cos \theta \]
What is \( \frac{d\theta}{dr} \) at \( r < R \) \( \frac{d\theta}{dr} = 0 \) at \( r > R \)

at \( r = R \) \( \frac{d\theta}{dr} = -\infty \)

Have a function \( \frac{d\theta}{dr} \) which is 0 except at 1 point \( (r = R) \) then it is \( \infty \).

Moreover \( \int_{R}^{\infty} \frac{d\theta}{dr} \, dr = \theta(\varepsilon) - \theta(-\varepsilon) \)

\( \varepsilon > 0 \), \( \varepsilon \ll R \)

A function of 1 variable that is 0 everywhere except at one point and which has integral = unity is called a \( \delta \) function.

\( \delta(x) = 0 \) if \( x \neq 0 \)

\( \delta(0) = \infty \)

\( \int_{-\infty}^{\infty} \delta(x) \, dx = 1 \)

\( \int_{a}^{b} \delta(x) \, dx = 0 \) if \( a > 0 \)

\( \int_{a}^{b} \delta(x) \, dx = 0 \) if \( b < 0 \)
Thus \( +Y \cdot M = -M_e \cos \theta \, \delta(r-R) \)

and \( \nabla^2 \Phi_m = -M_e \cos \theta \, \delta(r-R) \)

The only angular dependence is \( \cos \theta \)

so \( \Phi_m(r, \theta) = f(r) \cos \theta \)

No need to make expansion in Legendre polynomial

\[
\nabla^2 \Phi_m = \frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d}{dr} f \right) \cos \theta + \frac{f}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} f \right)
\]

\[
= \frac{d^2f}{dr^2} + \frac{2}{r} \frac{df}{dr} \cos \theta = \frac{2}{r^2} f(r) \cos \theta
\]

So \( \frac{d^2f}{dr^2} - \frac{2}{r} \frac{df}{dr} - \frac{2}{r^2} f = -\frac{M_e}{\delta(r-R)} \)

If \( r\leq R \) RHS = 0 and \( f(r) = 0 \) solves

If \( r > R \) RHS = 0 and \( f(r) = \frac{1}{r^2} \) solves

So function \( f \) looks like
Need only one const. derived \( R = r \)

Is not continuous \( f(r) = C \left[ \frac{R^3}{r^3} \right] \)

Integrate eq. (1) from \( r = R - \epsilon \) to \( R + \epsilon \)

\[
\frac{df}{dr}(R+\epsilon) - \frac{df}{dr}(R-\epsilon) = -2 \frac{f(R-\epsilon)}{R} - 0
\]

\[-\frac{2}{R^2} f(R)(R-\epsilon) = -M_0\]

So \( f'(R+\epsilon) - f'(R-\epsilon) = -M_0 \)

\[-\frac{2\pi C}{R^3} - \frac{C}{R} = -M_0\]

\[C = +\frac{M_0 R}{3}\]

\[f(r) = \frac{M_0 R}{3} \left[ \frac{r}{R^3} \frac{r < R}{r^2/R^2} \frac{r > R}{\hat{r}} \right] \]

Inside \( H = -\nabla \Phi_m = -\nabla \left( \frac{M_0 R}{3} \frac{\hat{r}}{R} \cos \theta \right) \)

\[\hat{r} = \nabla \left( -\frac{M_0}{3} \frac{\hat{r}}{R} \right) = -\frac{M_0}{3} \frac{\hat{r}}{R} \]
Inside

\[ \mathbf{B} = \mu_0 \mathbf{H} + \mathbf{H}_0 = \frac{2}{3} \mathbf{M} \mu_0 \]

Outside

\[ \mathbf{H} = -\mathbf{\nabla} \left[ \frac{1}{3} \mathbf{H}_0 \mathbf{R} \frac{R^2}{R^3} \cos \theta \right] \]

\[ = \frac{1}{3} \mathbf{H}_0 \frac{M_0 R^3}{r^4} \cos \theta \]

\[ = \frac{\mathbf{H}_0}{3} \frac{M_0 R^3}{r^3} \cos \theta \]

\[ \mathbf{B} = \mu_0 \mathbf{H} \text{ as seen} \]

\[ M = \frac{4\pi r}{3} R^3 M_0 \]
Exam Jan 31

Content HW1 HW2

Chapter 5, 6.1, 6.2

Equations for exam are posted. Will be removed out of the exam.

HW3 due Feb 5, you will have

When posted?

Dangerous 2, lecture not on exam.

but is on exam 2. 0 needed for HW3.
Current and resistance

We've done electrostatics & magnetostatics. Time to move on!

Chapter 7

Current and resistance

We learned in 2.1 that a conductor in equilibrium is equipotential. \( E = 0 \)

However, if there is a current it is not equipotential.

There must be a potential difference, \( \Delta V \) in the conductor. There must be a force pushing charge. Measurements tell us that current and potential difference are linearly related.

\[ V = I \cdot R \]

Ohm's Law

\[ R = \text{resistance units in SI} \]

\[ \text{[R]} = \frac{V}{I} = IR \]

\( I = \text{conductance} \)

Ohm's Law is an excellent approximation for many conductors, not universal.

Suppose have a conducting cylinder.

Radius \( R \)

\[ \frac{R}{A} = \frac{\rho L}{A} \]

not charge density = resistivity

\[ [\rho] = \text{S/m} \]
Pure metals $\sigma \approx 10^{-8} \text{ m}^2 \text{V}^{-1} \text{S}^{-1}$

Alloys $10^{-8} \text{ m}^2 \text{V}^{-1} \text{S}^{-1}$

Semiconductors: Ge $\sim 0.5 \Omega \text{m}$
Silicon (Si) $2300 \Omega \text{m}$

Insulators:
Wood $10^8 - 10^{11} \Omega \text{m}$
Glass $10^{10} - 10^{15} \Omega \text{m}$

Conductivity $\sigma = \frac{1}{\rho}$, $\rho \leftrightarrow \text{resistivity}$

So $R = \frac{1}{\sigma A}$

Insulators: energy to move electron from bound states in atoms into states they can move is very large — AMech property

There is a more basic eqn than $V = IR$

Local form of Ohm's Law

$\sigma E(\vec{r}) = \nabla \times \vec{J}$ is current density

$I = \int \vec{J} \cdot d\vec{A}$

low $\rho \rightarrow$ large $\sigma \rightarrow$ large current, again an approximation

Uniform cylinder $I = \int J A$
Example cylindrical resistor between two perfect conductors.

Current flows up.

Consider example to see where is change. Current's flow of charge, but system is neutral.
Cylindrical resistor area $A$ height $L$

About below perfect conductor $\rho = 0$ inside $\rho = \text{may vary with } z$

Goal compute resistance and charge distribution giving steady current $I$

breakup cylinder into disks of area $A$ height $dz$

For a length $dz$ we get a $dR$

$$dR = \frac{\rho(z) dz}{A}$$

Total resistance

$$R = \frac{I}{A} \int_{0}^{h} \rho(z) dz$$

If $\rho = \text{constant } \rho_0$ then $R = \frac{\rho_0 L}{A}$

What about charge dist. Use 321 method

to compute charge dist

$$\vec{J}(\vec{r}) = \frac{I}{A} \hat{z}$$

Potential diff at height $z$, $V(z) = \frac{I}{A} \int_{0}^{z} \rho(z) dz$

from $V = IR$ causes current flow

Potential of $\rho$ between bottom top
\[ E = - \nabla V \]
\[ = \frac{\rho(z)I}{A} \]
\[ \rho_{\text{free}} = \varepsilon \nabla \cdot E = \frac{\varepsilon I}{A} \frac{df}{dZ} \quad \text{q} \quad \rho = \text{const} \quad \text{q} \to 0 \]

\[ \sigma_{\text{free}} \text{ on surface at } Z = 0, \ h \quad \sigma = \varepsilon \varepsilon_n \]
\[ \sigma_{\text{free}}(h) \quad \varepsilon \in E_2(h) = \frac{\varepsilon I}{A} \rho(h) \]

If \( \rho \) is constant, charges on top and bottom are equal and opposite.

\[ \rho = \text{constant} \]

Sideview

\[ \text{no change in here} \]

If \( \rho \neq \text{constant} \), what happens?

\[ \text{at } Z = 0 \quad Q_{\text{free}} = \sigma_{\text{free}} A \varepsilon_n = \varepsilon I \rho(0) \]
\[ Z = h \quad = - \sigma_{\text{free}}(h) A = - \varepsilon I \rho(h) \]

\[ \text{In the resistor} \quad Q_{\text{free}} = \int_{\text{free}} \rho dV = \int dZ \frac{\varepsilon I df}{A dZ} \]
\[ = \varepsilon I (\rho(h) - \rho(0)) \]

\( \text{San of B} \) vanishes
Thursday Jan 24

5K: Electromagnetic Induction

5K10. Induced Currents and Forces

Wire and Magnet (5K10.15) — A single loop of wire is passed between poles of a large horseshoe magnet, causing current to flow (shown on a galvanometer). The faster the wire is moved, or the greater the number of loops, the larger the current.

Re: lecture demos for Tues Jan 22 9 am A118 and thursday Jan 24...

Coil Pendulum in Magnet (5K10.18) — A pendulum with a large coil for a bob swings between the poles of a large horseshoe magnet. A small light bulb wired to the coil flashes when the coil swings through the magnetic field.

Simple Coil and Bar Magnet (5K10.20) — A coil is connected to a galvanometer. A bar magnet is passed through the coil and the galvanometer measures the current.

10/20/40 Turn Coils with Magnet (5K10.21) — Coils of 10, 20, and 40 turns wired in series on a common stand, through which a permanent magnet is moved to produce small currents as shown on a galvanometer.

Mutual Induction (5K10.30) — Two coils slide on a track so that the distance between them can be varied. Current is pulsed into one coil with a switch, which induces current in the second coil. Meters show the currents in both coils, and show that there must be a changing current in the first coil to induce current in the second. Intensity of induced current changes with separation, and various metal cores can be inserted to determine their effect on the magnetic flux.
More on resistance

Resistance - an act of resisting opposition

\[ I_{\text{metals}} = \frac{V}{R} \frac{dE}{dF} \]

\[ I \frac{dV}{dF} \text{ is Newton wrong} \]

No, in metals & \text{force on electron}

How Vane to move through past other electrons

Anychee atoms in lattice

Basically in conductor, conductin electrons

lose energy by collisions with atoms

Energy lost is transfered as heat in the
form of kil, energy of atoms, power is
lost. How much??

The work per unit time as charge passes

through potential difference \( V \) is \( IV \). \[ P = \text{Work done} = \frac{\text{charge}}{\text{time}} \]

Energy conservation tells us that this power

is dissipated as heat \( P = IV = I^2R \)

Joule's law 1841 found by
Experimental measurement

<table>
<thead>
<tr>
<th>Device</th>
<th>Current A</th>
<th>Power W</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table Lamp</td>
<td>0.8</td>
<td>100</td>
</tr>
<tr>
<td>Hand Iron</td>
<td>8</td>
<td>1000</td>
</tr>
<tr>
<td>Starter motor of car</td>
<td>180</td>
<td>2200</td>
</tr>
</tbody>
</table>

Role of these devices

- Show demos

5K 10. Induced currents & force

<table>
<thead>
<tr>
<th>Test</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.15</td>
<td>Wire &amp; magnet</td>
</tr>
<tr>
<td>10.18</td>
<td>Coil &amp; pendulum in magnet</td>
</tr>
<tr>
<td>10.20</td>
<td>Coil &amp; bar magnet</td>
</tr>
<tr>
<td>10.21</td>
<td>Coil &amp; bar magnet</td>
</tr>
<tr>
<td>10.22</td>
<td>Mutual inductors</td>
</tr>
</tbody>
</table>

end of June 24 lecture