Now we do the general case of oblique angle

\[ E_1 \rightarrow E_2 \]

reflected wave

Incident wave

\[ \theta_i, \theta_r, \theta_e \]

also will want direction of \( E_2, n \)

this gives polarization of light

Will use the boundary conditions

tangential \( E \), tangential continues

normal \( D, B \) continue

First step: Enumerate fields

Incident
\[ E_0^i = E_0^e e^{i(k_1 z - \omega t)} \]

\[ B_0^i = \frac{\hat{E}_0^i \times \hat{B}_0^i}{\omega} \]

reflected
\[ E_0^r = E_0^e \frac{e^{i(k_2 z - \omega t)}}{\omega} \]

\[ B_0^r = \hat{a}_r \times \frac{\hat{E}_0^r}{\omega} \]

\( \hat{a}_r \) not \( \perp \) to \( \hat{a}_e \)

Hard to figure out where \( \hat{a}_e \) goes
Transmitted \[ E_T = E_0 e^{j(\mathbf{k}_0 \cdot \mathbf{r} - \omega t)} \]

\[ B_T = \frac{E_T}{\gamma_0} \times E_T \]

\[ \frac{E_T}{\gamma_0} \]

Notice the same \( e^{-j\omega t} \) is everywhere.

This is the only way to satisfy Boundary Condition advantage of complex notation. Get mag of \( E_1, T, \) \( \mathbf{J} \)

so we may write

\[ \mathbf{J} = \mathbf{E} = \frac{\mathbf{E}_1}{c} = \frac{\mathbf{E}_1}{c} \]

\[ \mathbf{J} = \frac{\mathbf{E}_1}{c} \]

Can get a lot of info from mere existence of boundary conditions BC

The interface is at \( z=0 \)

The BC must be satisfied at all values of \( x, y, z \) on the interface.

Why? IF not true BC won't be satisfied.

This means at \( z=0 \).

\[ \mathbf{E}_t \cdot \mathbf{r} = \mathbf{E}_r \cdot \mathbf{r} = \mathbf{E}_t \cdot \mathbf{r} \]

\[ \mathbf{r} \]

\[ \mathbf{r} \]

\( \mathbf{r} \) refers to position in complex plane.

Must be true at one place to satisfy BC. But that 126
If above is true at one point changing $r$ will make it not true

Another way to model problem

$$A e^{i\alpha x} + B e^{i\beta x} = C e^{i\gamma x}$$

(like our problem)

Show $A + B = C$ and $a = b = c$

Take $x = 0$

So

$$A (e^{i\alpha x} - e^{i\gamma x}) + B (e^{i\beta x} - e^{i\gamma x}) = 0$$

Take $\frac{d}{dx}$ at $x = 0$

$$A (a - c) = -B (b - c) \quad (1)$$

Take $\frac{d^2}{dx^2}$ at $x = 0$

$$A (a^2 - c^2) = -B (b^2 - c^2) \quad (2)$$

Take (2)

$$A (a + c) = (b + c)$$

(1) $\implies a = b$

$$A (a - c) = -B (b - c)$$

either $c = a$ or $A = -B$

if $A = -B$, $c = 0$ which violate given

so $c = a = b$
So use BC idea to relate I OR
\[ x(hR)_x + y(hR)_y = x(hT)_x + y(kT)_y = x(hT)_x + y(kT)_y \]
Can set \( x = 0 \)
\[ (hR)_y = (hT)_y = (kT)_y \]
Or \( y = 0 \)
\[ (hR)_x = hRx = hTx \]

Look at interface plane

Side:
Y axis out of page \( hR \) in \( x \), \( z \) plane, \( hT \) at

Thus \( hR_y = 0 = hT_y \)
can work in \( xz \) plane, plane of incidence
\( hR_x = hT \) in same plane just as drawn

Look as \( \cos \theta \) come
\[ (hR)_x = hR \sin \theta \]
\[ = (hT)_y = hT \sin \theta \]

Angle of incidence \( \theta I = \theta R \) since \( \sin \theta \) angles go

Also
\[ \frac{hR}{hT} \sin \theta = \frac{hT}{hT} \sin \theta \]
\[ \sin \theta = \sin \theta \]
Snell's Law

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

These 2 results are true for any type of wave: sound, seismic, EM.

Also, marching band moving at an angle to a boundary between low & high grass.

But EM is much richer. \( \mathbf{E} + \mathbf{B} \) can have x, y component.

Let's be specific about BC. There are no free charges or free currents.

Tangential \( \mathbf{E} \) is continuous. \( \mathbf{D} \) is not.

\[ (\mathbf{E}_{0x} + \mathbf{E}_{0y})_x = (\mathbf{E}_{0x})_x \]

Normal \( \mathbf{D} \) is continuous.

\[ \varepsilon_1 [\mathbf{E}_{0x} + \mathbf{E}_{0y}]_z = \varepsilon_2 (\mathbf{E}_{0z})_z \]

Because we have no free charges on the dielectric surface.

Don't bother with \( \mathbf{B} \)'s given \( \mathbf{E} \)'s known.
To proceed we need to specify polarization of light. There are two separate cases, depending on whether light polarizes parallel or perpendicular to the plane of incidence. Why different cases?

Physics is oscillating electric dipole at surface \( \mathbf{P} \propto \mathbf{E} \).

Suppose \( \mathbf{E} \) incidence light polarizes parallel to the plane of incidence means \( \mathbf{E} \) has only \( x, z \) components.

\[ E_{1o} \cos \theta_i + E_{2o} \cos \theta_i = E_{z} \cos \theta_i \]

Two components of \( \mathbf{E} \):

\[ E_{1} \left[ -E_{1} \sin \theta_i + E_{2} \sin \theta_i \right] = E_{2} E_{z} \]

Two equations, two unknowns.
Again we have 2 eqns unknowns.

Simplify eqns whatever.

Define $\beta = \frac{n_2}{n_1}$

$$d = \frac{\cos \beta}{\cos \theta}$$

Solve 2 eqns 2 unknowns

$$E_{OR} = \left( \frac{d-\beta}{d+\beta} \right) E_{OL}$$

$$E_{OL} = \frac{2}{d+\beta} E_{OL}$$

Fresnel eqns

Transmitted wave is phase with incident wave.

Reflected wave is in phase if $d > \beta$

or $180^\circ$ out of phase if $d < \beta$

$d$ & $\beta$ depend on indices of refraction & $\beta$ (obvious!)

Let's look at $d$

$$d = \sqrt{1 - \sin^2 \theta} = \sqrt{1 - \left( \frac{n_1}{n_2} \sin \theta \right)^2}$$
How can we check equal

normal incidence \( \theta_1 = 0 \) \( d = 1 \) \( \theta_2 = \alpha_2 \)

get the previous result \( \theta_1 = \frac{\pi}{2}, \alpha_2 = 0 \)

grazing incidence \( \theta_2 = 90^\circ \)

\[ d \to 0 \quad \text{entire wave is reflected} \]

\[ E_{0r} = E_{0i}, \quad B_{0r} = 0 \]

Can the reflected wave be eliminated

\( d = \beta \) means

\[ \frac{\cos \theta_1}{\cos \theta_2} = \frac{n_2}{n_1} = \sqrt{1 - \left( \frac{n_2 \sin \theta_2}{n_1} \right)^2} = \beta \]

\( \frac{n_2}{n_1} = \beta \)

when this can satisfied \( \theta_2 = \theta_B \) - Brewster angle

\[ \frac{n_2}{n_1} \cos^2 \theta_B = 1 - \frac{n_2^2 \sin^2 \theta_B}{n_1^2} \]

\[ \tan \theta_B = \frac{n_2^2}{n_1} \cos \theta_B = 1 - \frac{1}{n_2^2} \sin^2 \theta_B \]

\[ \frac{n_2}{n_1} \sin^2 \theta_B \]

\[ \frac{n_2^2}{n_1^2} - \frac{n_2^2 \sin^2 \theta_B}{n_1^2} = 1 - \frac{n_2^2}{n_1^2} \sin^2 \theta_B \]

\[ \frac{\beta^2 - 1}{\beta^2} = \sin^2 \theta_B = \frac{\beta^4 - \beta^2}{\beta^4 - 1} = \frac{\beta^2}{\beta^4 - 1} \]

\[ \frac{\beta^2 - 1}{\beta^2} \quad \beta > 1 \]

\[ \beta = \frac{n_2}{n_1} \quad \beta > 1 \]

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\[ \theta \text{ from air } n_1 = 1 \text{ to glass } n_2 = 1.5 \]

\[ \beta = 1.5 \]

\[ \theta \text{ out of phase} \]

\[ \theta_1 = 0 \]

\[ I_T = \frac{1}{2} E_{0z} E_{0z} \cos \theta_2 \]

Power per unit area, striking boundary

\[ S_{.5} \]

\[ R = \frac{I_R}{I_T} = \left| \frac{E_{0r}}{E_{0z}} \right|^2 = \left( \frac{d - \beta}{d + \beta} \right)^2 \]

\[ T = \text{more complicated} \]

\[ T = \frac{I_T}{I_T} = \frac{\beta^2}{T_1} \left( \frac{E_{0z}}{E_{0r}} \right)^2 \cos \theta_1 \cos \theta_2 = \frac{d \beta^4}{(d+\beta)^2} \]

\[ R + T = 1 \]

Won't do 1 case - nice extension for possible z
\[ E_I = B_{zo} \hat{y} e^{j(h_I \cdot r - \omega t)} \]
\[ B_I = \frac{\hat{z} \times E_I}{v} \]
\[ \hat{h}_I = \sin \theta \hat{x} + c \theta \hat{y} \]
\[ E_r = E_{zo} \hat{y} e^{j(h_I \cdot r - \omega t)} \]
\[ B_r = E_{zo} \hat{y} e^{j(h_I \cdot r - \omega t)} \]
\[ E_t = E_{to} \hat{y} e^{j(h_I \cdot r - \omega t)} \]

**BC** Tangent \( \vec{E} \) continuous

\[ E_{zo} e^{j \Phi_0} = B_{zo} \]

**Normal** \( \vec{B} \) is continuous

\[ \vec{B} = \vec{E} \] or \( \vec{E} \cdot \vec{B} \) has no \( \mu \) \( \Theta = 0 \)

**Normal component need other eq.**

**BC** Normal \( \vec{B} \)
In lectures we present physics in textbook fashion - in which it seems like everything is known. But this is not the case. 2 of Magnetic Monopoles is one case.

E&M appears everywhere in our world because it forms the basis for our technology. You might think everything is understood & there are no unanswered fundamental physics questions. True, the practical aspects are described by Maxwell's equations. At a deeper level, a quantum mechanical account given by QED predicts magnetic moment of electron to 10 sig figs. BUT there is one elementary aspect of E&M that is not understood - magnetic monopoles.

Magnets have 2 poles North & South - fact. But why shouldn't there be a magnet with a single North or South pole? Missing something? Or not found yet?

Maxwell's equations possess asymmetry between electric and magnetic...
Consider a region of space where $\rho = 0, \\overrightarrow{J} = 0$

$$\nabla \cdot \vec{E} = 0 \quad \nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \quad (1)$$

$$\nabla \cdot \vec{B} = 0 \quad \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} - \nabla \times \vec{B} = 0 \quad (2) \quad (\text{Minkowski} = \frac{1}{c^2})$$

There is a symmetry; the equations are invariant under the replacement $\vec{E} \rightarrow c\vec{B}$ and $\vec{B} = -\frac{\vec{E}}{c}$.

Under this replacement, we get:

$$\nabla \cdot \vec{B} = 0 \quad (1) \rightarrow \nabla \times c\vec{B} = \frac{1}{c} \frac{\partial \vec{E}}{\partial t} \quad (5)$$

$$\nabla \cdot \vec{E} = 0$$

$$\frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} - \nabla \times \vec{B} = 0 \quad (2) \rightarrow \frac{\nabla \times \vec{B} - \frac{\partial \vec{E}}{\partial t}}{c} = 0 \quad (7)$$

Physicists love symmetry. But this symmetry is lost when charges and currents are around:

$$\nabla \cdot \vec{E} = \rho / \varepsilon_0 \quad \text{and} \quad \nabla \cdot \vec{B} = 0$$

The symmetry can be maintained if we postulate the existence of magnetic charges and currents.

Then we have
\[ \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \quad \nabla \cdot \vec{B} = \frac{\partial \rho}{\partial t} \]

\[ \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} - \nabla \times \vec{B} = -\frac{\partial \vec{D}}{\partial t} \]

\[ \frac{\partial \vec{B}}{\partial t} + \nabla \times \vec{E} = -\frac{\partial \vec{D}}{\partial t} \quad \text{(3)} \]

Invariance under \( \vec{E} \rightarrow -\vec{E}, \quad \vec{B} \rightarrow -\vec{B}, \quad \rho \rightarrow -\rho, \quad \vec{J} \rightarrow -\vec{J} \)

Recall that the displacement current \( \vec{D} \) arose from the need to have \( \nabla \cdot \vec{J} = 0 \)

Similarly, the magnetic current \( \vec{J} \) is conserved:

\[ \nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0 \]

Take \( \nabla \cdot \) of Eq.(3) \( \frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial t} = -\frac{\partial \vec{D}}{\partial t} \)

This symmetry is called duality symmetry.

The existence of \( \vec{J} \) means the analog of Coulomb's Law would occur.

Maxwell's theory is compatible with the existence of magnetic charges, but were not included. This because experiments suggested that these charges did not exist.
Experiments in Maxwell's time were simply involved with cutting long magnets to pieces. The assumption of no magnetic charges still holds—not found yet. But we still do not understand why.

Let's try to understand how a magnetic monopole would work.

\[
\mathbf{B} = \frac{\mu_0}{4\pi} \frac{q}{r^2}
\]

Charged particle of charge \( q \) moves along an almost straight line with speed \( v \). At time \( t \), charge is directly above \( q \).

\[
r = \sqrt{b^2 + v^2 t^2}
\]

\( e \) feels force:

\[
\mathbf{F} = e\mathbf{v} \times \mathbf{B} = euv \mathbf{B} \sin \theta \mathbf{g} \quad \text{(out of page)}
\]

\[
F_y = \frac{euv \mu_0 g}{q\pi} \left( \frac{1}{b^2 + v^2 t^2} \right)^{3/2}
\]

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Let's compute $\Delta P_y$ caused by $F_y$

$$\Delta P_y = \int_{-\infty}^{\infty} dt \left[ e^{\mu_0 q \cdot b} \right] \frac{2}{(b^2 + t^2)^{3/2}}$$

$$= \left[ e^{\mu_0 q \cdot \infty} \right] \frac{1}{(b^2 + \infty)^{3/2}} \Rightarrow 2/\pi b$$

$$\Delta P_y = \frac{e \mu_0 q}{2\pi b}$$

Particle is deflected in y direction out of page.

There is a change in angular momentum

$$L = r \times p \Rightarrow \Delta L_z = b \Delta P_y$$

$$\Delta L_z = \frac{e \mu_0 q}{2\pi} \text{ independent of } b$$

Now if we think about quantum mechanics, angular momentum is quantized.

$$\Delta L_z = n \hbar \quad n = \text{integer}$$

We get

$$\frac{e \mu_0 q}{2\pi} = n \hbar$$

Dirac quantization condition.
There are amazing consequences

1) given a monopole of charge \( q \)
we see that electric charges is quantized.

2) given \( e \) is known for electron
\[
\frac{e^2}{\hbar c} = \frac{1}{137}
\]
\[
\frac{q}{M_0 e} = \frac{2\pi \hbar}{M_0 e}
\]

Magnetic force between two monopoles is 4700 times as strong as Coulomb force between two electrons.

Monopoles came into theoretical physics in 1970's. Grand Unified Theories (GUTs) postulate unification of strong, weak, and electron forces, and GUTs involve set of equations

\( \rightarrow \) existence of magnetic monopoles
\( \rightarrow \) proton decay
Mass \( = 10^7 \) proton mass

Cosmic Conundrum
Magnetic monopoles are stable
only disappear when a positive mass charge
hits a negatively charged monopole

before \( \Phi \rightarrow \psi \) after \( \phi \) (photons)

Pair annihilation

If monopoles were produced any time in history of
universe, we would still be present.

Based on each universe very hot monopoles
made as universe cooled down, density
decreased by pair annihilation. Once
sufficiently depleted, density is low
in can't find partners so still around

According to traditional Big Bang theory—

Temperature was very high, no inflationary
epoch—monopoles would have
light mass of \( 10^{-15} \) e proton mass

Otherwise their mass would exceed
matter density of universe
This mass well below GUT prediction rules out GUTs.

"Monopole problem" motivation for the inflationary epoch. Expansion would have diluted monopole density to low levels. A Guth 1980

At about that time an experiment appeared to discover magnetic monopole.

Basic idea: Suppose you have a monopole moving along and there is a conducting ring.

What happens when \( q \) goes through ring? There is a \( \frac{d\Phi}{dt} \), and a current. You suddenly see a current as if magic!

Next explain in more detail.
Ring is superconducting.

Perfect conductor: \( \mathbf{J} = 0 \) \( \mathbf{E} \), \( \mathbf{E} = \mathbf{J}/\sigma \)

\( \sigma \rightarrow 0^+ \) \( \mathbf{E} \rightarrow 0 \) for current to be finite.

From \( \nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t \)

\( \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} \)

Superconductor: \( \mathbf{B} = 0 \) inside SC

Further current is on surface only, \( \mathbf{J} \) inside

\( \nabla \times \mathbf{E} + \partial \mathbf{B}/\partial t = -\mu_0 \frac{\mathbf{J}_M}{c} \)

Now the experiment relies on t-dep magnetic flux creating a current

\( \mathbf{J}_M = \frac{\partial \mathbf{\Phi}}{\partial t} \)

\( \mathbf{\Phi} \) fixed

Take an area in the plane of the ring

\( \int d\mathbf{a} \cdot (\nabla \times \mathbf{E}) + \frac{d\mathbf{\Phi}}{dt} = -\mu_0 \int d\mathbf{a} \cdot \mathbf{q} \delta(\mathbf{z} - \mathbf{z}(t)) \sin \psi \)

\( \int \mathbf{E} \cdot d\mathbf{a} + \frac{d\mathbf{\Phi}}{dt} = -\mu_0 \mathbf{q} \delta(\mathbf{z}) \)

\( \mathbf{E} = 0 \) in superconductor
\[ \frac{d\Phi}{dt} = -\mu_0 q \delta(t) \]

Integrate over time from \( t = -\infty \) to \( t = \infty \).

We get \( \Phi(t) = -\mu_0 q \Theta(t) \)

\[ \int_{-\infty}^{\infty} \Theta(t) = t \]

The total flux thru the surface \( S \) is the contribution from monopole and \( \Phi_s \) from the induced supercurrent

\[ \Phi_s = -I(t) \cdot L \]

\( L = \text{self inductance} \)

\[ \Phi_g(t) = \int \mathbf{B}_g \cdot d\mathbf{a} \]

Radius of ring = \( bR \)

\( \mathbf{B}_g = \frac{\mu_0 q}{4\pi} \frac{\mathbf{r}}{r^3(t)} \)

\[ \Phi_g(t) = \int d\mathbf{a} \cdot \frac{\mu_0 q}{4\pi} \frac{\mathbf{r}}{r^3} \]

\[ d\mathbf{a} \cdot \mathbf{r} = 0 \text{ for } \mathbf{r} \text{ constant} \]
Need to treat $t < 0 > t > 0$ as separate cases.

$t < 0$  Flux through ring $> 0$

Need $\mathbf{B} \cdot d\mathbf{a}$

$$d\mathbf{a} = \int_0^{2\pi} r db db$$

$$\mathbf{B} \cdot d\mathbf{a} = |\mathbf{B}| \cos \theta 2\pi db db$$

$$\cos \theta = \frac{1}{|\mathbf{B}|} \frac{1}{R} \frac{1}{(b^2 + u^2)^{3/2}}$$

$$\Phi_g(t, < 0) = \pm \frac{M_o g}{4\pi} \int_0^{R} db db \frac{u |t|}{(b^2 + u^2)^{3/2}}$$

$$= \frac{M_o g}{2} u |t| \left[ - \frac{1}{R} \left( \frac{u}{(b^2 + u^2)^{3/2}} \right)_0^R \right]$$

$$= \frac{M_o g}{2} u |t| \left[ - \frac{1}{R} \left( \frac{u}{(b^2 + u^2)^{3/2}} \right) + \frac{1}{u |t|} \right]$$

$$= \frac{M_o g}{2} \left[ \frac{u |t|}{\sqrt{R^2 + b^2}} + 1 \right]$$

$t > 0$  $\Phi_g(t) = - \frac{M_o g}{4\pi} \int_0^{R} db db \frac{u |t|}{(b^2 + u^2)^{3/2}}$

$$= \frac{M_o g u |t|}{2} \left[ \frac{1}{(b^2 + u^2)^{3/2}} - \frac{1}{u |t|} \right] = \frac{M_o g u |t|}{2} \left[ \frac{u |t|}{(b^2 + u^2)^{3/2}} - 1 \right]$$
Combining these gives

\[ \Phi_g(t) = \frac{M_0 g}{2} \left[ 1 - 2 \theta(t) + \frac{u \cdot t}{(R + u^2 t^2)^2} \right] \]

all times \( t \)

\[ \Phi_s(t) = -I(t) L \quad \text{so} \quad \Phi = \Phi_s + \Phi_g \]

\[ -\frac{M_0 g}{2} \theta(t) = -I(t) L + \frac{M_0 g}{2} \left[ 1 - 2 \theta(t) + \frac{u \cdot t}{(R + u^2 t^2)^2} \right] \]

\[ I(t) = \frac{1}{L} \left[ \frac{M_0 g}{2} \right] \left[ 1 + \frac{u \cdot t}{(R + u^2 t^2)^{1/2}} \right] \]

Slope at \( t=0 \):

\[ \frac{2L}{M_0 g} \]

Prediction of monopole

Sudden persistent current
This event occurred after 157 days

Peter kept running

Expected 1.5 events/ year

Never saw another

Search for monopoles carried out

Galactic magnetic fields

Cosmic rays

Moon rocks

Antarctic

Don't exist in universe today

Can make in collider experiments

LHC: \( p + p \rightarrow p + p + m + \bar{m} \)

Works for electric charges

Nothing seen

More details, refs in Physics Today 69 (10) to (2016)

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