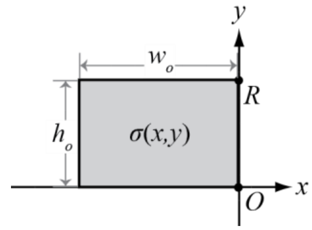
Name ______________________________________ **Student ID** _______________ **Score**________
last *first*

V. [22 points total] Tutorial question.

A thin rectangular sheet is located on the *xy*-plane (shown at right) with non-uniform surface charge density $\sigma(x, y) = a_o x^2$ where a_o is a positive constant. The dimensions of the sheet are width w_o along the *x*-axis and height h_o along the *y*-axis, with the origin located at point *O* at the bottom right corner.



a. [4 pts] Is the net charge of this charge distribution *positive*, *negative*, or *zero?* Explain your reasoning **in words**.

Positive. The charge distribution is always positive at all locations (except the y-axis where it is zero), so the net charge is positive.

b. Recall that the point-charge form of the dipole moment is given by $\vec{p} = \sum Q_i \vec{r}_i$.

i. [5 pts] Determine an integral expression for the ***x*-component** of the dipole moment for this charge distribution, in terms of given variables. You do not need to evaluate the integral, but it should be explicit enough for a computer to obtain a value.

Thus full integral should look like $p_x = \int_{-w_o}^{0} \int_0^{h_o} \{(a_o x^2 dxdy)x\}$ *or* $\int_{-w_o}^{0} \int_0^{h_o} a_o x^3 dxdy$

(The integral $\int_0^{h_o} dy$ *can be replaced by* h_o*, as the density is constant in y.)*

N.b., the question does not ask you to show your work, but here is the derivation anyways:

In integral form, the dipole moment looks something like $\vec{p} = \int (dq)\vec{r}$*.*

To find the x-component, we can make the substation: $\vec{r} \to x$ *(as* $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$*).*

We can express dq in terms of density by using the relationship: $dq = \sigma(x, y)dA$

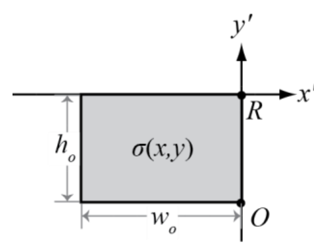
The infinitesimal area is two-dimensional in Cartesian, so $dA = dxdy$

Finally we should show the appropriate bounds based on the location of the charge: x from $-w_o$ *to 0 and y from 0 to* h_o*.*

ii. [3 pts] From your expression above, identify which factor(s) best represent Q_i from $\vec{p} = \sum Q_i \vec{r}_i$.

Q_i *represents each individual point charge, so in an integral we are looking for something that resembles an infinitesimal point charge. That would be* (σdA) *or* $(a_o x^2 dxdy)$ *in this case.*

c. Consider the act of choosing a new origin located at point *R* at the top right corner of the sheet without changing the charge distribution on the sheet, with $\sigma'(x, y) = a_o x^2$.



i. [5 pts] Is the *x'*-component of the new dipole moment *greater than, less than,* or *equal to* the *x*-component of the old dipole moment? Explain your reasoning.

Equal to. Using $\int (dq)\vec{r}$*, the charge distribution is unchanged, so dq is unchanged. With this shift of origin, x = x', so the x-component of* $\vec{r}$ *doesn't change either. Thus the x-component of the dipole moment remains the same. In words, the charge distribution's x-position relative to the origin never changes, so the dipole moment should not change.*

ii. [5 pts] Is the *y'*-component of the new dipole moment *greater than, less than,* or *equal to* the *y*-component of the old dipole moment? Explain your reasoning.

Less than. Using $\int (dq)\vec{r}$*, the charge distribution is unchanged (and always non-negative), but now the y-component of* $\vec{r}$ *has switched from always being positive to always being negative. Thus, the y-component of the dipole moment has decreased from being some positive number to being some negative number.*