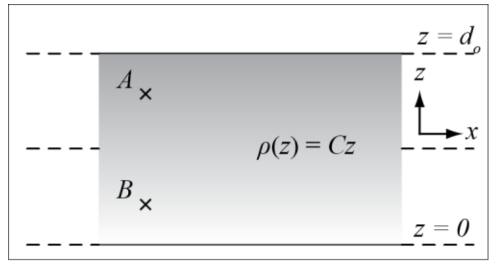
Name ______________________________________ **Student ID** _______________ **Score**________
last *first*

IV. [25 points total] Tutorial question.

A charged slab with variable charge density $\rho(z) = Cz$ is shown at right, where C is a positive constant. The slab is infinite in the xy-plane, and extends from $z = 0$ to $z = d_o$. Assume that the electric field is zero at $z < 0$.



Non-uniform charged slab

A. [6 pts] What type(s) of symmetry does this slab have (*i.e.* continuous rotational symmetry about the x-axis)? Explain how you know.

There is continuous translational symmetry along the x- and y- axes. If the slab were moved along either of these axes, the apparent charge distribution would look identical.

Technically, there is also continuous rotational symmetry around any z-axis, coupled by radial invariance in the xy-plane.

B. [6 pts] Describe a Gaussian surface for which the flux integral will be easy to simplify. You may use sketches or diagrams in your answer. Explain why you chose this surface.

Any vertical right prism would work: such as a cylinder, rectangular prism, "pill box", etc.

In order to simplify a flux integral, we need faces at good angles (0, 90, or 180) to the electric field. In order to take the magnitude of E out of the integral, we need to make the non-zero surfaces along constant E.

C. [8 pts] How does electric field within the slab depend on z (*i.e.* constant with z, proportional to $1/z$, z^2, etc.)? Explain your reasoning.

Proportional to z^2.

Using a Gaussian surface with the bottom below z = 0, we can consider $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_o}$ for various heights. The charge enclosed will be proportional to the xy-area of the shape and z^2 because the charge integral involves ρdV, which includes zdz.

One can also use the differential form $\nabla \cdot E = \frac{\rho}{\varepsilon_o}$ and ignore the x and y derivatives due to symmetry. This also leads to an integral including zdz.

How does the electric field within the slab depend on x (*i.e.* constant with x, proportional to $1/x$, x^2, etc.)? Explain your reasoning.

Constant with x.

The charge distribution has continuous translational symmetry along the x-direction, which tells us that the field should as well.

D. [5 pts] Is the divergence of the electric field at point *A greater than, less than,* or *equal to* the divergence of the electric field at point *B?* Explain your reasoning.

Greater. By Gauss' Law in differential form given by $\nabla \cdot E = \frac{\rho}{\varepsilon_o}$, we can compare the charge density at the two locations. Looking at $\rho(z) = Cz$ shows us that point A has more charge density, and thus a higher divergence of electric field.