# PHYSICS 321
# CLASSICAL ELECTRODYNAMICS

25 Oct. 2019 Problem Set 4 These problems are due on Thursday , Oct 31

1. *Parallel Plates* Consider a very large conducting plate at potential $V_0$ suspended a distance $d$ above a very large grounded plane. Find the potential between the plates. The plates are large enough so that they may be considered to be infinite. This means that one can neglect fringing fields.

2. *Forces and pressure* Suppose that the forces of mutual attraction cause the plates of a parallel-plate capacitor to move together by an infinitesimal distance $d$.

(a) Use the idea that the pressure on the plates, $P = \frac{\epsilon_0}{2}E^2$ to express the work done by electrostatic forces, in terms of the field $E$ and $A$ the area of the plates.

(b) Use the idea that the energy per unit volume is $\frac{\epsilon_0}{2}E^2$ to determine the energy loss by the field in this process. Should the answers to parts (a) & (b) be the same?

3. *Charge above a conducting plane* A positive point charge $q$ ia fixed at the point $(0, 0, z_0)$ above a grounded conducting plane at $z = 0$.

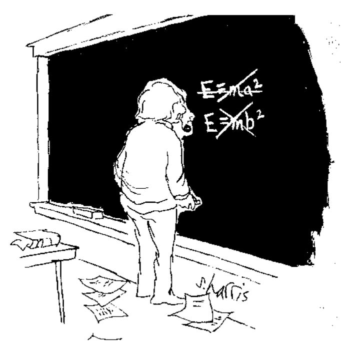
(a) What is the total charge induced on the plane?

(b) Determine the radius of the circle centered at the origin in the $xy$ plane which enclose half of the total charge induced on the plane.

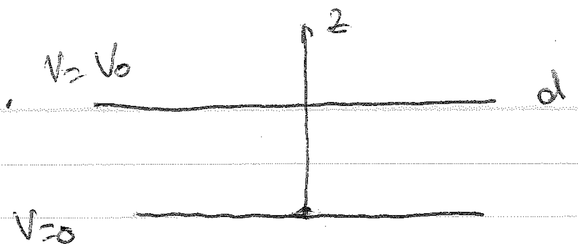
(c) Determine the force on the charge.

(d) Determine the energy in the electric field. How does this compare with the energy of two charges $+q$ and $-q$ at a separation of $2z_0$

4. *Boundary value problem* Two infinitely long grounded conducting plates at $y = 0$ and $y = L$ are connected at $x = \pm L/2$ by metal strips maintained at a potential $V_0 y/L$. Find the potential inside the resulting rectangular pipe.



1.




This is a one-dimensional problem

$$\frac{d^2V}{dz^2} = 0$$

$$V = a + bz$$

$$V(z=0) = 0 \Rightarrow a = 0$$

$$V(z=d) = V_0 = bd$$

$$b = V_0/d$$

$$V = V_0 z/d$$

2a $P = \frac{F}{A} = \frac{\epsilon_0 E^2}{2}$, $F = \frac{\epsilon_0 E^2}{2} A$

This is the force on the area of the plate

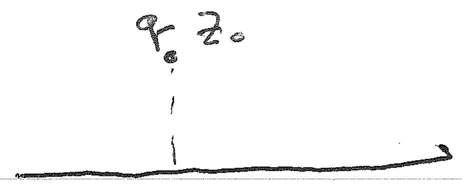
$W = Fd = \frac{\epsilon_0 E^2}{2} Ad$

b. The energy loss is the energy per unit volume times the volume lost. The energy per unit volume is $\epsilon_0 E^2/2$, the volume lost is $A\,d$.

Energy loss $= \frac{\epsilon_0 E^2}{2} Ad =$ Work done

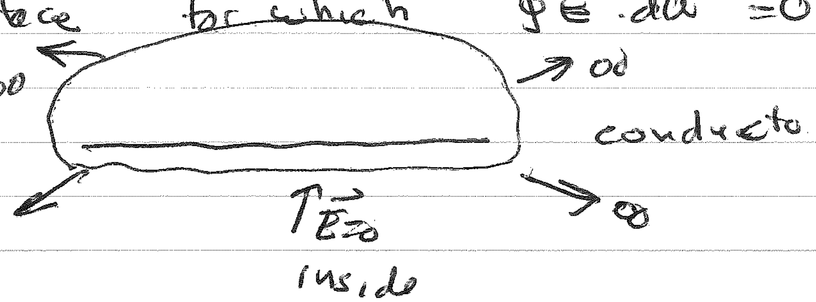
These answers should be the same

3




a) The charge induced on the plane is $-q$

This is because we can construct a gaussian surface for which $\oint \vec{E} \cdot d\vec{a} = 0$




b) The surface charge density induced on the plane is given by

$$\sigma_{(s)} = \frac{-q}{4\pi} \frac{2D}{(D^2+s^2)^{3/2}}$$

with $D = z_0$

$s^2 = x^2+y^2$

(shown in class and text book)

The charge inside a circle of radius $R$ is given by

$$Q(R) = \int_0^R 2\pi s\,ds\,\sigma(s)$$

$$= -\frac{q}{4\pi} 2\pi \int_0^R \frac{s\,ds\,2z_0}{(s^2+z_0^2)^{3/2}}$$

$$= -\frac{q}{4} 2z_0 (-)(s^2+z_0^2)^{-1/2} \Big|_0^R$$

$$= \frac{q z_0}{} \left[ \frac{1}{z_0} - \frac{1}{(R^2+z_0^2)^{1/2}} \right]$$

$$= -q \left( 1 - \frac{z_0}{(R^2+z_0^2)^{1/2}} \right)$$

we want $Q(R) = -\frac{q}{2} = -q\left(1 - \frac{z_0}{(R^2+z_0^2)^{1/2}}\right)$

so

$$\frac{z_0}{(R^2+z_0^2)^{1/2}} = \frac{1}{2} \,; \qquad \boxed{z_0 = \frac{R}{\sqrt{3}}}$$

c) The force on the charge:

The charge $q$ is attracted to the plane because of the negative induced charge. The potential in the vicinity of $q$ is the same as the one with $+q$ and $-q$ and no conductor, so is the $\vec{E}$

so $\vec{F} = -\frac{q^2}{4\pi\epsilon_0} \frac{1}{(2d)^2} \hat{z}$

d) The energy in the electric field is given by

$$W = -\frac{q^2}{16\pi\epsilon_0 d^2}$$

See page 127 of your text

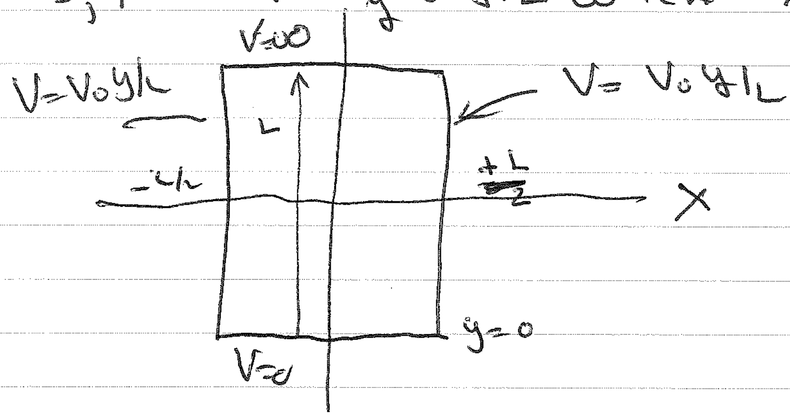
4 We have $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) V = 0$ See Fig 3.20 in text

and boundary conditions

1. $V = 0$ when $y = 0$

2 $V = 0$ when $y = L$

3, 4 $V = V_0 y/L$ when $x = \pm L/2$




$$V(x, y) = \sin \frac{n\pi y}{L}\left[A \cosh \frac{n\pi x 2}{L} + B \sinh \frac{n\pi x 2}{L}\right]$$

but there is a symmetry about $x = 0$
so choose the cosh sol'n

So $V(x, y) = \sum_{n=1} A_n \sin \frac{n\pi y}{L} \cosh \frac{2n\pi x}{L}$

when $x = \pm L/2$ $V = V_0 y/L$ so we have

$$V_0 y/L = \sum_{n=1} A_n \sin \frac{n\pi y}{L} \cosh n\pi$$

This is a Fourier sine series

$$A_m \cosh m\pi = \frac{2}{L}\int_0^L V_0 \frac{y}{L} \sin \frac{m\pi y}{L}\, dy$$

$$= 2V_0 \int_0^1 u \sin m\pi u \, du$$

$$= \frac{2V_0 (-1)^{m+1}}{m\pi}$$

So $$V(x,y) = V_0 \sum_{m=1} \left( \frac{2(-1)^{m+1}}{m\pi} \frac{1}{\cosh m\pi} \sin \frac{m\pi y}{L} \cosh \frac{2m\pi x}{L} \right)$$