PHYSICS 321- Fall 2019
CLASSICAL ELECTRODYNAMICS

Oct 3   Problem Set 2   These problems are due 10 am Thursday, Oct 10 in the boxes outside the north entrance of PAB (same as tutorial boxes). Please label your homework with name, section, and HW#.

1. *Comparing gravitational and electrostatic forces*

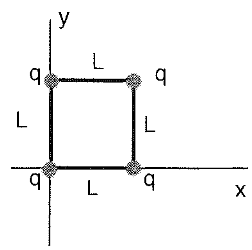
(a) Calculate the gravitational force on a proton at the surface of the Sun.

(b) How many electrons placed at the center of the Sun yield the same electrostatic force on the proton?

2. *Force between point charge and line charge*
A uniform linear distribution (of infinite extent) of charge of λ Coulombs/meter is situated at a distance r from a point charge Q of opposite sign.

(a) Calculate the force of attraction. (b) Show that the force is the same as if the linear distribution were replaced by a single charge $Q' = 2\lambda r$ situated at the foot of the perpendicular drawn from Q.

3. Four point charges, each of charge q and mass m are located at the four corners of a sqaure of side L.



(a) Determine the magnitude of the force on one of the charges.(Hint: does it matter which one?)

(b) Use the force in part (a) to find the velocity of one of the charges a long time after the four charges are released from rest in the original confifuration.

4. This problem concerns the electric field a distance D *above the center* of a flat circular plate of radius R. The surface charge density is given as σ, so the total charge on the plate is $Q = \pi\sigma R^2$. Define the vertical direction to be the z-axis.

(a) Determine the x and y components of E.

(b) Find an expression for $\mathbf{E}$ as a two-dimensional integral.

(c) Carry out the integral to determine $\mathbf{E}(D)$.

(d) Check that your expression has the correct limit for $D \to \infty$.

(e) Check that your expression has the limit for correct limit for $D \to 0$.

1. Assuming spherical symmetry of

a) solar mass distribution the force is given by

$$\vec{F} = - \frac{G M_{\odot} m_p}{R_{\odot}^2} \hat{r}$$

$M_{\odot}$ = solar mass = 2×10^{30} kg
$R_{\odot}$ = solar radius = 7×10^{8} m

m_p = proton mass = 1.67×10^{-27} kg

$G = 6.7 \times 10^{-11}$ N m²/kg Use these to get

$|\vec{F}| = 4.6 \times 10^{-25}$ Newton

(b) Place N electrons at the center of the sun (of charge (-e))

$$|F| = \frac{N e^2}{4\pi\epsilon_0} \frac{1}{R_{\odot}^2}$$

$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N m^2}$, $e = 1.6 \times 10^{-19}$ C

$$N \frac{(1.6 \times 10^{-19})^2}{4\pi (8.85 \times 10^{-12})} \frac{1}{(7 \times 10^{8})^2} = 4.6 \times 10^{-25} \text{ Newtons}$$

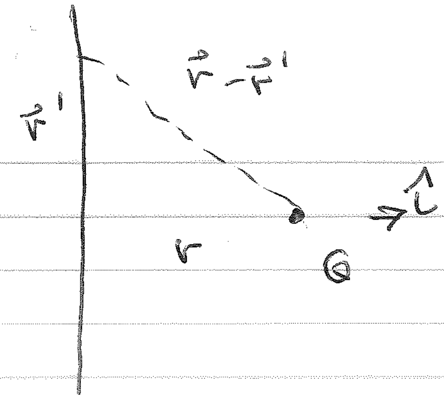
$$\boxed{N = 9.7 \times 10^{20}}$$

Aside the mass of an electron is 9.1×10^{-31} kg

so the electric force of $\approx 10^{-9}$ g is equivalent to the grav force of the mass of the sun

2

(a)



$$\vec{F}_{on\,Q} = \frac{Q}{4\pi\epsilon_0}\int_{-\infty}^{\infty}\frac{\lambda\, dz'\,(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}$$

$$\vec{r} = r\,\hat{\imath} \qquad \vec{r}\,' = z\,\hat{k}$$

$$\vec{r}-\vec{r}\,' = r\,\hat{\imath} - z'\,\hat{k}$$

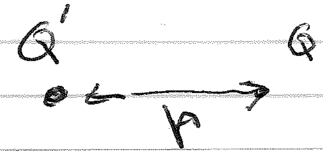
$$|\vec{r}-\vec{r}\,'| = \sqrt{r^2+z'^2}$$

$$\vec{F} = \frac{Q}{4\pi\epsilon_0}\,\lambda\int_{-\infty}^{\infty}\frac{dz'\,[r\,\hat{\imath} - z'\,\hat{k}]}{(r^2+z'^2)^{3/2}}$$

$$\vec{F} = -\frac{|Q||\lambda|}{4\pi\epsilon_0}\,\hat{\imath}\; r\int_{-\infty}^{\infty}\frac{dz'}{(r^2+z'^2)^{3/2}}$$

$$= -\frac{|Q||\lambda|}{4\pi\epsilon_0}\,\hat{\imath}\; r\left[\frac{2}{r^2}\right] = -\frac{|Q||\lambda|\,\hat{\imath}}{2\pi\epsilon_0}\,\frac{1}{r}$$

(b) Equivent charge (EC)



rewrite $\vec{F}$ as $-\dfrac{|Q|\,|\lambda|}{2\pi\epsilon_0}\,\dfrac{1}{r} \times \dfrac{2r}{2r}\,\hat{\imath}$

$$= \frac{-|Q|\;2|\lambda|\,r}{4\pi\epsilon_0\, r^2}\;,\; |Q'| = 2\lambda r$$

3. a) The magnitude of the force on any of the charges is the same. This is because of the symmetry of the problem.

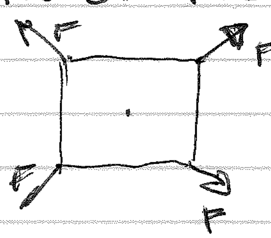
Consider the charge on the upper-right-hand corner due to the other three charges is given by

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{L^2}\left[\hat{i} + \hat{j} + \frac{\hat{i}+\hat{j}}{\sqrt{2}\,2}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q^2}{L^2}\left[\sqrt{2} + \frac{1}{2}\right]\frac{\hat{i}+\hat{j}}{\sqrt{2}}$$

$$|\vec{F}| = \frac{1}{4\pi\epsilon_0}\frac{q^2}{L^2}\left[\sqrt{2} + \frac{1}{2}\right]$$

(b) The charges all feel the same magnitude of force in a direction diagonally away from the center of the square.



After release

The configuration remains a square, but of increasing length L'. To determine the velocity, start with the acceleration a

$$a = \frac{F}{m} = \frac{dv}{dt} = \frac{dv}{dL'}\frac{dL'}{dt} \quad (1)$$

. The center of the square remains fixed and the distance r from the center of the square is related to L' by $r = L'/\sqrt{2}$

Then $\frac{dL'}{dt} = \sqrt{2}\,\frac{dr}{dt} = \sqrt{2}\,v$

$$a = \sqrt{2}\,v\,\frac{dv}{dL'} \quad (2)$$

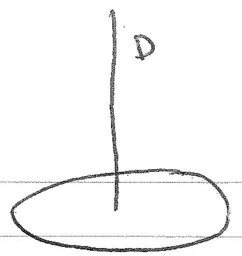
, (1) & (2) give

$$v\frac{dv}{dL'} = \frac{1}{\sqrt{2}\,m}\frac{q^2}{4\pi\epsilon_0}\left(\frac{1+2\sqrt{2}}{2}\right)\frac{1}{L'^2} = \frac{1}{2}\frac{dv^2}{dL'}$$

$$\int_0^{v_{final}} dv^2 = \frac{1}{\sqrt{2}\,m}\frac{q^2}{4\pi\epsilon_0}(1+2\sqrt{2})\int_L^\infty \frac{dL'}{L'^2}$$

$$\boxed{v_{final}^2 = \frac{1}{\sqrt{2}\,m}\frac{q^2(1+2\sqrt{2})}{4\pi\epsilon_0}\frac{1}{L}}$$

4



$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{da'\,(\vec{r}-\vec{r}\,')\,\sigma}{|\vec{r}-\vec{r}\,'|^3}$$

here $\vec{r} = \hat{k}\,D$

$\vec{r}\,' = s(\cos\phi'\,\hat{\imath} + \sin\phi'\,\hat{\jmath})$ cylindrical coordinates

$da' = s\,ds\,d\phi'$

$|\vec{r}-\vec{r}\,'| = D^2 + s^2$ so

$$\vec{E}(\vec{r}) = \frac{\sigma}{4\pi}\int_0^R s\,ds\int_0^{2\pi} d\phi'\,\frac{[D\hat{k} - s(\cos\phi'\,\hat{\imath} + \sin\phi'\,\hat{\jmath})]}{(D^2+s^2)^{3/2}}$$

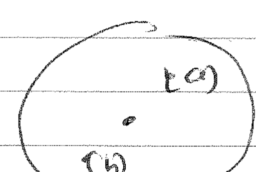
a) $E_x(\vec{r}) = 0$, $E_y = 0$, because

$$\int_0^{2\pi} d\phi'\,\cos\phi' = 0, \quad \int_0^{2\pi} d\phi'\,\sin\phi' = 0$$

This can be seen also from symmetry

the x, and y components ~~cancel~~ coming from

the opposite points a, b cancel



b) $$\vec{E}(\vec{r}) = \frac{\sigma D\hat{k}}{4\pi\varepsilon_0}\int_0^R s\,ds\int_0^{2\pi} d\phi\,\frac{1}{(D^2+s^2)^{3/2}}$$

c) $$\vec{E}(\vec{r}) = \frac{\sigma D\,\hat{k}}{2\varepsilon_0}\int_0^R \frac{s\,ds}{(D^2+s^2)^{3/2}} = \frac{\sigma D}{2\varepsilon_0}\left[\frac{1}{D} - \frac{1}{(R^2+D^2)^{-1/2}}\right]$$

$$\vec{E}(\vec{r}) = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{D}{(R^2+D^2)^{1/2}}\right)$$

(d)
Take $D \to \infty$ $\quad \frac{1}{(R^2+D^2)^{1/2}} = \frac{1}{D}\frac{1}{\left(1+\frac{R^2}{D^2}\right)^{1/2}} \approx \frac{1}{D}\left(1 - \frac{R^2}{2D^2}\right)$

Then $\lim_{D\to\infty} \vec{E}(\vec{r}) = \frac{\sigma D}{2\epsilon_0}\left[\frac{1}{D}\right]\left[1 - \left(1 - \frac{R^2}{2D^2}\right)\right]\hat{k}$

$$= \frac{\sigma \pi R^2}{2\pi\epsilon_0 \, 2D^2}\hat{k} = \frac{Q}{4\pi\epsilon_0}\frac{\hat{k}}{D^2}$$

This is what is expected for a point charge

(e) $D \to 0$ $\quad \frac{1}{(R^2+D^2)^{1/2}} = \frac{1}{R}\left(1 - \frac{1}{2}\frac{D^2}{R^2}\right)$

$\vec{E}(\vec{r}) = \frac{\sigma}{2\epsilon_0}$ as expected. This is the expected result. This is like the field above an infinite plane