PHYSICS 321
CLASSICAL ELECTRODYNAMICS

7 Nov. 2019   Problem Set 6   These problems are due on Thursday , Nov. 14

1. *Uniform surface charge* Consider a circular disk of radius R and constant surface charge density σ. See Figure 2.34c on page 87. Set up a coordinate system in which there is azimuthal symmetry.

(a) Determine the potential $V(r,\theta)$ for positions with distances r from the center of the disk such that $r > R$.

(b) Determine the potential $V(r,\theta)$ for positions, not on the disk, with distances r from the center of the disk such that $r < R$.

2. *Cylindrical symmetry* A very, very long charged line (along the z-axis) with constant linear charge density λ is the simplest cylindrically symmetric system. The electric field can be obtained from the Gauss law and the potential V can be obtained by integration. For an infinitely long line charge $V(s) = -\frac{\lambda}{2\pi\epsilon_0}\ln s/s_0$, where the value of s_0 is arbitrary. This potential has the unrealistic property that it is infinite for $s = 0$ and for $s = \infty$. This is because the line is of infinite length and of zero width. However, the work required to take a unit charge from s_1 to s_2 is finite: $V(s_2) - V(s_1) = (\lambda/2\pi\epsilon_0)\ln s_1/s_2$.

(a) Check that the stated potential $V(s)$ yields the correct electric field, $\mathbf{E}$.

Here we'll find the same result for $V(s)$ by using the solution to Laplace's equation. There are two symmetries-translational invariance in z and rotational invariance in ϕ the azimuthal angle. Therefore the potential can depend only on s, the perpendicular distance to the line. In cylindrical coordinates Laplace's equation becomes $\frac{1}{ds}\frac{d}{s}\left(s\frac{dV}{ds}\right) = 0$ which has the general solution $V(s) = A\ln(s/s_0) + B$, where A and B are constants and s_0 is an arbitrary length.

(b) Determine A and B for the infinitely long line charge of constant λ.

Next consider a conducting cylinder of radius a which carries a constant line charge density λ spread uniformly on its surface

(c) Determine the potential for $s \geq a$.

(d) Consider two concentric conducting cylinders. The potential of the inner cylinder, of radius a, is set at V_0; the outer cylinder of radius b is grounded ($V = 0$). Determine the potential $V(s)$ and the electric field for $a \leq s \leq b$.

(e) Determine the capacitance per unit length.

3. *Force between two dipoles* Two dipoles, $\vec{p}_1$ and $\vec{p}_2$ are separated by a displacement $\vec{r}$. Determine the force between these objects. These are "pure" dipoles in the sense of Griffiths: their only relevant property is to have a dipole moment. Is your result consistent with Newton's third law?