PHYSICS 321
CLASSICAL ELECTRODYNAMICS

30 Oct. 2019 Problem Set 5 These problems are due on Thursday , Nov. 7

1. *Potential on a sphere* The potential on the surface of a sphere of radius R is given by $V(\theta) = V_0 \cos 4\theta$. There is no charge inside or outside the sphere.

(a) Determine the potential $V(r, \theta)$ for positions inside and outside the sphere.

(b) Determine the surface charge density $\sigma(\theta)$ on the sphere.

2. *Image charge and capacitance* Consider a long wire of radius R with charge per unit length λ. The wire is suspended a distance $d \gg R$ above a very large grounded plane that can be approximated as infinitely large. Determine the capacitance per unit length.

3. *Multipole expansion* Three charges are arranged in a linear array. A charge $-2q$ is placed at the origin, and two charges, each of $+q$ are placed at $(0, 0, L)$ and $(0, 0, -L)$. Consider the multipole expansion

$$V(r, \theta) = \frac{1}{4\pi\epsilon_0} \sum_{l=0}^{\infty} \frac{B_l}{r^{l+1}} P_l(\cos\theta)$$

$$B_l \equiv \int d^3r' \rho(\mathbf{r}') r'^l P_l(\cos\theta') = \sum_n q_n r_n^l P_l(\cos\theta_n),$$

with the second expression for B_l applying for the case of point charges.

(a) Determine the multipole coefficients that are non-vanishing.

(b) Compute the non-vanishing B_l for the two lowest values of l.

4. *Boundary value problem* The potential on the surface of a sphere of radius R is given by:

$$V(R, \theta) = V_0,\ 0^\circ \leq \theta \leq 90^\circ; \quad V(R, \theta) = -V_0,\ 90^\circ \leq \theta \leq 180^\circ$$

(a) Find the potential $V(r, \theta)$ outside and inside the sphere. You may express your answer in terms of a well-defined integral of a Legendre polynomial, $I_n = \int_0^1 dx P_n(x)$.

(b) Find the surface charge density on the sphere.

(c) Find the electric dipole and octupole moments of the sphere by (i) identifying them from the Legendre polynomial expansion of the potential. (ii) integrating over the surface charge density.