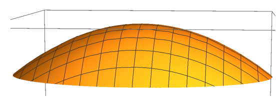
PHYSICS 321- Fall 2019
CLASSICAL ELECTRODYNAMICS

Sep't 26 Problem Set 1 These problems are due 10 am Thursday, Oct 3 in the boxes outside the north entrance of PAB (same as tutorial boxes). Please label your homework with name, section, and HW#.

1. Consider a surface $\mathcal{S}$ bounded at the top by a spherical cap, $x^2 + y^2 + z^2 = 3a^2$, with $z > a$, together with its base: $x^2 + y^2 \leq 2a^2$, $z = a$. See the figure. The vector $\vec{v} = \mathbf{v} = xz\mathbf{i} - yz\mathbf{j} + y^2\mathbf{k} = xz\hat{\mathbf{x}} - yz\hat{\mathbf{y}} + y^2\hat{\mathbf{z}}$. Compute the flux of $\mathbf{v}$ through the surface $\mathcal{S}$ directly (a) and (b) using the divergence theorem (1.56). The divergence in spherical coordinates is given on the inside front cover.



2. The function ϕ is given by $\phi = \frac{1}{r}$, and $\mathbf{v} = -\vec{\nabla}\phi$. Let $\mathcal{P}$ be a curve in the xy plane ($z = 0$) such that $x^2 + y^2 = a^2$. Compute $\oint_{\mathcal{P}} \mathbf{v} \cdot d\mathbf{l}$ directly and using Stokes theorem with the surface $\mathcal{S}$ defined by the hemisphere of radius of radius a, bounded by $\mathcal{P}$, with $z \geq 0$.

3. (a) Evaluate $\int_{\mathcal{V}} \mathbf{r} \cdot (\mathbf{d} - \mathbf{r})\delta(\mathbf{r} - \mathbf{e})d\tau$, where $\mathbf{d} = (0, 1, 2)$, $\mathbf{e} = (2, 1, 0)$ and $\mathcal{V}$ is a sphere of radius 2 centered at (1,1,1).

(b) Evaluate the integral $J = \int_{\mathcal{V}} e^{-r/a}(1 + \frac{r}{a}) \left(\boldsymbol{\nabla} \cdot \frac{\hat{\mathbf{r}}}{r^2}\right) d\tau$, where $\mathcal{V}$ is a sphere of radius R, centered at the origin. Use two different methods as in Ex. 1.16.

4 (a) Consider a river in which the water velocity v is proportional to the distance from the bottom, and the flow is in the z-direction according to $v_x = 0, v_y = 0, v_z = cx$, where c is a given constant. Find $\boldsymbol{\nabla} \times \mathbf{v}$.

(b) Sketch the field lines (pages 67,68) for the vector function $\mathbf{F}(x, y, z) = \mathbf{i}x + \mathbf{j}y$. Explain why $\mathbf{F}$ does or does not have a divergence and curl.

(c) Sketch the field lines for the vector function$\mathbf{G}(x, y, z) = \frac{-\mathbf{i}y + \mathbf{j}x}{\sqrt{x^2+y^2}}$. Explain why $\mathbf{G}$ does or does not have a divergence and curl.

5. *Maxwell's Equations* (a) Show that the general form of Maxwell's Equations are consistent with the equation $\boldsymbol{\nabla} \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0$.

(b) Use (a) to show that the time rate of change of the charge inside a given fixed volume is equal to the current flowing through the surface. Thus the equation in (a) is a statement of current and charge conservation