if point charges

$$B_\ell = \sum_i (r_i')^\ell q_i P_\ell(\cos\theta_i)$$

$$B_\ell = \int d^3r' \, \rho(\vec{r}') \, r'^\ell \, P_\ell(\cos\theta')$$

multipole coefficients ~~with~~ (azimuthal symmetry)

general meaning

$\ell = 0$ $\quad B_0 = \int d^3r' \, \rho(\vec{r}') = Q$ = charge

$P_0(\cos\theta') = 0$

monopole term

$\ell = 1$ $\quad B_1 = \int d^3r' \, \rho(\vec{r}') \, r' \cos\theta'$

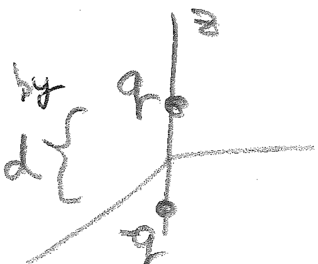
$P_1(\cos\theta') = 1$

$$= \int d^3r' \, \rho(\vec{r}') \, z'$$

$$= p_z$$

compondent of dipole moment

Example: ρ is given by point charges



$\cancel{B_1 =} \int d^3r' \, \rho(\vec{r}') \, F(\vec{r}') \Leftrightarrow \sum_i q_i \, F(\vec{r}_i)$

$B_0 = 0$ ask

$$B_1 = q\frac{d}{2} + (-q)\left(-\frac{d}{2}\right) = qd = p_z$$

$B_2 = ?$

octopole

$$B_3 = q\left(\frac{d}{2}\right)^3 + (-q)\left(-\frac{d}{2}\right)^3 = \frac{qd^3}{4}$$

The given charge distribution has ∞ many multipole coefficients B_1, B_3, B_5

$$V(\vec{r}) = \sum_{n=odd} \frac{B_n}{r^{n+1}} P_n(\cos\theta)$$

In general for neutral systems the dipole is the most important

$$\lim_{r \to \infty} V \sim \frac{1}{|\vec{r}|^2}$$ others fall off faster.

There is a term used by Griffith, puredipole

means all other moments completely ignorable

$l=1$ term is called dipole

$$V_{dipole} = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\int d^3r'\,\rho(\vec r\,')\,r'\,\hat r\cdot\hat r\,'$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\hat r}{r^2}\cdot\int d^3r'\,\rho(\vec r\,')\,\vec r\,'$$

$$\vec p = \text{dipole moment} = \int d^3r'\,\rho(\vec r\,')\,\vec r\,'$$

$$\boxed{V_{dipole} = \frac{1}{4\pi\varepsilon_0}\frac{\hat r\cdot\vec p}{r^2}} = \frac{\vec p\cdot\vec r}{4\pi\varepsilon_0 r^3}$$

Skip

$r>0$

$$\vec E_{dipole} = -\vec\nabla V_{dip}$$

$$= -\frac{\vec p}{4\pi\varepsilon_0 r^3} - \vec p\cdot\vec r\,\vec\nabla\frac{1}{r^3}$$

$$\vec\nabla\frac{1}{r^3} = \hat r\frac{\partial}{\partial r}\frac{1}{r^3} = \frac{-3\hat r}{r^4}$$

$$\vec\nabla\,\vec p\cdot\vec r = \hat e_n\frac{\partial}{\partial x_n}p_m x_m$$

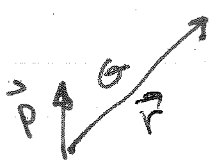
$$= \hat e_m p_m$$

$$\vec\nabla f(r) = \hat r\frac{d}{dr}f(r)$$

$$\vec E_{dipole} = -\frac{\vec p}{4\pi\varepsilon_0 r^3} + \frac{3\vec p\cdot\hat r\,\hat r}{r^4\,4\pi\varepsilon_0}$$

$$\vec E_{dipole} = \frac{1}{4\pi\varepsilon_0}\frac{(3\,\vec p\cdot\hat r\,\hat r - \vec p)}{r^3}$$

Fixed r as function of θ



when is $|\vec E|$ biggest?

$\theta=0$	$\lvert 2\vec p\rvert$
$\theta=90^\circ$	$\lvert -\vec p\rvert$
$\theta=180$	$= \lvert -2\vec p\rvert$

Electric fields in matter

We've talked about conductors.
In those some electrons are free
to move. $\vec{E}$ inside $= 0$ in a static
situation

There is another kind of matter - the

opposite - insulator or dielectric

All electrons are bound in atoms

Conductors (copper) o dielectrics (glass) is simplification

Much matter in between semi conductors

- ~~H~~ very important in electronics

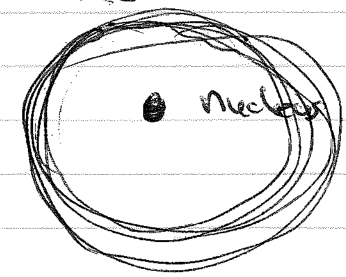
But our job here is to understand

dielectrics

Suppose apply an $\vec{E}$ field to dielectric

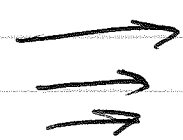
charges move a minute distance

$\vec{E} = 0$



spherical charge dist $\vec{p} = 0$
$Q = 0$

$\vec{E} \neq 0$



electrons go left
nucleus goes right
a smaller distance
nucleus is much heavier

A dielectric in which this occurs

is <u>polarized</u>

Some molecules possess a permanent

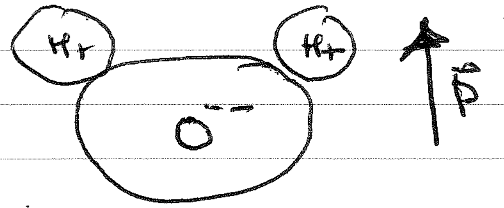
dipole moment — they are <u>polar</u>

2 basic mechanisms

1. Under $\vec{E}$ center of mass of electronic distribution moves slightly

electronic polarization

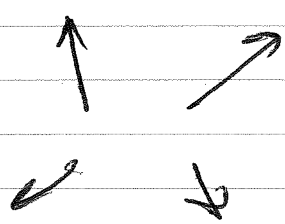
2 Polar molecules H_2O



$\vec{p} \neq 0$

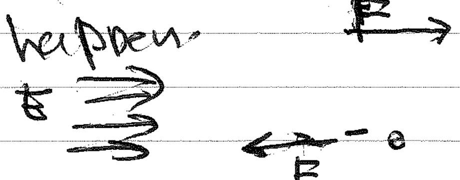
but without $\vec{E}$ applied

have dipoles in random direction



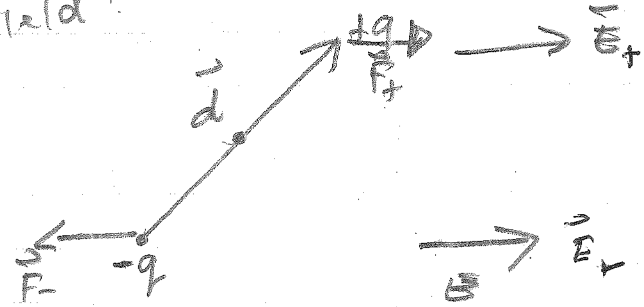
$\sum \vec{p} = 0$

Expose dipole to $\vec{E}$ field what happen.



dipoles line up with $\vec{E}$

What happends when $\vec{p}$ placed in $\vec{E}$ field? $\vec{E}$ not so strong as to dest. dipole



$\underline{\vec{E} = \text{constant}}$, $\vec{E}_+ = \vec{E}_-$

If $\vec{E}$ varies there will be a force $\vec{F} = (\vec{p} \cdot \vec{\nabla})\vec{E}$

Total Force = 0 $\quad \vec{F}_+ + \vec{F}_- = 0$

$$\vec{N} = \vec{r}_+ \times \vec{F}_+ + \vec{r}_- \times \vec{F}_-$$

Take origin to be cm $\frac{1}{2}$ d displacement

$$= \frac{\vec{d}}{2} \times q\vec{E} + \left(-\frac{\vec{d}}{2}\right) \times (-q\vec{E})$$

$$= q\vec{d} \times \vec{E} = \vec{p} \times \vec{E}$$

$\vec{N}$ makes dipole line up ll to $\vec{E}$

$\vec{p}$ line up with $\vec{E}$. So in general dipole ll $\vec{E}$

Suppose $\vec{E}$ is not constant $\vec{E}_+ \neq \vec{E}_-$

$$\vec{F} = \vec{F}_+ + \vec{F}_- = q(\vec{E}_+ - \vec{E}_-)$$

Assuming dipole is small in size
Taylor Series expansion

~~$d\vec{E} = \vec{E}(0) + \vec{\nabla}$~~

$$d\vec{E}(\vec{r} + d\vec{l}) = \vec{E}(\vec{r}) + \vec{\nabla}\vec{E}(\vec{r}) \cdot d\vec{l}$$

Expand about ~~center~~

$$\vec{E}_+ - \vec{E}_- = \vec{d} \cdot \vec{\nabla} \vec{E}$$

potential energy

$$\text{So} \quad \vec{F} = (\vec{p} \cdot \vec{\nabla})\vec{E} = -\vec{\nabla} U$$

$$U = -\vec{p} \cdot \vec{E}$$

energy min if $\vec{p} \parallel \vec{E}$

H 11-7

A polarized object has its own field. Total field is external plus field of dipole

In many situations dipole moment of atom is proportional to applied $\vec{E}$

$\vec{p} = \alpha \vec{E}$ $\quad\quad \alpha =$ atomic polarizability

Units of α

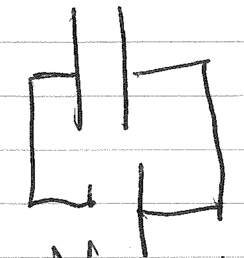
$$[\alpha] = \frac{[p]}{[E]} = \frac{\text{charge} \times \text{length}}{\frac{\text{charge}}{4\pi\epsilon_0 \text{ length}^3}} = 4\pi\epsilon_0 \text{ Volume}$$

$\alpha / (4\pi\epsilon_0 10^{-30} m^3)$ $\quad\quad 10^{-10}$ m = radius of atom

H	He	Li	Be	C	Ne	Na
.667	.205	24.3	5.60	1.76	0.396	24.1

Understand the numbers

H atom in $\parallel$ plate capacitor



$l = 1\,mm$

$V = 500$ Volts

radius of H atom $= a_0 = 0.5 \times 10^{-10}\,m$

Find displacement d

$$p = \alpha E = q d \qquad q = 1.6 \times 10^{-19}\,C$$

$$\frac{d}{a_0} = \frac{\alpha E}{q a_0} \ ; \quad E = \frac{V}{d} = 5 \times 10^{5}\,V/m$$

$$= \frac{4\pi \epsilon_0 (.667)\, 10^{-30} \times 5 \times 10^{5}}{1.6 \times 10^{-19} \quad (.5 \times 10^{-10}\,m)}$$

$$\epsilon_0 = 8.85 \times 10^{-12}\ C^2/N m^2$$

$$\frac{d}{a_0} = 4.6 \times 10^{-6}$$

Dielectric matter

$\vec{E}_{ext} \rightarrow$ dipole moments

1.) permanent dipoles line up along $\vec{E}$

2) dipoles created $\parallel$ to $\vec{E}$

dipole moments causes new $\vec{E}_d$

total $\vec{E} = \vec{E}_{ext} + \vec{E}_d$

today comput $\vec{E}_d$

<u>Demos</u>

How much polarization

$\vec{P} \equiv$ dipole moment per unit volume

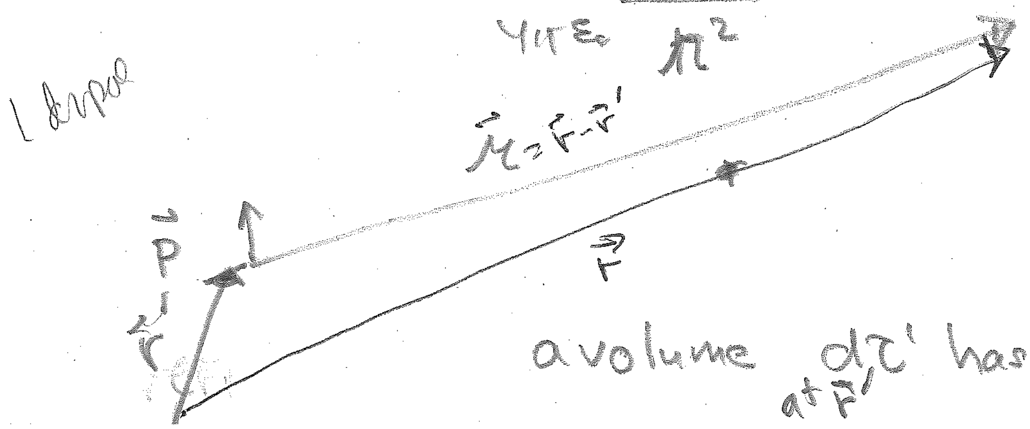
$[P] = ? \; \frac{e\,l}{l^3} = \frac{\text{charge}}{\text{area}}$

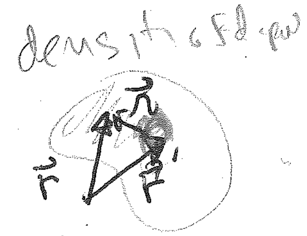
what field does this cause?

Compute potential due to pol

Recall V due to single dipole $\vec{p}$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \frac{\hat{\mathscr{r}} \cdot \vec{p}}{\mathscr{r}^2}$$





a volume $d\tau'$ at $\vec{r}'$ has $\vec{p} = \vec{P}\, d\tau'$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_V \frac{\hat{\mathscr{r}} \cdot \vec{P}(\vec{r}')}{\mathscr{r}^2}\, d\tau' \qquad \mathscr{r} = |\vec{r} - \vec{r}'|$$

Can simplify the result.

we know $-\vec{\nabla} \frac{1}{\mathscr{r}} = + \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$, $-\vec{\nabla} \frac{1}{|\vec{r}-\vec{r}'|} = \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$

$\mathscr{r} = |\vec{r} - \vec{r}'|$

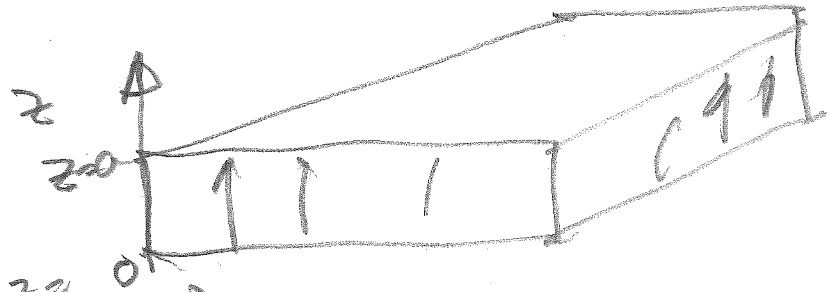
$$\vec{\nabla}' \frac{1}{\mathscr{r}} = -\vec{\nabla} \frac{1}{\mathscr{r}} = + \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int d\tau'\, \vec{P}(\vec{r}') \cdot \vec{\nabla}' \frac{1}{\mathscr{r}}$$

Integrate by parts.

Looks kind of funny - <u>example</u>

Rectangular slab



$\vec{P} = P\hat{z}$ inside

$= 0$ outside

$$\vec{P}(\vec{r}') \cdot \vec{\nabla}' \frac{1}{|\vec{r}-\vec{r}'|} = \vec{\nabla}' \cdot \left(\vec{P} \frac{1}{|\vec{r}-\vec{r}'|} \right)$$

($\vec{r}'$ inside)

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_V d\tau' \, \vec{\nabla}' \cdot \left(\frac{\vec{P}}{|\vec{r}-\vec{r}'|} \right)$$

where V
which S

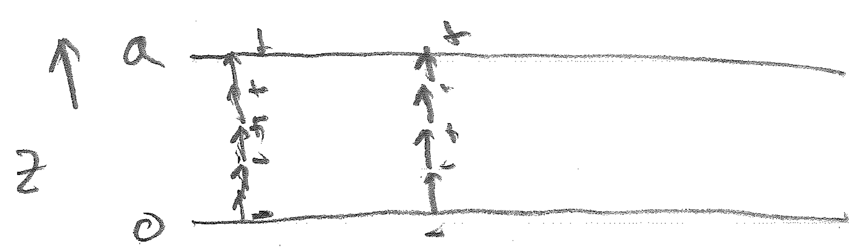
$$= \frac{1}{4\pi\varepsilon_0} \int_S da' \, \frac{\hat{n}' \cdot \vec{P}}{|\vec{r}-\vec{r}'|}$$

$\hat{n}' \cdot \vec{P}$ acts as surface charge distribution $\hat{n}' \cdot \vec{P} = +P$ at $z=a$

$-P$ at $z=0$

The dipole cause a potential that is coming from surface charge
+ on top − on bottom

Physical interpretation



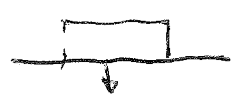
What's $\vec{E}$ outside

0

What's inside

$$\vec{E} = \frac{\sigma_B}{\epsilon_0}\hat{z} = \frac{P}{\epsilon_0}\hat{z}$$

Gaussian surfa



$$E\, dA = \sigma_B$$

$$\vec{E}\cdot\hat{n}\, dA = -dA\frac{\sigma_B}{\epsilon_0}$$

$$-E = -\frac{\sigma_B}{\epsilon_0}$$

15

How did $\vec{P}$ get there?

Often External $\vec{E}_{ext}$ caused this $\vec{P}$

total field is sum of these $\vec{E}$'s + external $\vec{E}_{ext}$

Usually if $\vec{E}_{ext} \Rightarrow 0$ $\vec{P} \Rightarrow 0$

Some special material exists

electret - some dielectrics retain polarization for long periods

certain polymers have lifetimes of sev. thousand years at room temp. Such a dielectric is called an electret. ~~is a nob~~

electret is electrical equivalent of bar magnet.

Example

poly vinylidine fluoride PVF_2

Chains $CH_2 - CF_2$ units

$P = 50$ milli Coul / m^2

Useds in ~~transducer~~s microphones

90% of all microphones today
1 billion made / year
based on Foil Electret Principle
Electret film on metal
Sound waves deflect the electret
which generates output voltage.

From Patent

Sound converted to electrical signals
Example
transducer is device that
Converts input energy to output energy

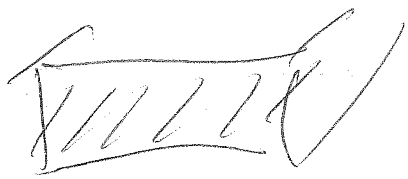
~~energy~~

Next do general case
$\vec{P}(\vec{r}')$ not constant
go back do $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int d\tau' \, \vec{P}(\vec{r}') \cdot \vec{\nabla}' \frac{1}{|\vec{r}-\vec{r}'|}$

Insulators dielectrics had

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int d^3r' \, \vec{P}(\vec{r}')\cdot\nabla'\frac{1}{|\vec{r}-\vec{r}'|}$$

$\vec{P}$ is dipole mom/unit volume



$\vec{P}$ = const inside 0 outside

moved ∇' thru

$\vec{P}\cdot\nabla' = \nabla'\cdot\vec{P}$

if P = constant

Then used divergence theorem

got surface charge density

Now suppose $\vec{P}$ is not constant

Int by parts

Thm in G will prove

Integration by parts in 3D imensions

In General:

$$\vec{\nabla}\cdot(\vec{B}f) = (\vec{\nabla}\cdot\vec{B})f + \vec{B}\cdot\vec{\nabla}f$$

$$\int_{\mathcal{V}} d\tau\, \vec{\nabla}\cdot(\vec{B}f) = \int_{\mathcal{S}} d\vec{a}\cdot\vec{B}f$$

$$= \int_{\mathcal{V}} d\tau \left(\vec{\nabla}\cdot\vec{B}\, f + \vec{B}\cdot\vec{\nabla} f\right)$$

what we want with Integration "differentiation over $\vec{r}\,'$:

$$\vec{B} = \vec{P}(\vec{r}\,'), \quad f = \frac{1}{|\vec{r}-\vec{r}\,'|}$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_{\mathcal{S}} \frac{d\vec{a}\,'\cdot\vec{P}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|} + \int_{\mathcal{V}} \frac{d\tau'}{|\vec{r}-\vec{r}\,'|}\left(-\vec{\nabla}'\cdot\vec{P}(\vec{r}\,')\right)$$

$\vec{P}(\vec{r}\,') = \sigma_B(\vec{r}\,')$ bound charge surface density

$-\nabla'\cdot\vec{P}(\vec{r}\,') = \rho_B(\vec{r}\,') =$

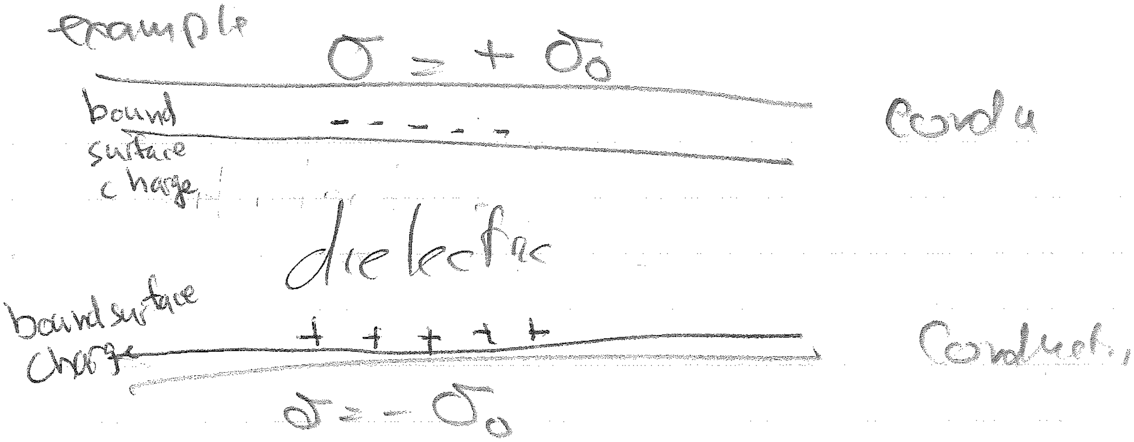
Volume bound charge density

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\oint_S da'\,\frac{1}{|\vec{r}-\vec{r}\,'|}\,\sigma_b(\vec{r}\,')$$

$$+ \frac{1}{4\pi\varepsilon_0}\int_V d^3r'\,\frac{\rho_B(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}$$

So we now know how to deal with bound charge. ρ_B suppose there is also free charge ρ_f

example



$\rho = \rho_f + \rho_B$ $\qquad\qquad$ $\rho_f \to E \to \vec{P}$ $\to$ new total $\vec{E}$

Want an $\qquad\qquad$ to study

Gauss Law including dielectrics

$$\vec{\nabla} \cdot \epsilon_0 \vec{E} = \rho = \text{total charge}$$

$$= \rho_b + \rho_f \quad \leftarrow f = \text{free}, \; b = \text{bound}$$

$$= -\vec{\nabla} \cdot \vec{P} + \rho_f$$

$$\vec{\nabla} \cdot (\epsilon_0 \vec{E} + \vec{P}) = \rho_f$$

$$\vec{D} = \text{displacement vector} \qquad \text{electric displacement}$$

$$= \epsilon_0 \vec{E} + \vec{P}$$

H| n|n