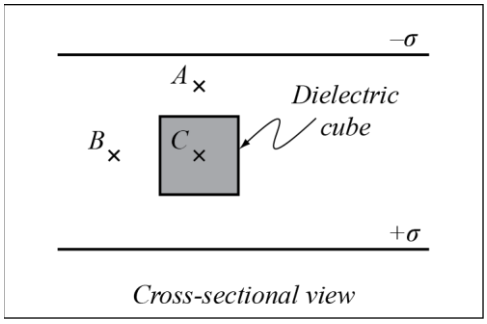
Name _____ **Student ID** _____ **Score**_____
last *first*

IV. [25 points total] Tutorial question.

A dielectric cube is placed between two charged sheets as shown at right. The sheets produce an upward electric field $\vec{E}_{\text{sheet}} = E_o\hat{z}$ and an upward displacement field $\vec{D}_{\text{sheet}} = D_o\hat{z}$.



You may use the approximation that the dielectric cube responds uniformly to the field of the sheet, and ignore any higher-order corrections to only the polarization.

Let $|\vec{E}_P|$ and $|\vec{D}_P|$ represent the magnitude of the net electric and displacement fields at a specified point *P*, respectively.

A. [6 pts] In what direction is the dipole moment of the cube? If the dipole moment is zero, state so explicitly. Explain your reasoning.

Upwards.

Positive bound charges appear on the top part and negative bound charges appear on the bottom. Dipole moment points from negative to positive, which can be justified using $\sum Q_i\vec{r}_i'$.

Alternatively, polarization points in the direction if external or net electric field, which is pointing upward. As polarization is defined as the dipole moment per volume, the polarization will also point up.

B. [6 pts] At point *C*, is $|\vec{D}_C|$ *greater than, less than,* or *equal to* $\varepsilon_o|\vec{E}_C|$? Explain your reasoning.

Greater than.

Using $\vec{D} = \varepsilon_o\vec{E} + \vec{P}$*, both net electric field and polarization are pointing up within the cube, so the magnitude of displacement field is larger than* $\varepsilon_o\vec{E}$ *or* $\vec{P}$ *individually.*

C. [6 pts] At point *A,* is $|\vec{E}_A|$ *greater than, less than,* or *equal to* E_o*?* Explain your reasoning.

Greater than.

Positive bound charges appear on top of the cube, and negative bound charges appear on the bottom. Point A is closer to the positive than the negatives, so the induced electric field from the bound charges point upward, in the same direction as the original $\vec{E}_{sheet} = E_o\hat{z}$*. Thus the net electric field is greater than* E_o.

D. [7 pts] At point *B* is $|\vec{D}_B|$ *greater than, less than,* or *equal to* D_o*?* Explain your reasoning.

Less than.

The upward polarization has curl out of the page (counter-clockwise) on the left side of the cube, and curl into the page (clockwise) on the right side. As $\vec{\nabla}\times\vec{D} = \vec{\nabla}\times\vec{P}$*, the displacement field from the curl makes an induced displacement field pointing downward on the left side of the cube. This points in the opposite direction as the original* $\vec{D}_{sheet} = D_o\hat{z}$*, so the net displacement field is smaller.*

One could first compare $|\vec{E}_B|$ *and* E_o *as well, in a similar manner to part C. In this case, the induced electric field points downward, away from the positive bound charges and toward the negative bound charges. This makes* $|\vec{E}_B|$ *less than* E_o*. As point B is in a region with no polarization,* $\vec{D} = \varepsilon_o\vec{E}$ *will also tell us that* $|\vec{D}_B|$ *is less than* D_o.