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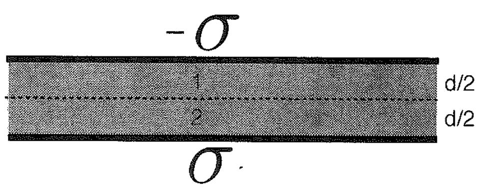

1. Two isolated square, very-large conducting plates with separation d are charged with surface densities $+\sigma > 0$ on the lower plate and $-\sigma$ on the upper plate. Two dielectric slabs, each of thickness $d/2$ are inserted between the plates, one slab above the other as in the figure. The permeabilities are ϵ_1 and ϵ_2.

(a) (6) Determine: **D** everywhere between the plates.

Gauss Law $\int \vec{D}\cdot d\vec{a} = q_{free} \Rightarrow D\,A = q_{free}$

$D = q_{free}/A = \sigma$ D points up

(b) (7) Write the boundary conditions that **D** and **E** must satisfy on the boundary between the two dielectrics. Define the quantities in region 1 as $\mathbf{D}_1$, $\mathbf{E}_1$, and those in region 2 as $\mathbf{D}_2$, $\mathbf{E}_2$ The normal component of D is continuous

$\vec{D}_1\cdot\hat{n} - \vec{D}_2\cdot\hat{n} = 0$ with $\hat{n}$ pointing up

Tangential component of $\vec{E}$ is continuous

$\vec{E}_1\cdot\hat{t} - \vec{E}_2\cdot\hat{t} = 0$ with $\hat{t}\cdot\hat{n} = 0$

(c) (6) Determine **E** everywhere between the plates. $D_{1,2} = E_{1,2} * (\epsilon_{1,2})$

$E_1 = \sigma/\epsilon_1$

$E_2 = \sigma/\epsilon_2$

(d) (6) Determine the bound surface charges on the interface between the two dielectrics (dashed line). $\vec{P}_{1,2}$ are needed $\vec{D}_1 = \epsilon_0\vec{E}_1 + \vec{P}_1 \Rightarrow P_1 = \sigma - \frac{\epsilon_0}{\epsilon_1}\sigma$

$P_1 = \frac{\epsilon_1-\epsilon_0}{\epsilon_1}\sigma$ $\vec{P}_2 = \frac{\epsilon_2-\epsilon_0}{\epsilon_2}\sigma$, $\vec{P}_{1,2}$ both point up

$\sigma_b = -\vec{P}_1\cdot\hat{n} + \vec{P}_2\cdot\hat{n}$, with $\hat{n}_2$ up $= \left(-\frac{(\epsilon_1-\epsilon_0)}{\epsilon_1} + \frac{\epsilon_2-\epsilon_0}{\epsilon_2}\right)\sigma = \epsilon_0\left[-\frac{1}{\epsilon_2}+\frac{1}{\epsilon_1}\right]\sigma$

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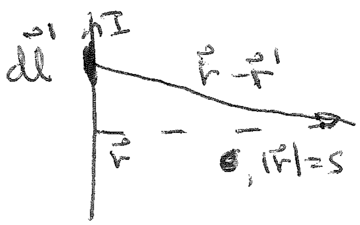
2. A very long wire is placed along the z axis, with current I flowing upward. A point P is a distance s away from the wire.

(a) (6) Obtain an expression for the magnetic field **B** at the point P. Your answer should be in the form of a one-dimensional integral over a line in which **every** symbol is defined using a sketch. Do not evaluate the integral.



$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{d\vec{l}' \times (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|}$$

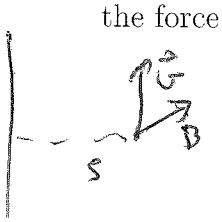
(b) (4) The magnetic field has at the point P has a magnitude $B_0 = \frac{\mu_0 I}{2\pi s}$. State the direction of the magnetic field.

Magnetic field points into the page

(c) (7) A particle of mass m and charge $Q < 0$ is placed at rest at the point P. Determine what happens to the charged particle, and state your answer in **one** circled sentence.

There is no force on the particle. It just stays at rest at the point P.

(d) (8) At $t = 0$ particle of mass m and charge $Q < 0$ is placed at the point P, moving upward with an initial speed v_0. Describe the initial force on the particle. Then describe the force a short time after that.



The $\vec{v} \times \vec{B}$ points inward but the force points outward with magnitude $|Q| v_0 B$. The particle accelerates radially outward so after a short time there is also a downward force

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3. A region of charge, with unknown charge density $\rho(\vec{r})$, is localized around an origin $\vec{r} = 0$. The charge density vanishes for all values of r greater than a given distance L. The potential $V(\vec{r})$ (in spherical coordinates) in the region outside the charge distribution ($r > L$) is:

$$V(r,\theta) = \frac{1}{4\pi\epsilon_0}\left[\frac{B\cos(\theta)}{r^2} + C\frac{(3\cos^2(\theta)-1)}{r^3}\right], \quad (1)$$

where B, C are given positive constants. Hint: $\frac{1}{|\vec{r}-\vec{r}\,'|} = \sum_{l=0}^{\infty} \frac{(r')^l}{r^{l+1}} P_l(\hat{r}\cdot\hat{r}\,')$, if $r > r'$.

(a) (5) Is the charge density spherically symmetric. Explain why yes, or why no. No

a spherically symmetric charge distribution will produce a $V(\vec{r})$ that depends only on $r = |\vec{r}|$

(b) (6) Consider a sphere, S, of radius $2L$, centered at the origin. Compute the integral: $\oint \mathbf{E}\cdot d\mathbf{a}$.

$\oint \vec{E}\cdot d\vec{a} = q_{enclosed}/\epsilon_0 = 0$ The charge is 0 because $V(r,\theta)$ has no term proportional to $1/r$

OR doing the integral is 0 because the angular integrals of $\cos\theta$ and $3\cos^2\theta - 1$ are both 0

(c) (7) Compute the electric field (magnitude and direction) along the z-axis for $z = 2L$.

$\vec{E} = -\vec{\nabla}V = -\hat{r}\frac{\partial V}{\partial r} - \frac{\hat{\theta}}{r}\frac{\partial V}{\partial \theta}\Big|_{\theta=0}$

on the z-axis $\theta = 0$, $\sin\theta = 0$ so the second term has no effect

$\vec{E} = -\hat{z}\frac{\partial V}{\partial r}\Big|_{\theta=0} = \frac{\hat{z}}{4\pi\epsilon_0}\left[\frac{B}{(2L)^3} + \frac{3C\cdot 2}{(2L)^4}\right]$

(d) (7) Compute the integral $\int d^3r\rho(\vec{r})(3z^2 - r^2)$.

$V(r,\theta) = \frac{1}{4\pi\epsilon_0}\int d^3r'\frac{\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|} = \frac{1}{4\pi\epsilon_0}\int d^3r' \sum_l \frac{r'^l}{r^{l+1}} P_l(\hat{r}\cdot\hat{r}')$

compute this $V(r,\theta=0)$ compare with given. Take $l=2$

$V(r,\theta=0) = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}\int d^3r'\,\rho(\vec{r}\,')\,r'^2\left(\frac{3\cos^2\theta'-1}{2}\right) = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}\int d^3r'\rho(\vec{r}\,')\frac{3z'^2-r'^2}{2}$

$= 2C$, so $\int d^3r\,\rho(\vec{r})(3z^2 - r^2) = \underline{4C}$

from (1)