IV. [20 points total] Tutorial question.

A charged rod with non-uniform charge density $\rho(\vec{r}) = C_0 s$ (while inside the rod) is shown at right, where $C_0$ is a positive constant. There is no charge outside of the rod. The rod is infinitely long.

Point $A$ is located near the center of the rod, whereas point $B$ is located just outside the surface.

A. Suppose you were to evaluate the magnitude of the electric field at point $B$ using Gauss’ Law in integral form.

i. [3 pts] Describe and/or sketch a Gaussian surface for which the electric flux integral will be easy to simplify. If there is no Gaussian surface possible, state so explicitly.

A closed cylinder concentric with the rod such that point $B$ lies on the curved surface.

ii. [5 pts] Explain in words why the flux integral will be (or cannot be) easily simplified. Any math must be paired with a verbal interpretation of each step.

The flux simplification process requires going from $\int \vec{E} \cdot d\vec{A} \rightarrow |\vec{E}| \int dA \rightarrow |\vec{E}| \int dA$ to evaluate the magnitude of the electric field at point $B$. For the first step, we need to have the direction of the electric field be perpendicular or parallel to the area vectors on each surface. For the second step, we need to have the magnitude of the electric field be constant along each surface where the flux is non-zero.

B. [4 pts] How does electric field within the rod depend on $z$ (e.g., constant with $z$, proportional to $1/z$, $z^2$, etc.)? Explain your reasoning.

The electric field is independent of (does not depend on, is constant with) $z$.

This describes field nature, so we use the continuous translational symmetry along the $z$-axis to show that regardless of the $z$-position of the point of interest, the behavior should be the same.

C. [8 pts] Is the divergence of the electric field at point $A$ greater than, less than, or equal to the divergence of the electric field at point $B$? If the divergence at either point is zero, state so explicitly. Explain your reasoning.

Greater than, with the divergence at point $B$ being zero.

The divergence of the electric field describes the charge density. Point $A$ is a location with positive charge density, while point $B$ is outside of the rod and has no charge.