Subject: demos for Tues Oct 8, 830 am A118
From: "Gerald A. Miller" <miller@uw.edu>
Date: 10/6/19, 3:02 PM
To: lectdemo@phys.washington.edu

HI Steven, Please have:

Thanks

Electrostatic Rods and Cloth (5A10.10) -- Glass rods and silk (positive charge), clear acrylic rods and wool (positive charge), red acrylic rods and wool (negative charge), or rubber and fur (negative charge)

Electrostatic Attraction and Repulsion (Charged Rods on Pivot) (5A20.10) -- Charged rods (5A10.10) on a pivoting stand are free to rotate and show attraction or repulsion between charges. Note: See also Magnetic Attraction and Repulsion (5H20.10).

Aluminized Ping-Pong Balls (Electrically Connected) -- Two aluminized ping-pong balls hang from wires attached to a metal plate. Touch a charged rod to the plate and the balls separate.

Aluminized Ping-Pong Balls (Electrically Insulated) -- Two aluminized ping-pong balls hang from a plastic rod so that they may be given opposite charges to demonstrate attraction.

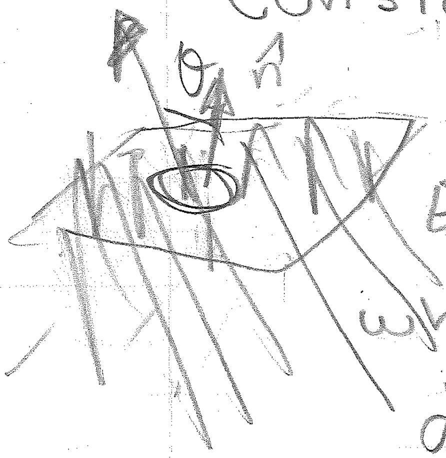
Three Ping-Pong Balls -- Three aluminized and electrically connected ping-pong balls hang from a common point on thin wires so that they just touch. The balls are now charged with a Wimshurst generator, causing them to fly apart by electrostatic repulsion. A camera is mounted above the ping-pong balls and shows that the direction of the repulsive force is outwards from the center of the three balls.

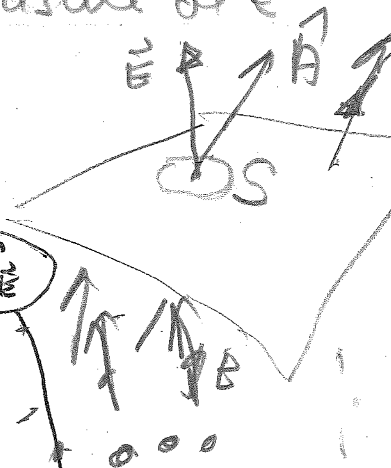
--
Prof. Gerald A. Miller
Univ. of Washington, Dep't of Physics
Seattle, WA 98195-1560
206-543-2995

Tuesday Oct 8

Other ways to get $\vec{E}$

Consider a ... measure of E





$$\Phi_E = \int_S \vec{E} \cdot d\vec{a}$$

Measures strength & direction of $\vec{E}$

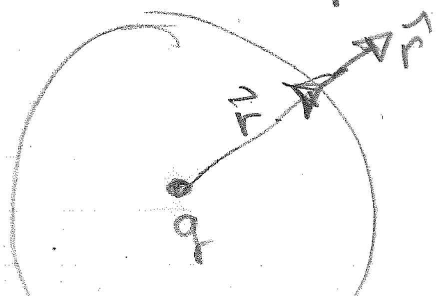
what does this mean

$$d\vec{a} = \hat{n}\, da$$

choice of 2 in general

$$\vec{E} \cdot d\vec{a} = \vec{E} \cdot \hat{n}\, da$$

Example q at center of sphere of radius r



$$\oint d\vec{a} \cdot \vec{E}$$

closed surf

For closed surface $\hat{n}$ points out

here $\hat{n} = \hat{r}$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} q \frac{\vec{r}}{r^3}$$

$$\vec{E} \cdot \hat{r}\, da = \frac{1}{4\pi\epsilon_0} q \frac{\vec{r} \cdot \hat{r}\, da}{r^3} = \frac{q}{4\pi\epsilon_0} \frac{1}{r^2} da$$

$$da = r^2 \sin\theta \, d\theta \, d\phi$$

$$= r^2 \, d\Omega \qquad d\Omega = \frac{da}{r^2}$$

$$\oint \vec{E} \cdot \hat{r} \, da = \frac{q}{4\pi\varepsilon_0} \frac{1}{r^2} \int r^2 \, d\Omega$$

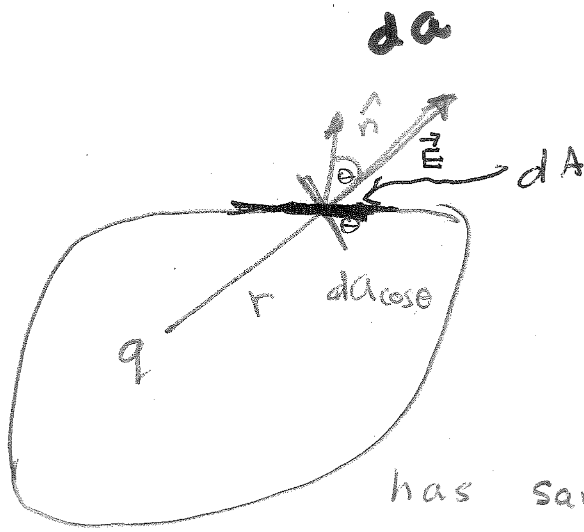
$$= \frac{q}{\varepsilon_0}$$

Suppose surface not sphere

q is not at center or

Same answer?

YES



$\vec{E}\cdot\hat{n}\,da = E\cos\theta\,da$

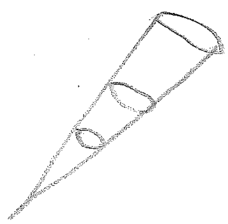
$\cos\theta\,da$ is projection of area on to plane normal to $\vec{E}$. This has same area as piece of sphere

$$\vec{E}\cdot\hat{n}\,da = \frac{q}{4\pi\epsilon_0}\,\frac{(\cos\theta\,da)}{r^2}$$

$$\frac{(da\cos\theta)}{r^2} = \frac{da_{sphere}}{r^2} = d\Omega$$

= solid angle

$d\Omega$ ind. of r^2



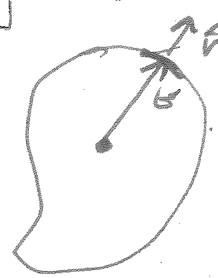
ratio invariant $= d\Omega$

No charge

$$\oint d\Omega = \frac{4\pi r^2}{r^2} = 4\pi$$

here $\boxed{\oint \vec{E}\cdot d\hat{a} = q/\varepsilon_0}$

For a single point charge



Same as for sphere

$\vec{E}\cdot\hat{n}\,da$ always +

But suppose now

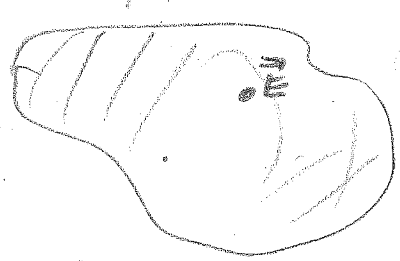
$$\oint \vec{E}\cdot d\vec{a} = \frac{\sum q_i}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$ total charge inside

suppose no charge inside what's $\oint \vec{E}\cdot d\vec{a}$?

$\boxed{\text{Gauss law}}$ ask class does mean $\vec{E} = 0$?

just get $q \to$

Example



q

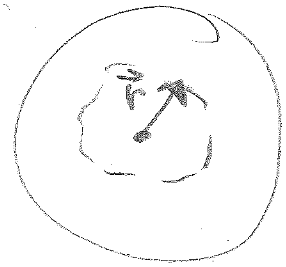
$\oint \vec{E}\cdot da = 0$ is $E = 0$

have integral thm

To get $\vec{E}$ need something extra
some other information - symmetry

Example — sphere of uniform charge

Ask them outside



density ρ, radius R

what's $\vec{E}(\vec{r})$ $r < R$

Guess how does E vary w r

1. $\vec{E}$ is spherically symmetric $\vec{E} \parallel \hat{r}$ $\vec{E} = E\,\hat{r}$

2. Consider fictitious surface a Gaussian surface a sphere

$$\oint_{sph} \vec{E} \cdot d\vec{a} = \frac{q}{\varepsilon_0} \text{ (inside)} = \frac{\rho \text{ Volume inside}}{\varepsilon} = \rho \frac{4\pi r^3}{3\varepsilon_0}$$

$$\vec{E}\, d\vec{a} = E\, da$$

$$E\, 4\pi r^2 = \frac{\rho}{\varepsilon_0} \frac{4\pi r^3}{3}$$

Suppose

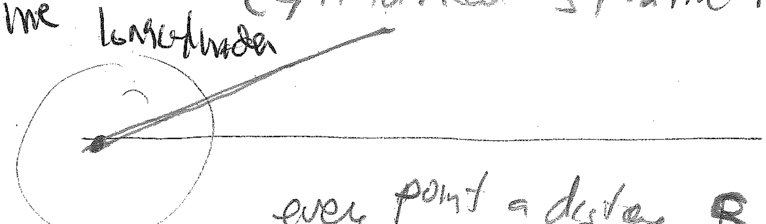
what does G.L. tell of $\vec{E}$

after $\vec{E} = \frac{\rho \vec{r}}{3\varepsilon_0}$ $r < R$

$\boxed{\text{spherically symmetric}}$ all points a distance r away from center are the same

Other examples -
cylindrical symmetry

very long line ∞ long cylinder



axis

every point a distance s away from axis same

Suppose cylinder is finite in length? Use Gauss Law?

So we have seen 2 ways to get $\vec{E}$

1) given $\rho(\vec{r}')$, $\sigma(\vec{r}')$ $\lambda(\vec{r}')$ do appropriate integrals

2). Gauss Law

In each the vector nature of $\vec{E}$ is evident. Can't avoid vector

Can we get $\vec{E}$ from a scalar quantity.

YES

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{d^3r'\,\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}(\vec{r}-\vec{r}\,')$$

Better

Proof consider $\vec{R} = r - \vec{r}\,'$

$$\vec{\nabla}_R \frac{1}{R} = -\frac{\hat{R}}{R^2}$$

$$\left(\hat{R}\frac{\partial}{\partial R} + \;\right)\frac{1}{R} = -\frac{\hat{R}}{R^2} = \frac{\vec{R}}{R^3} =$$

$-\hat{R}$

Fact
$$\frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|^3} = -\vec{\nabla}\frac{1}{|\vec{r}-\vec{r}\,'|}$$

only acts on $\vec{r}$

$\vec{r} \neq \vec{r}\,'$

Talk

Proof $\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$

$\boxed{\vec{\nabla}\, r^2 =}\; 2\vec{r}$

Here

$$\frac{1}{|\vec{r}-\vec{r}\,'|} = \left[(x-x')^2+(y-y')^2+(z-z')^2\right]^{-1/2}$$

$$\vec{\nabla}\frac{1}{|\vec{r}-\vec{r}\,'|} = \left[\hat{i}\,(-\tfrac{1}{2})(2)(x-x') + \hat{j}\,(-)(y-y') + \hat{k}\,(-)(z-z')\right]\left[(x-x')^2+(y-y')^2+(z-z')^2\right]^{-3/2}$$

$$= -\frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}$$

. We can take $\vec{\nabla}$ out of integral

$$\vec{E} = -\vec{\nabla}\frac{1}{4\pi\varepsilon_0}\int d^3r'\,\frac{\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|} = -\vec{\nabla}V$$

V is named scalar potential or Electric potential

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{d^3r'\,\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}$$

Mathematical point

$$\vec{E} = \vec{\nabla} V$$

$$\vec{\nabla} \times \vec{E} = 0 \,?$$

$$\vec{\nabla} \times \vec{\nabla} = 0$$ Vector cross itself not 0

object?

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{vmatrix} = 0$$

$$\boxed{\vec{\nabla} \times \vec{E} = 0}$$ Electrostatics

Follows from Coulomb's Law

Do you rember what

should be on right hand side?

Electric potential

$$\vec{E} = -\vec{\nabla} V$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int d^3r' \frac{\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}$$

For surface charges $\quad \rho(\vec{r}\,')d^3r' \Rightarrow \sigma(\vec{r}\,')da'$

line charges $\quad \rho(\vec{r}\,')d^3r' \Rightarrow \lambda(\vec{r}\,')dl'$

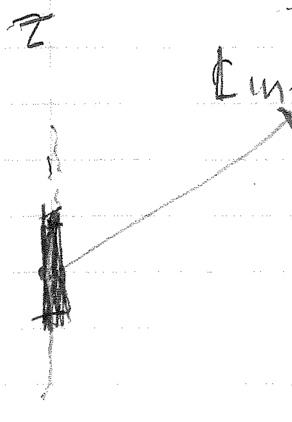
point charges $\quad V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{|\vec{r}-\vec{r}_i|}$

Major advantage – get V to compute $\vec{E}$. Can get V by integration or as we'll see later by solving a PDE

Given $\vec{E}$ we can get V by integration

Not such a good way to get V because if you have $\vec{E}$ you know everything

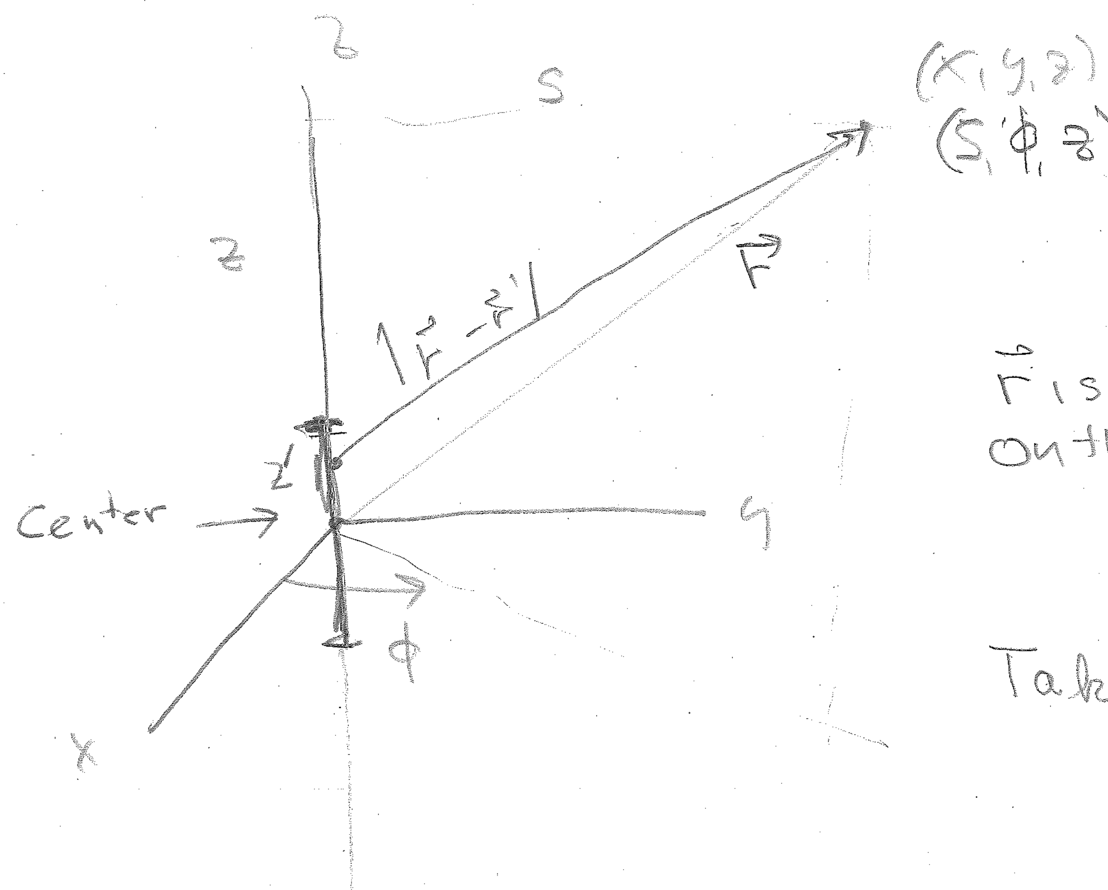
Example of computing V



Line charge Finite length L $\lambda = Q/L$ const
get V

If L were infinite how could you get $\vec{E}$ directly?

what is the symmetry? S polar and top hat



$\vec{r}$ is not on the line.

Take $z > L/2$

Will potential depend on ϕ?

$V(S,z) = V(S, -z)$

$z > L/2$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_{-\frac{L}{2}}^{L/2}\frac{dz'\lambda}{|\vec{r}-\vec{r}'|}$$

$$\vec{r}-\vec{r}' = \sqrt{(z-z')^2 + S^2}$$

from hypotenuse of triangle

$$V = \frac{\lambda}{4\pi\varepsilon_0}\int_{-L/2}^{L/2} dz' \frac{1}{\sqrt{(z-z')^2+S^2}}$$

$$V = \frac{\lambda}{4\pi\varepsilon_0}(-)\log\left((z-z') + \sqrt{(z'-z)^2+S^2}\right)\Bigg|_{-\frac{L}{2}}^{L/2}$$

$\log = \ln$

$z - z' > 0$

$$= \frac{\lambda}{4\pi\varepsilon_0} \log \frac{+\frac{L}{2} + z + \sqrt{\left(\frac{L}{2}+z\right)^2 + s^2}}{\cancel{+}\frac{L}{2} + z + \sqrt{\left(\frac{L}{2}+z\right)^2 + s^2}}$$

$z > L/2$

limit $L \to 0$ $V = 0$ with λ held fixed

$$= \frac{\lambda}{4\pi\varepsilon_0} \log \frac{\frac{L}{2} + z + \sqrt{\left(\frac{L}{2}+z\right)^2 + s^2}}{z - \frac{L}{2} + \sqrt{\left(z - \frac{L}{2}\right)^2 + s^2}}$$

Independent of ϕ

$$\lim_{s \to 0} \quad \frac{\lambda}{4\pi\varepsilon_0} \ln \frac{\frac{L}{2} + z}{z - \frac{L}{2}}$$

$\ln\left(\frac{1+x}{1-x}\right)$ $x<1$

$= 2\left(x + \frac{x^3}{3} + \frac{x^5}{5}\right)$

$$\lim_{z \to \infty} = \frac{\lambda}{4\pi\varepsilon_0} \ln \frac{1 + \frac{L}{2z}}{1 - \frac{L}{2z}}$$

$\ln(1+\varepsilon) = \varepsilon$

$$= \frac{\lambda}{4\pi\varepsilon_0} 2\left(\frac{L}{2z} + \frac{1}{3}\left(\frac{L}{2z}\right)^3 + \cdots\right)$$

$$V \to \frac{\lambda L}{4\pi\varepsilon_0 z} = \frac{Q}{4\pi\varepsilon_0 z}$$

suppose $z < L/2$ expression is not correct

$s \to 0$ lim is ln negative number

must redo integral

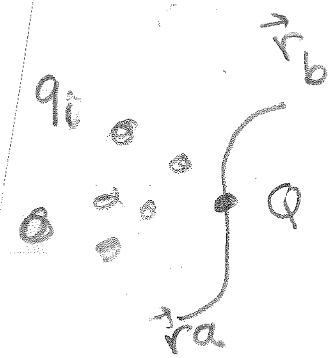
$$\text{if } z < 0 \quad \ln \frac{\frac{L}{2} - z + \sqrt{\left(\frac{L}{2} - z\right)^2 + s^2}}{-\frac{L}{2} - z + \sqrt{\left(\frac{L}{2}+z\right)^2 + s^2}}$$

$$\vec{E} = -\vec{\nabla} V = -\hat{s}\frac{\partial}{\partial s} V - \hat{\phi}\frac{\partial}{s\partial\phi} V - \hat{z}\frac{\partial}{\partial z} V$$

here

$$= -\hat{s}\frac{\partial V}{\partial s} - \hat{z}\frac{\partial V}{\partial z}$$ Mess but can be done

Physical meaning of V



work that you have to do to move a test charge Q from $\vec{r}_a$ to $\vec{r}_b$, without changing kinetic energy

$$\vec{F}_{on}(Q) = Q\vec{E}$$

$\vec{F}_{you}$

Force you apply $= -Q\vec{E}$

$$W = \int_{\vec{r}_a}^{\vec{r}_b} \vec{F}\cdot d\vec{l}$$

$$W = -Q\int_{\vec{r}_a}^{\vec{r}_b} \vec{E}\cdot d\vec{l}$$

what is $\vec{E}\cdot d\vec{l} = -\vec{\nabla} V\cdot d\vec{l}$

$d\vec{l} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$

$\vec{\nabla} V = \frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}$

$$= -\frac{\partial V}{\partial x}dx - \frac{\partial V}{\partial y}dy - \frac{\partial V}{\partial z}dz$$

$$= dV$$

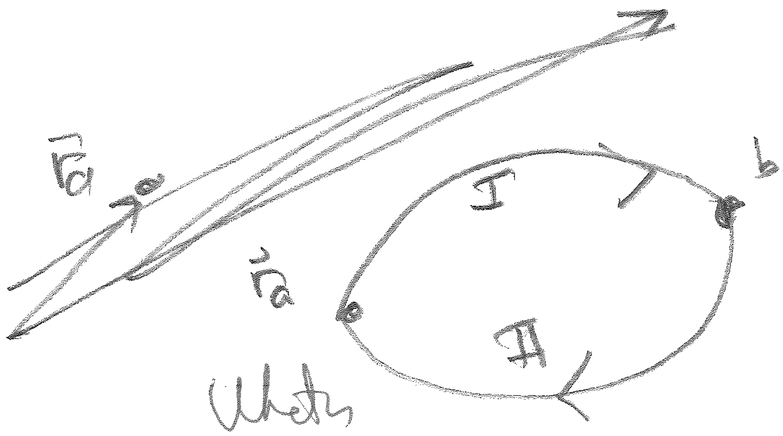
$$W = -Q\left[\int_a^b dV\right] = +Q\left[V(\vec{r}_b) - V(\vec{r}_a)\right]$$ independent of path

$$\frac{W}{Q} = V(\vec{r}_b) - V(\vec{r}_a)$$ PE difference is work done.

Work in moving charge from r_a to r_b

$$\frac{W}{Q} = V(\vec{r}_b) - V(\vec{r}_a) = -\int_{r_a}^{r_b} \vec{E} \cdot d\vec{l}$$

Independent Path?



When

$$\oint \vec{E} \cdot d\vec{l} = 0$$

why

$$\oint \vec{E} \cdot d\vec{l} = \int (\vec{\nabla} \times \vec{E}) \cdot d\vec{a}$$

$$\vec{E} = -\vec{\nabla} V$$

$$\vec{\nabla} \times \vec{\nabla} V = 0$$

$$\int_{I\,r_a}^{r_b} \vec{E} \cdot d\vec{l} + \int_{II\,r_b}^{r_a} \vec{E} \cdot d\vec{l} = 0$$

$$\int_{I\,r_a}^{r_b} \vec{E} \cdot d\vec{l} = \int_{II\,r_a}^{r_b} \vec{E} \cdot d\vec{l}$$

→ Very useful
many applications
outside E&M

$\boxed{\text{Derive PDE for } V}$

avoids integration

Use another integral theorem

based on Gauss Law

$$\oint_S \vec{E}\cdot d\vec{a} = \frac{Q}{\varepsilon_0} \quad (\text{enclosed in } S)$$

$$= \frac{1}{\varepsilon_0}\int_V d\tau\, \rho \qquad = \frac{1}{\varepsilon_0}\int d^3r\, \rho(\vec{r})$$

Divergence theorem

$$\oint_S \vec{E}\cdot d\vec{a} = \int_V d\tau\, \vec{\nabla}\cdot\vec{E} = \int_V d\tau\, \rho/\varepsilon_0$$

$$\int_V d\tau\,(\vec{\nabla}\cdot\vec{E} - \rho/\varepsilon_0) = 0$$

$$V \to 0$$

$$\boxed{\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0}$$ 1st Maxwell

$$\vec{E} = -\vec{\nabla}V$$

$$\vec{\nabla}\cdot(\vec{\nabla})V = \boxed{-\vec{\nabla}^2 V = \rho/\varepsilon_0}$$ Poisson's eq

$$\vec{\nabla}^2 = \vec{\nabla}\cdot\vec{\nabla} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)$$

$$= +\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) \quad \text{Laplacian}$$

if $\rho = 0$

$$\nabla^2 V = 0$$ Laplace's equation

Example In a region of space

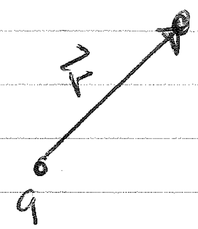
$$V = -\frac{E_0}{L} X$$

what is ρ

what is $\vec{E}$

what is special about X direction

Example Meaning of point charge



V of point charge at origin is

$$V(\vec{r}) = \frac{q}{4\pi\varepsilon_0 r}$$

$$\nabla^2 V = -\rho(\vec{r})/\epsilon_0$$ Poissons eq

$$\nabla^2 \frac{1}{r} = -4\pi\delta(\vec{r})$$

$$\nabla^2 V = -\frac{q}{\epsilon_0}\delta(\vec{r}) = -\frac{\rho(\vec{r})}{\epsilon_0}$$

$$\boxed{\rho(\vec{r}) = \delta(\vec{r})\, q}$$

electron is a point charge

proton is not a point charge