Hi Steven, Please have:

Thanks

Electrostatic Rods and Cloth (5A10.10) -- Glass rods and silk (positive charge), clear acrylic rods and wool (positive charge), red acrylic rods and wool (negative charge), or rubber and fur (negative charge)

Electrostatic Attraction and Repulsion (Charged Rods on Pivot) (5A20.10) -- Charged rods (5A10.10) on a pivoting stand are free to rotate and show attraction or repulsion between charges. Note: See also Magnetic Attraction and Repulsion (5H20.10).

Aluminized Ping–Pong Balls (Electrically Connected) -- Two aluminized ping-pong balls hang from wires attached to a metal plate. Touch a charged rod to the plate and the balls separate.

Aluminized Ping–Pong Balls (Electrically Insulated) -- Two aluminized ping-pong balls hang from a plastic rod so that they may be given opposite charges to demonstrate attraction.

Three Ping–Pong Balls -- Three aluminized and electrically connected ping-pong balls hang from a common point on thin wires so that they just touch. The balls are now charged with a Wimshurst generator, causing them to fly apart by electrostatic repulsion. A camera is mounted above the ping-pong balls and shows that the direction of the repulsive force is outwards from the center of the three balls.

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Other ways to get $E$

Consider a surface of charge $Q$.

\[ \Phi_E = \int_S \mathbf{E} \cdot d\mathbf{a} \]

What does this mean?

$d\mathbf{a} = \hat{n} \, d\mathbf{a}$

Choose two general surfaces.

\[ \mathbf{E} \cdot d\mathbf{a} = \mathbf{E} \cdot \hat{n} \, d\mathbf{a} \]

Example: a point charge $q$ at center $r$.

\[ \Phi_E = \int_{closed \, \text{surf}} \mathbf{E} \cdot d\mathbf{a} \]

For closed surfaces, points out:

\[ q = \frac{1}{4\pi \varepsilon_0} \frac{1}{r^2} \]

Here $\hat{n} = \hat{r}$.
\[ da = r^2 \sin \theta \, d\phi \]
\[ \Rightarrow \frac{d\theta}{r^2} = r \, d\theta \\
\quad \frac{d\phi}{1} = \frac{da}{r^2} \]

\[ \oint_{S_0} \epsilon_0 \, da = \frac{q}{4\pi \epsilon_0} \cdot \frac{1}{r^2} \int r^2 \, dr \]

\[ = \frac{q}{\epsilon_0} \]

Suppose surface net sphere

Same answer?

YES
\[ \mathbf{E} \cdot \mathbf{n} \, dA = \mathbf{E} \cdot \cos \theta \, dA \]

\[ \cos \theta \, dA \text{ is projection of area on to plane normal to } \mathbf{E}. \text{This} \]

\[ \frac{dA \cdot \cos \theta}{r^2} = \frac{dA_{\text{spine}}}{r^2} = d\Omega \]

\[ \text{Ratio invariant: } d\Omega \]

\[ dR \text{ ind of } r^2 \]
\[ \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\varepsilon_0} \]

For a single point charge, suppose how

\[ \oint \mathbf{E} \cdot d\mathbf{A} = 4\pi \varepsilon_0 \]

Gauss Law

Example:

\[ \mathbf{E} = \mathbf{0} \]

\[ \oint \mathbf{E} \cdot d\mathbf{A} = 0 \]
To get $E$ need some information – symmetry

Example

To find uniform charge density $p$, radius $R$ what is $E(r)$, $r < R$? [Gauss law $\oint E \cdot dA = 2\pi r E$]

1. $E$ is spherically symmetric $E \propto \frac{1}{r}$

Suppose $E = \frac{p}{2\pi r}$, $p$ Vol inside

\[ \oint E \cdot dA = \frac{p}{2\pi r} \cdot 2\pi r \cdot 4\pi r^2 = \frac{p}{\varepsilon_0} \]

Other examples – cylindrical symmetry

Every point a distance $r$ away from center are the same

Suppose cylinder is finite in length? Use Gauss Law $E = \frac{p}{2\pi r}$
So we have seen 2 ways to get $\vec{E}$

1) given $P(\vec{r})$, $\delta(\vec{r}) \geq (\vec{r})$ do appropriate integral

2) Gauss Law

In each the vector nature of $\vec{E}$ is evident. Can't avoid vector.

Can we get $\vec{E}$ from scalar quantity.

Yes
\[
\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \int \frac{d^3r'}{|\mathbf{r} - \mathbf{r}'|^3} \mathbf{p}(\mathbf{r}')
\]

**Fact**

\[
\frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} = -\nabla \left( \frac{1}{|\mathbf{r} - \mathbf{r}'|} \right)
\]

**Proof**

\[
\nabla \left( \frac{1}{|\mathbf{r} - \mathbf{r}'|} \right) = \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3}
\]

\[
\nabla \cdot \left( \frac{1}{|\mathbf{r} - \mathbf{r}'|} \right) = \frac{2\mathbf{r} - 2\mathbf{r}'}{|(\mathbf{r} - \mathbf{r}')^2 + (y - y')^2 + (z - z')^2|^{3/2}}
\]

\[
\nabla \cdot \left( \frac{1}{|\mathbf{r} - \mathbf{r}'|} \right) = -\frac{1}{|\mathbf{r} - \mathbf{r}'|^2}
\]

\[
V = \frac{1}{4\pi\varepsilon_0} \int \frac{d^3r'}{|\mathbf{r} - \mathbf{r}'|} \mathbf{p}(\mathbf{r}')
\]

**Proof** (continued)

\[
\nabla \cdot \left( \frac{1}{|\mathbf{r} - \mathbf{r}'|} \right) = -\frac{1}{|\mathbf{r} - \mathbf{r}'|^2}
\]

**Electric field**

\[
\mathbf{E} = -\frac{1}{4\pi\varepsilon_0} \int \frac{\mathbf{p}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} d^3r'
\]

**Potential**

\[
V = \frac{1}{4\pi\varepsilon_0} \int \frac{\mathbf{p}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3r'
\]

**Electric field**

\[
\mathbf{E} = -\nabla V
\]

**Potential**

\[
V = \frac{1}{4\pi\varepsilon_0} \int \frac{\mathbf{p}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3r'
\]

**Electric potential**

\[
V = \frac{1}{4\pi\varepsilon_0} \int \frac{\mathbf{p}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3r'
\]

**Electric field**

\[
\mathbf{E} = -\nabla V
\]
Mathematical point

\[ \mathbf{E} = \nabla \mathbf{V} \]

\[ \nabla \times \mathbf{E} = \mathbf{0} \]

\[ \nabla \times \nabla \mathbf{\phi} = \mathbf{0} \quad \text{Vector cross itself not 0} \]

Object

\[
\begin{vmatrix}
\mathbf{a} & \mathbf{b} & \mathbf{c} \\
\mathbf{d} & \mathbf{e} & \mathbf{f} \\
\mathbf{g} & \mathbf{h} & \mathbf{i}
\end{vmatrix}
= 0
\]

\[ \nabla \times \mathbf{\xi} = 0 \quad \text{Electrostatic} \]

Follows from Coulomb's Law

Do you remember what should be on right hand side?
Electric potential

\[ E = -\nabla V \]

\[ V(\mathbf{r}) = \frac{1}{4\pi \varepsilon_0} \int \frac{d^3 r'}{|\mathbf{r} - \mathbf{r}'|} p(\mathbf{r}') \]

For surface charges \( p(\mathbf{r}') d^3 r' \Rightarrow \sigma(\mathbf{r}') d\mathbf{a}' \)

For line charges \( \phi(\mathbf{r}') d^3 r' \Rightarrow \lambda(\mathbf{r}') d\mathbf{l}' \)

For point charges \( V(\mathbf{r}) = \frac{1}{4\pi \varepsilon_0} \sum \frac{q_i}{|\mathbf{r} - \mathbf{r}_i|} \)

Major advantage - get \( V \) to compute \( E \). Can get \( V \) by integration or as we'll see later by solving a PDE.

Given \( E \) we can get \( V \) by integration.

Not such a good way to get \( V \) because you have \( E \) you know everything.

Example of computing \( V \):

Line charge length \( L \):

\[ \text{Finite} \quad \lambda = \frac{Q}{AL} \]

If \( L \) were infinite how could you get \( E \) directly?
What is the symmetry of polar coordinate system?

If $F$ is not on the line:

$$ V = \frac{1}{q \varepsilon_0} \int \frac{\phi'^2 \, d^2 \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|} $$

$$ F - z' = \sqrt{(x - x')^2 + (y - y')^2} $$

$$ V = \frac{1}{4 \pi \varepsilon_0} \log \frac{1}{(x - x')^2 + (y - y')^2} $$

Take $z > \frac{1}{2}$ from hypothesis of triangle.

$$ V = \frac{1}{4 \pi \varepsilon_0} \log \left( \frac{1}{(x - x')^2 + (y - y')^2} \right) \bigg|_{z > \frac{1}{2}} $$

$$ z - z' > 0 $$
\[
\log \frac{1}{2} + z + \sqrt{\left(\frac{1}{2} + z\right)^2 + z^2} = S^2
\]

\[
\lim_{z \to 0} \quad \text{with } z \text{ held fixed}
\]

\[
\ln \frac{1}{2} + z + \sqrt{\left(\frac{1}{2} + z\right)^2 + z^2} = \frac{1}{2} + \sqrt{\left(\frac{1}{2}\right)^2 + z^2}
\]

\[
\ln \left(1 + \frac{1}{z}\right) = 3
\]

\[
\lim_{z \to 0} \frac{z}{\ln z} = \frac{1}{\ln 2}
\]

\[
V = \frac{1}{4} + 2 = \frac{3}{4}
\]

Suppose \( \epsilon \leq 1 \) and that \( z \) is a negative integer.

\[
\lim_{z \to 0} \ln \left(\frac{1}{2} + z + \sqrt{\left(\frac{1}{2} + z\right)^2 + z^2}\right)
\]
\[
\vec{E} = -\nabla V = -\frac{\partial V}{\partial x} \hat{i} - \frac{\partial V}{\partial y} \hat{j} - \frac{\partial V}{\partial z} \hat{k}
\]

\[
\text{where} \quad \frac{dV}{dz} = -\frac{1}{\varepsilon_0} \frac{\partial V}{\partial z} = -\frac{\delta^2 V}{\delta z^2}
\]

\[
\text{Physical meaning of } V
\]

\[
\oint \overrightarrow{E} \cdot d\overrightarrow{l} = \int F \cdot dl = W
\]

\[
\text{Work you have to do to move a test charge } Q \text{ from } a \text{ to } b \text{ without changing kinetic energy}
\]

\[
W = -Q \int_{a}^{b} \overrightarrow{E} \cdot d\overrightarrow{l}
\]

\[
= \int_{a}^{b} \frac{dV}{dz} dz
\]

\[
= \int_{a}^{b} \nabla V \cdot \hat{z} dz
\]

\[
W = -Q \left[ \int_{a}^{b} V(\vec{r}_b) - V(\vec{r}_a) \right] \text{ independent of path}
\]

\[
\frac{\partial V}{\partial z} = \frac{\partial V}{\partial z}
\]

\[
\text{Pe del} \text{ line crossed}
\]
\[
W = \frac{1}{Q} V(F_b) - V(F_a) = -\sum \mathbf{E} \cdot d\mathbf{l}
\]

Indepnd. Path ?

\[\mathbf{E} \times \nabla V = 0\]

\[\mathbf{E} \cdot d\mathbf{l} = 0\]

\[\mathbf{E} \cdot d\mathbf{l} = \mathbf{E} \times \nabla V \cdot d\mathbf{a}\]

\[\mathbf{E} = -\nabla V\]
Derive PDE for $V$

Avoids integration

Use another integral theorem

Based on Gauss Law

$$\int_S \mathbf{E} \cdot d\mathbf{a} = \frac{Q}{\varepsilon_0} \quad \text{(enclosed in S)}$$

Divergence Theorem

$$\int_V \nabla \cdot \mathbf{E} \, dV = \int_S \mathbf{E} \cdot d\mathbf{a} = \int_V \nabla \cdot \mathbf{p} \, dV = \frac{1}{\varepsilon_0} \int_S \mathbf{p} \cdot d\mathbf{a}$$

$$\int_V \mathbf{E} \cdot dV = \frac{1}{\varepsilon_0} \int_S \mathbf{E} \cdot d\mathbf{a}$$

$\nabla \cdot \mathbf{E} = \frac{\partial \mathbf{p}}{\partial t}$

$\int_V (\nabla \cdot \mathbf{E} - \mathbf{p}/\varepsilon_0) = 0$

$\nabla \cdot (\nabla V) \equiv \nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$

Laplacian

$\nabla^2 V = -\frac{\mathbf{p}}{\varepsilon_0}$

Poisson's

$\mathbf{E} = \frac{\mathbf{p}}{\varepsilon_0}$

$\mathbf{E} = -\nabla V$

$\nabla \cdot (\nabla V) \equiv \nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$

Laplacian

$\nabla^2 V = -\frac{\mathbf{p}}{\varepsilon_0}$

Poisson's
\[ \nabla^2 V = 0 \quad \text{Laplace's equation} \]

Example: In a region of space
\[ V = -\frac{E_0 \cdot x}{L} \]

What is \( V \)?
What is \( E \)?
What is special about \( x \) direction?

Example: Meaning of point charge
\[ V(\mathbf{r}) = \frac{q}{4\pi \varepsilon_0 r} \]

\[ \nabla^2 V = -\frac{\rho(\mathbf{r})}{\varepsilon_0} \quad \text{Poisson's eq} \]

\[ \nabla^2 \frac{1}{r} = -4\pi \delta(\mathbf{r}) \]

\[ \rho(\mathbf{r}) = \delta(\mathbf{r}) \cdot q \]

electron is a point charge
proton is not a point charge