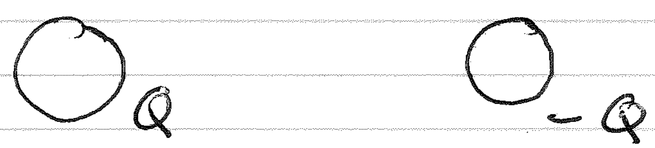
Last time - energy of electric field and then properties of conductors

Today we'll start by combining the two ideas: Conductors used in energy storage and also in circuits. Used a

Capacitor is 2 terminal electrical component that stores energy in $\vec{E}$-field

Any two conductors separated by insulator (or vacuum) form a capacitor



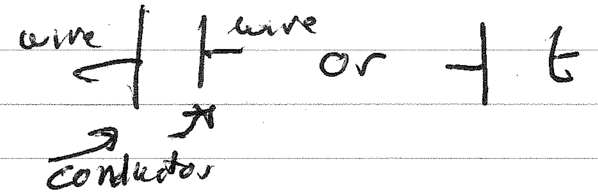
In practical applications, each conductor initially has 0 net charge and electrons transformed from one to the other.

Then two conductors have equal and opposite charge and total charge on entire system remains 0

Cap. with charge Q means one has $+Q$ other $-Q$

Capac. used as components of electric circuits. Block DC but allow AC

Capacitors are used in resonant circuits to stabilize power flow. In circuits they are written as



One way to charge capacitor is to connect the two wires to opposite terminals of battery. Once Q & $-Q$ established battery is disconnected this gives a pot'l difference between conductors

$$V_+ - V_- = -\int_{(-)}^{(+)} \vec{E}\cdot d\vec{l} \equiv V$$

V is proportional to Q, more charge means more potential diff

Charge of cap. both

$$C = Q/V = \text{Capacitance}$$

Units $[C] = \dfrac{\text{Coulomb}}{\text{Volt}} = \text{Farad}$ F

practical $\mu F = 10^{-6}\,F$

$pF = 10^{-12}\,F$

~~Fa~~ C is a geom quantity

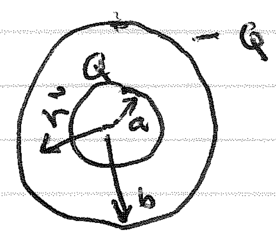
determined by sizes shape, separation

of conductors

Worthwhile to do example

Two concentric spherical metal

shells radii a, b



whats C

Step 1 - whats $\vec{E}$

Gauss Law Gaussian surface
Spherical symmetry

if $a < r < b$ $\quad \vec{E} = \dfrac{Q}{4\pi\epsilon_0} \dfrac{\hat{r}}{r^2}$

$$V = -\int_b^a \vec{E} \cdot d\vec{l} = +\int_a^b \vec{E} \cdot d\vec{l}$$ path independent

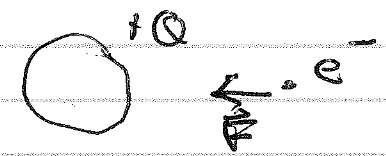
take straight line, $d\vec{l} = dr\,\hat{r}$

$$V = +\frac{Q}{4\pi\epsilon_0} \int_a^b \frac{dr}{r^2} = -\frac{Q}{4\pi\epsilon_0} \frac{1}{r}\Big|_a^b$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{Q}{4\pi\epsilon_0} \frac{b-a}{ab}$$

$$C = \frac{Q}{V} = 4\pi\epsilon_0 \frac{ab}{b-a}$$

How much energy is stored in a capacitor? To charge up cap remove electrons from one & carry them to the other. Do this means you fight against $\vec{E}$ which pulls electrons back



How much work to get to final charge q? Suppose at intermediate stage charge on positive plate is q, $V = q/C$

The work to transport next piece of charge dq, is $V\,dq$

$$dW = \frac{q}{C}\,dq$$

$$W = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2} C V^2$$

Next topic <u>Compute potentials</u>

Two basic equations

Poissons $\nabla^2 V = -\rho/\epsilon$

if $\rho = 0$ Laplace $\nabla^2 V = 0$ Harmonic function

$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, $\nabla^2 \equiv$ Laplacian

Now Laplace equation has one very easy solution $-$ $V = 0$

How can $V \neq 0$ when $\nabla^2 V = 0$

Charges $\rho \neq 0$

$\uparrow \rho = 0$ over here

Laplace eq enters in gravitation magnetism heat conduction, even quantum mechanics.

A version thats similar with similar solution techniques is Helmholtz equation $(\nabla^2 + k^2) V = 0$

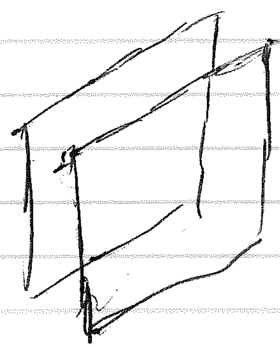
We'll follow & discuss

solutions of L. eq in 1, 2, 3 dimensions

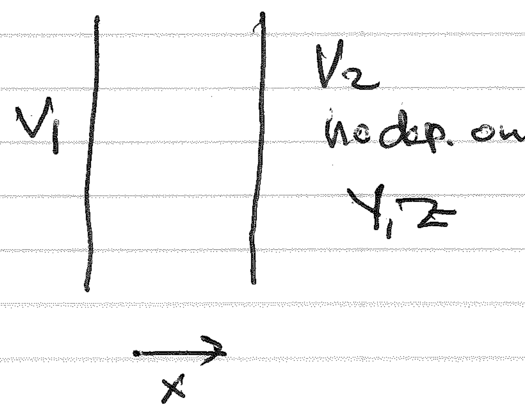
1, dimension - physics independent

of 2 dimensions - example

two very large conductors



side view



$$\nabla^2 V = 0 \Rightarrow \frac{d^2}{dx^2} V = 0$$

$$V = ax + b$$

2nd order diff eq. 2 constants

needed to specify potential

Physics is not - at $x = 2\,m$ $V(x) = 5V$

$x = 4m$ $V(x) = 9V$

What is a, b

$$V(4) - V(2) = a(4-2)m \Rightarrow 2m\,a = 4V$$

$$a = 2V/m \quad , \quad V(2m) = 5V = 2\frac{V}{m}(2) + b$$

$$b = 1V$$

2 apparent features of 1 Dim case are general

1) $V(x)$ is average of $V(x+c)$ & $V(x-c)$

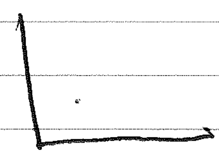
$$V(x) = \frac{1}{2}\left(V(x+c) + V(x-c)\right)$$

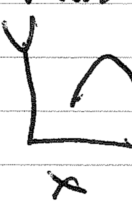
$$= \frac{1}{2}\left[a(x+c) + b + a(x-c) + b\right]$$

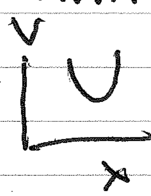
$\to 0$

$$= \frac{1}{2}(2ax + 2b) = ax + b$$

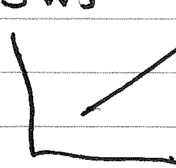
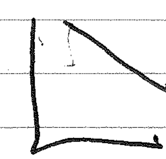
2) Solution of Laplace eq has no local max or min. Max or min at end point



Local max or min looks like 

or 

These are not solutions

Solution look like lines  

More generally reasoning if $V(x)$ were at a max it would be higher than at $x \pm c$ and could not be the average

Two dimensions

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

This is a PDE.

∞ many solutions

$$V = (mx+b)(ny+c)$$

or $e^{-\alpha y} \sin \alpha x$

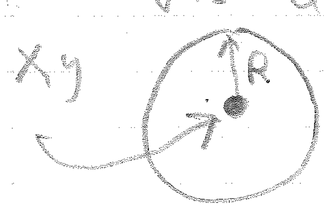
$e^{-\beta x} \sin \beta y$

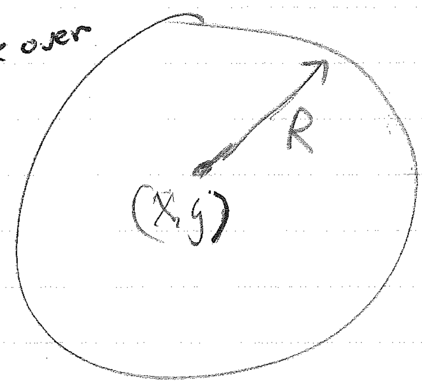
More guess a couple

Can't be discussed in as simple form as in 1 dim. case

Same two properties are true

1 V is an average over circle

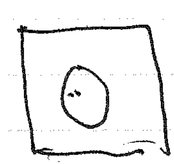




$$V(x,y) = \frac{1}{2\pi R} \oint_{\text{circle}} V \, dl$$

Indep R

This is an average over circle

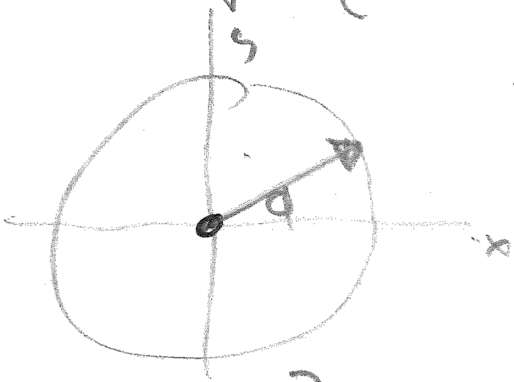


X=R
y=...

Example suppose

$V = (mx + a)(my + b)$

$V(0,0) = ab$



$x = R\cos\phi$

$y = R\sin\phi$

call point in question the origin

$$\frac{1}{2\pi R}\oint_0^{2\pi} R\,d\phi\,(mR\cos\phi + a)(mR\sin\phi + b)$$

$$= \frac{1}{2\pi R} R\int_0^{2\pi} d\phi\; ab = ab = V(0,0)$$

$V(0,0) = ab$

This average suggests a method for computer ~~simulation~~ solutions of L eq

Start with V on boundary, guess V for grid of interior points. Iteration first pass reassigns to each point the average of its nearest neighbors, 2nd pass repeats

Method of relaxation is name

Again V has no local max or min. If it did V could not be the average of nearby points

3 Dimensions

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

example $V = (mx+a)(ny+c)(lz+d)$

$$\sin \alpha x \sin \beta y \; e^{\pm\sqrt{\alpha^2+\beta^2}\, z}$$

again the property that V is average of nearby points holds