

Purpose of lecture

Explain material in Griffiths w different point of view Also

Math Material not in Griffiths on exams
Derivations will be shown - techniques needed to solve problems

allow you to ask questions

allow me to ask questions

often answer is 0

For A118 Thursday *:30

Demo

Subject: For A118 Thursday *:30
From: "Gerald A. Miller" <miller@uw.edu>
Date: 9/24/19, 5:55 PM
To: lectdemo@phys.washington.edu

Cenco 3-meter Transmitter (5N10.60) -- A small tube-style dipole transmitter sends radio waves to a pickup antenna, lighting a small bulb in the center of the antenna. Moving the antenna away from the transmitter dims the bulb, and rotating the antenna at right angles extinguishes the bulb, showing the polarized nature of the radio waves.

--

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This gives electromagnetic waves &

Modern technology - TV, microwave oven

energetical
Light from sun gives us life on earth

E & M is a classical theory

quantum version applies to atoms & electron

Very successful

Serves as a model & theories of new

forces

All from solving Maxwell Equs

You need the math to solve!

So lecture is review of Math in

Chapt 1 We are not kidding

This is review - expect you've seen
this before, but now use a lot

Gradient Operator

$\vec{\nabla}$ is differential operator

$$\vec{\nabla} = \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z}$$

in Cartesian coordinates

$\vec{\nabla}$ acts on a scalar function to give a vector

$$\vec{\nabla} f(x, y, z) = \hat{i} \frac{\partial f}{\partial x} + \hat{j} \frac{\partial f}{\partial y} + \hat{k} \frac{\partial f}{\partial z}$$

Griffiths writes

$$\begin{aligned} \hat{i} &\Rightarrow \hat{x} \\ \hat{j} &\Rightarrow \hat{y} \\ \hat{k} &\Rightarrow \hat{z} \end{aligned}$$

Geom interpretation:

consider how f changes with position Taylor series

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz$$

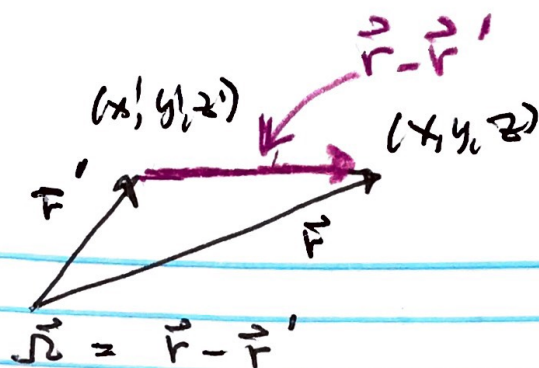
$$= \vec{\nabla} f \cdot d\vec{x}$$

$$= |\vec{\nabla} f| |d\vec{x}| \cos \theta$$

← angle between

$\vec{\nabla} f \parallel d\vec{x}$ The dot product maxes if $\cos \theta = 1, \theta = 0$
 so $\vec{\nabla} f$ gives direction of max change of function
 $|\vec{\nabla} f|$ gives magnitude

Hill example - find direction of steepest ascent this is direction of $\vec{\nabla} f$ where f is height of hill



Determine $\vec{\nabla} \Omega^2$

$$\Omega^2 = r^2 - 2\vec{r} \cdot \vec{r}' + r'^2$$

physics is not
a spectator sport
write Ω^2 in terms
of $\vec{r}, \vec{r}'$

$$\vec{\nabla} r^2 = \nabla (x^2 + y^2 + z^2) = 2x\hat{i} + 2y\hat{j} + 2z\hat{k} = 2\vec{r}$$

$$\begin{aligned}\vec{\nabla} (-2\vec{r} \cdot \vec{r}') &= -2\vec{\nabla} (xx' + yy' + zz') \\ &= -2x'\hat{i} - 2y'\hat{j} - 2z'\hat{k} \\ &= -2\vec{r}'\end{aligned}$$

Thus

$$\vec{\nabla} (\Omega^2) = 2(\vec{r} - \vec{r}')$$

Vectors have dot products and cross products

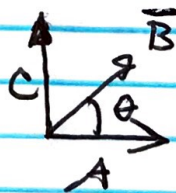
dot product $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

$$= |\vec{A}| |\vec{B}| \cos \theta$$



cross product

$$\vec{A} \times \vec{B} = \vec{C}$$



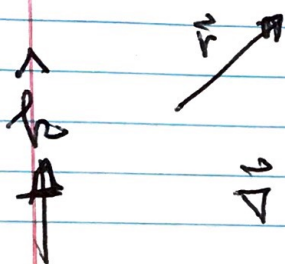
$$|\vec{C}| = |\vec{A}| |\vec{B}| \sin \theta$$

direction of $\vec{C}$ is $\perp$ to A & B in right hand sense

$\vec{\nabla}$ is a vector and it can have dot products
a cross product

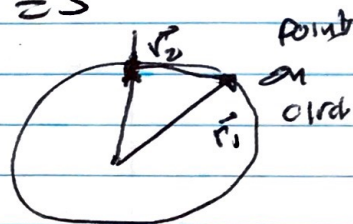
$\vec{\nabla} \cdot \vec{A}$ is divergence of vector $\vec{A}(x, y, z)$
(3 functions)
 $= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = \text{a scalar}$

$\vec{\nabla} \cdot \vec{A}$ is a measure of how a vector
spreads out from a given point



$$\vec{\nabla} \cdot \vec{r} = \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z} = 3$$

$$\vec{\nabla} \cdot \hat{k} = \frac{\partial}{\partial z} 1 = 0$$



$\vec{\nabla} \times \vec{A}$ is cross product = a vector

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \hat{i} \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \hat{j} \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \hat{k} \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right]$$

$\vec{\nabla} \times \vec{A}$ is measure of how much the vector $\vec{A}$ swirls about a given point

Physics is not - compute

$$\vec{\nabla} \times \vec{r} = \hat{i} \left(\frac{\partial z}{\partial y} - \frac{\partial y}{\partial z} \right) + \hat{j} \left(\frac{\partial x}{\partial z} - \frac{\partial z}{\partial x} \right) + \hat{k} \left(\frac{\partial y}{\partial x} - \frac{\partial x}{\partial y} \right) = 0 + 0 + 0 = 0$$

Compact way of writing $\vec{\nabla} \times$

ϵ_{ijk} = Levi-Civita tensor

$\epsilon_{123} = 1$ all even permutations = 1
all odd permutations = -1

$$\epsilon_{312} = 1$$

$$\epsilon_{213} = -1 \quad \epsilon_{231} = -1$$

$$\epsilon_{231} = 1$$

$$\epsilon_{321} = -1$$

if any 2 indices are the same = 0

$$A_x = A_1 \quad A_y = A_2 \quad A_z = A_3$$

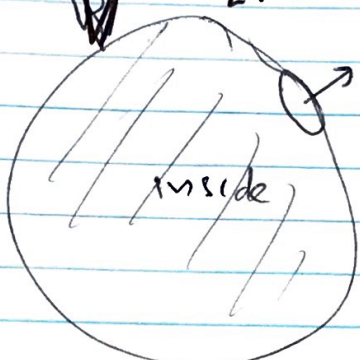
$$\text{Then } (\vec{\nabla} \times \vec{A})_i = \sum_{j,k} \epsilon_{ijk} \frac{\partial A_k}{\partial x_j} = \epsilon_{ijk} \frac{\partial A_k}{\partial x_j}$$

More

Quick review of chapter 1
You've had this in math courses

$\vec{\nabla}$ Vector

Theorems

$$\int_{\text{LHS}} (\vec{\nabla} \cdot \vec{v}) d\tau = \int_{\text{RHS}} \vec{v} \cdot d\vec{a}$$


$d\vec{a} = da \hat{n}$ $\hat{n} \perp$ to surface outwards

Example: Sphere of radius a
 $\vec{v} = \vec{r}$

Compute LHS & RHS

ask $\vec{\nabla} \cdot \vec{v} = \frac{\partial}{\partial x} x + \frac{\partial}{\partial y} y + \frac{\partial}{\partial z} z = 3$

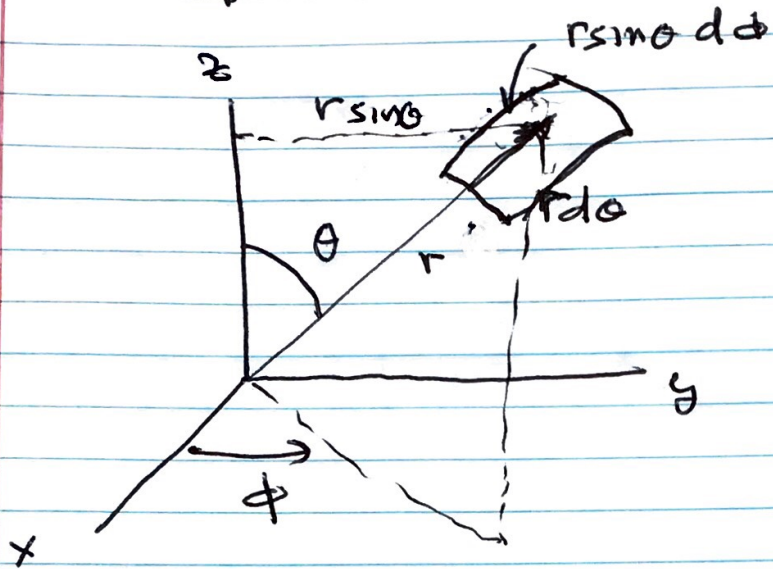
Volume $\text{LHS} = 3 \left(\frac{4\pi a^3}{3} \right) = 4\pi a^3 = 3 \text{ volume} = 3 \left(\frac{4\pi a^3}{3} \right) = 4\pi a^3$

RHS $\vec{v} \cdot d\vec{a} = r da$

$$\int \vec{v} \cdot d\vec{a} = \int_a r da = a (4\pi a^2)$$

where does 4π come from? $da = r^2 d\Omega = r^2 \sin\theta d\theta d\phi$ (r mode) $\times$ (r dphi)

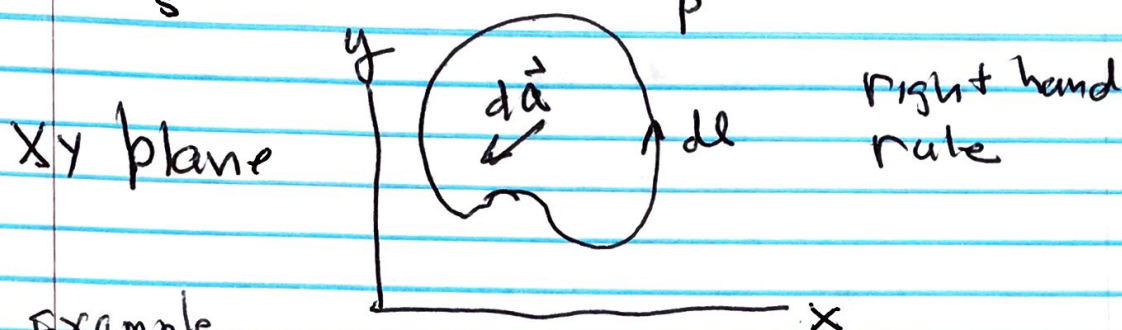
Spherical coordinates



$$\begin{aligned} dA &= r \sin \theta d\theta r d\phi \\ &= r^2 \sin \theta d\theta d\phi \\ &\equiv r^2 d\Omega \end{aligned}$$

Stokes theorem

$$\int_S (\nabla \times \vec{v}) \cdot d\vec{a} = \oint_P \vec{v} \cdot d\vec{l}$$



Example

Given $\vec{v} = 4y\hat{i} + x\hat{j} + 2z\hat{k}$

Compute $\int_S (\nabla \times \vec{v}) \cdot d\vec{a}$

when S is a hemisphere

(like HW problem)

$$x^2 + y^2 + z^2 = R^2, \quad z \geq 0$$



$$d\vec{a} = \hat{r} d\Omega$$

$$\begin{aligned} \nabla \times \vec{v} &= \hat{i} \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \right) \\ &+ \hat{j} \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \right) \\ &+ \hat{k} \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) \end{aligned}$$

$$= -3\hat{k}$$

3 ways to do problem

a) evaluate expression

$$\int_S -3\hat{h} \cdot d\vec{a} = -3 \int \hat{h} \cdot \hat{r} da$$

$$= -3 \int_0^{2\pi} \cos\theta R^2 \sin\theta d\phi \int_0^{\pi/2}$$

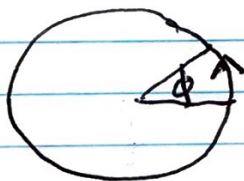
$$= -3 R^2 \int_0^{2\pi} d\phi \int_0^{\pi/2} \sin\theta \cos\theta d\theta$$

$$= -3 R^2 (2\pi) \left[\frac{\sin^2\theta}{2} \right]_0^{\pi/2}$$

$$= -3 R^2 \pi$$

b) Stokes theorem

$d\vec{l}$ is circle in plane $z=0$



$$\vec{v} \cdot d\vec{l} = v_x dx + v_y dy$$

$$= \oint \vec{v} \cdot d\vec{l}$$

$$d\vec{l} = R [-\sin\phi \hat{i} + \cos\phi \hat{j}]$$

$$= -y \hat{i} + x \hat{j}$$

$$\vec{v} \cdot d\vec{l} = -4y^2 + x^2 = R^2 (-4\sin^2\phi + \cos^2\phi)$$

$$\int_0^{2\pi} \sin^2\phi d\phi = \int_0^{2\pi} \cos^2\phi d\phi = \pi \quad \left[\int_0^{2\pi} \sin^2\phi d\phi = \int_0^{2\pi} \cos^2\phi d\phi = \pi \right] \text{ So } \oint \vec{v} \cdot d\vec{l} = R^2 \pi (-4 + 1) = -3\pi R^2$$

c) Choose plane area - in plane
Stoke's theorem



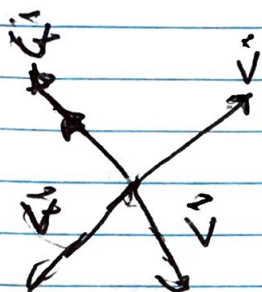
$$d\vec{a} = \hat{n} da$$

$$(\vec{\nabla} \times \vec{v}) \cdot d\vec{a} = -3 da$$

$$\oint (\vec{\nabla} \times \vec{v}) \cdot d\vec{a} = -3 \int da = -3\pi R^2$$

Dirac Delta function

Suppose $\vec{V} = \frac{1}{r^2} \hat{r} = \frac{\vec{r}}{r^3}$



$\vec{\nabla} \cdot \vec{V}$ using

$\vec{\nabla}$ in spherical

$$\vec{\nabla} \cdot \vec{V} = \frac{1}{r^2} \frac{d}{dr} (r^2 v_r) + \dots$$

Doesn't look like $\vec{0} = 0$ Very strange!

Use $\vec{\nabla} \cdot$ theorem

$$\oint \vec{v} \cdot d\vec{a} = \int_{\text{Volume}} \vec{\nabla} \cdot \vec{v} d\tau$$

sphere

radius R

$$= \int \frac{\vec{r}}{r^2} \cdot \hat{r} r^2 d\Omega = 4\pi \quad \text{Not zero}$$

not zero - we are supposed to

have equation of volume integral of $\vec{\nabla} \cdot \vec{v}$ with $\vec{v}$ surface integral

Problem $\vec{v} = \frac{1}{r^2} \hat{r}$
infinity at $r=0$ -

goes to
not allowed to divide
by zero

The origin is a singular point
not allowed to divide by zero
 $\vec{\nabla} \cdot \vec{u} = 0$ except at origin.

Have function that $= 0$ everywhere
except at one point

The function is infinite at that
point in such a way that integral is
well defined

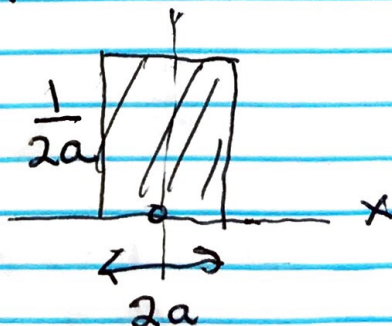
Need to formalize this

1 Dimensional Dirac delta function

$$\delta(x) = \begin{cases} 0 & \text{if } x \neq 0 \\ \infty & \text{if } x = 0 \end{cases}$$

$$\int_{-\infty}^{\infty} dx \delta(x) = 1$$

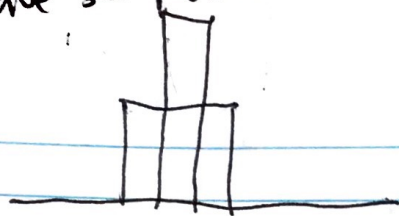
$\delta(x)$



$$\text{area} = \frac{1}{2a} \cdot 2a = 1$$

Then let $a \rightarrow 0$

have sequence



etc

Can put ^{singular} point anywhere on axis

$$\delta(x-a) = \begin{cases} 0 & x \neq a \\ \infty & x = a \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(x-a) dx = 1$$

$$\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = f(a) \int_{-\infty}^{\infty} \delta(x-a) dx = f(a)$$

assumes $f(a)$ is well defined

3-dimensional delta function

$$\delta(\vec{r}) = \delta(x) \delta(y) \delta(z)$$

$$\int_{\text{all space}} \delta(\vec{r}) d\vec{r} = 1$$

$$\int_{\text{all space}} f(\vec{r}) \delta(\vec{r}-\vec{a}) = f(\vec{a})$$

The divergence of $\vec{U} = \frac{\hat{r}}{r^2}$ is proportional to delta function

$$\vec{\nabla} \cdot \vec{U} = 4\pi \delta(\vec{r})$$

Then all is ok

$$\vec{\nabla} \cdot (\hat{r}/r^2) = 4\pi \delta(\vec{r})$$

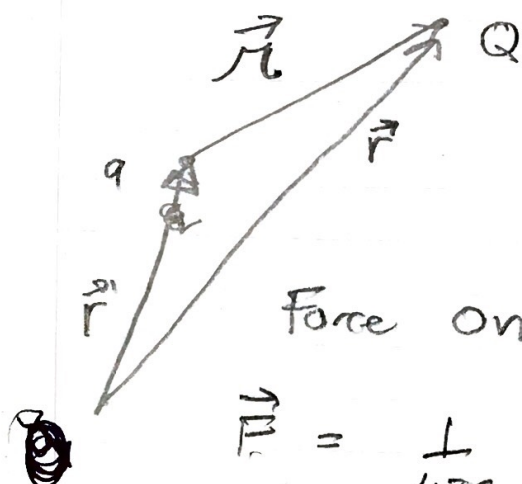
note $\vec{\nabla} \frac{1}{r} = -\frac{1}{r^2} \hat{r}$

so we have $\vec{\nabla} \cdot \vec{\nabla} \frac{1}{r} = -4\pi \delta(\vec{r})$

$$\boxed{\nabla^2 \frac{1}{r} = -4\pi \delta(\vec{r})}$$

Poisson's eq for point particle

Coulomb's Law



Force on Q due to charge q

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \hat{r}$$

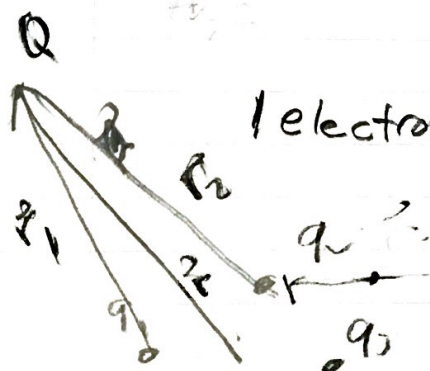
$$\vec{r} - \vec{r}' = \vec{r}$$

Griffiths

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{qQ}{|\vec{r} - \vec{r}'|^2} \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$$

$$\vec{r} \propto \vec{r} - \vec{r}'$$



$$1 \text{ electron} = -1.6 \times 10^{-19} C$$

which is
 $\vec{r}$

Superposition
Force on Q

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots$$

$$= \frac{1}{4\pi\epsilon_0} Q \left[\frac{q_1}{|\vec{r} - \vec{r}_1|^3} (\vec{r} - \vec{r}_1) + \frac{q_2}{|\vec{r} - \vec{r}_2|^3} (\vec{r} - \vec{r}_2) + \dots \right]$$

$$\vec{F} = Q\vec{E}$$

Property of charge distribution

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{|\vec{r} - \vec{r}_i|^2} (\vec{r} - \vec{r}_i)$$

$\vec{E}$ = Force per charge

— what's field fills space —

We shall want to calculate $\vec{E}$

For a variety of distribution of charge
Example what's $\vec{E}$? — ask students



Each field has magnitude E
 But need vectors sum

$$\begin{aligned} E_x &= 0 \\ E_y &= 2E \cos \alpha = E \\ &= 2E \cos 45^\circ = E = 0 \end{aligned}$$

E at a point ^{above L} above one of the charges

Comp. $\perp$ axis = 0
Continuous charge distribution.

Many charges.

$$\sum q_i \rightarrow \int dq$$

$$\begin{aligned} \vec{r}_i &\rightarrow \vec{r}' \\ q_i &\rightarrow dq'_{\text{cont}} \end{aligned}$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{dq'}{|\vec{r} - \vec{r}'|^2} (\vec{r} - \vec{r}')$$

$\vec{r}'$ is pos. of charge

3

3 integrals

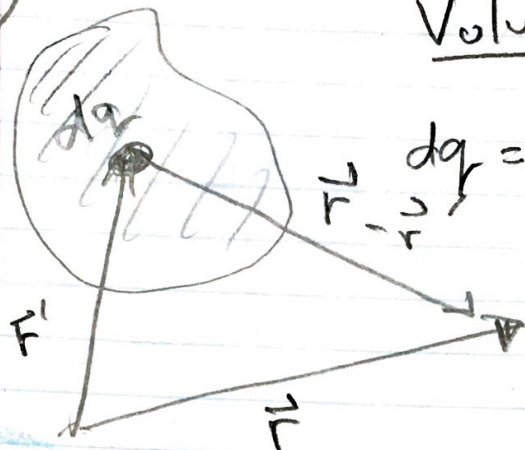
today 2.1, 3, 2.14, 2.2

Frid 2.2, 2.3

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_{\text{Volume}} dq \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} = \frac{1}{4\pi\epsilon_0} \int dq \frac{\hat{r}}{r^2}$$

Volume

Localized



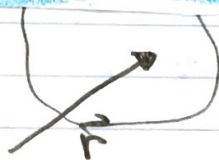
$$dq = \rho(\vec{r}') d^3r' = \rho(\vec{r}') d\tau'$$

Griffiths

$$\vec{E}(\vec{r}) = \frac{1}{4\pi} \int \rho(\vec{r}') \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3} d\tau'$$

In integral

What happens when $\vec{r} = \vec{r}' \rightarrow \infty$?



so that

$\vec{r}' = \vec{r}$ at one point

Choose $\vec{r}$ in dist. at choose origin to coincide with $\vec{r}'$

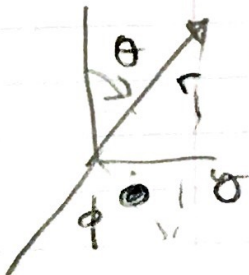
$$\vec{E}(\vec{r}=0) = \frac{1}{4\pi\epsilon_0} \int d^3r' \rho(\vec{r}') \frac{(-\vec{r}')}{|\vec{r}'|^3}$$

look at math at $r'=0$

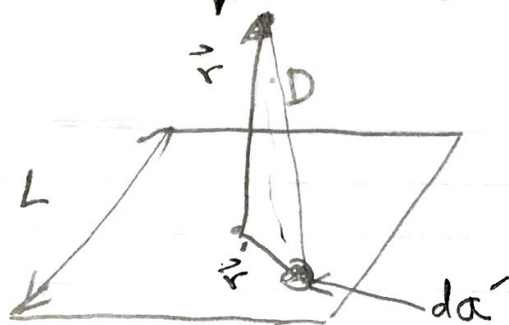
$$d^3r' = r'^2 dr' \sin\theta' d\theta' d\phi' = r'^2 dr' d\Omega'$$

$$\vec{E}(\vec{r}=0) = \frac{1}{4\pi\epsilon_0} \int r' d\Omega' \frac{(-\vec{r}')}{|\vec{r}'|^3} \rho(\vec{r}')$$

3 powers of r' in numerator



Surface charge distribution on a square $dq = \sigma da$
~~evenly~~ uniform
 $\sigma = Q/L^2$



$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \sigma da' \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$

Ask

Value of $\vec{E}$ Does $\vec{E}$ have horizontal component?

how Setup integral $\vec{r} = (0, 0, z)$

$$dq = \sigma da'$$

$$\vec{E}(\vec{r}) = \frac{\sigma}{4\pi\epsilon_0} \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_{-\frac{L}{2}}^{\frac{L}{2}} dx' dy' \frac{[-x'\hat{i} - y'\hat{j} + z\hat{k}]}{[x'^2 + y'^2 + z^2]^{3/2}}$$

What is x component of $\vec{E}$?

Why?

What is y component of $\vec{E}$?

$$\vec{E}(\vec{r}) = \frac{\sigma}{4\pi\epsilon_0} \hat{k} z \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_{-\frac{L}{2}}^{\frac{L}{2}} dx' dy' \frac{1}{[x'^2 + y'^2 + z^2]^{3/2}}$$

Integrate to get answer - math or table

Let's get do limits what happens as $z \rightarrow \infty$

Should get $\vec{E}$ of point charge

$z \rightarrow \infty$ $z^2 \gg x'^2 + y'^2$ always
larger w $2L^2$

$$\vec{E}(\vec{r}) = \frac{\sigma}{4\pi\epsilon_0} \frac{1}{z^3} \int_{-L}^L \int_{-L}^L \frac{\vec{r}}{r^3} dx' dy' \quad L^2 = \frac{Q}{4\pi\epsilon_0} \frac{1}{z^2}$$

Suppose position below plane
what would result? look at

What about $z \rightarrow 0$?

From
122

Field at infinity, plane = $\vec{E}(\vec{r}) = \frac{1}{2} \frac{\sigma}{\epsilon_0}$

How to get what went wrong

$x'^2 + y'^2 \rightarrow 0$ then integrand $\approx \frac{1}{0}$

Can $\sim \frac{1}{0^3}$ if take $z \rightarrow 0$ too early
Integrate on y' $\frac{1}{0^3}$ integrate on y' $\frac{1}{0^3}$

$$\vec{E}(\vec{r}) = \frac{\sigma}{4\pi\epsilon_0} \frac{1}{z} \int_{-L}^L dx' \left[\frac{y'}{x'^2 + z^2} \frac{1}{\sqrt{x'^2 + y'^2 + z^2}} \right]_{y'=-\frac{L}{2}}^{y'=\frac{L}{2}}$$

$$= \frac{\sigma}{4\pi\epsilon_0} \frac{1}{z} \int_{-L}^L dx' \frac{L}{(x'^2 + z^2) \sqrt{x'^2 + z^2 + L^2/4}}$$

$$x' = z \tan \theta$$

$$dx' = z d\theta \cos^2 \theta \quad \text{OK OK}$$

$$\vec{E}(\vec{r}) = \frac{\sigma}{4\pi\epsilon_0} \frac{1}{z} \int_{-\pi/2}^{\pi/2} d\theta \frac{L}{\cos^2 \theta z^2 (1 + \tan^2 \theta) \sqrt{\frac{L^2}{4} + z^2 (1 + \tan^2 \theta)}}$$

$$= \frac{\sigma}{4\pi\epsilon_0} \frac{1}{z} \int_{-\pi/2}^{\pi/2} d\theta \frac{L}{\sqrt{L^2 + 4z^2}}$$

$$\frac{\sigma}{4\pi\epsilon_0} \oint \hat{n} \cdot \nabla \tan^{-1} \frac{X'}{Z} \Big|_{-L/2}^{L/2}$$

$$\frac{\sigma}{4\pi\epsilon_0} \hat{n} \left[\frac{\pi}{2} - \left(-\frac{\pi}{2} \right) \right] = \frac{\sigma}{2\epsilon_0} \hat{n}$$

Color chalk

SKIP

Went out by

Computing E

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{d\vec{q}'}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}')$$

$$d\vec{q}' = \rho(\vec{r}') d^3r' \quad \text{Volume}$$

Why do I ask - always gotta do

2. Charge on plane

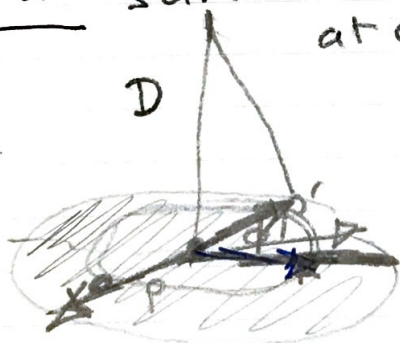
$$d\vec{q}' = \sigma(\vec{r}') d\vec{a}'$$

$$3 \text{ or line } d\vec{q}' = \lambda(\vec{r}') d\vec{l}'$$

Example

Take $\vec{E}$
 $\sigma = \text{constant}$
 > 0

at center
a bone a
surface charge dist over o.
at center



$$d\vec{a}' = \hat{r} dr' d\phi$$

$$\vec{E} = \frac{\hat{z}}{4\pi\epsilon_0} \int_0^R \int_0^{2\pi} \frac{r' dr' d\phi}{|\vec{r} - \vec{r}'|^3} \sigma(r')$$

Today 2.1.3, 2.1.4

22

Friday 2.2, 2.3

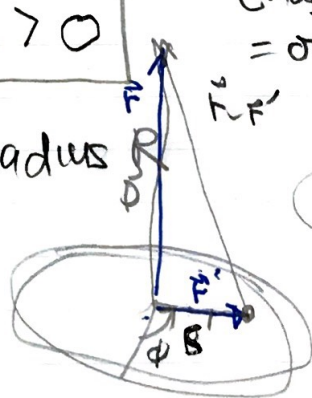
Physicists

Soker

Yes $\vec{E}$ have horizontal component

$D > 0$

Radius R



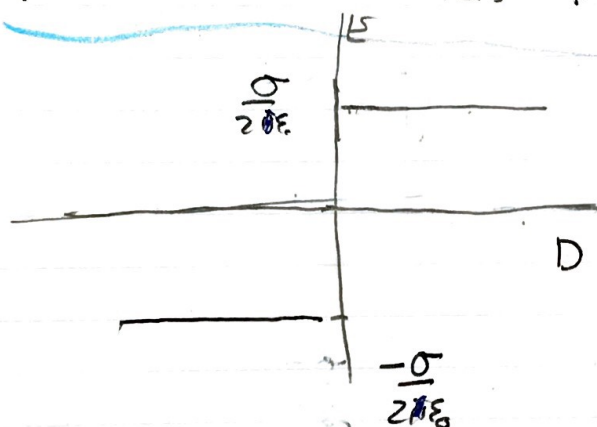
Start charge per
 $= \sigma = \rho_{\text{surf}}$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{R^2} \left[\frac{1}{D} - \frac{1}{D} + \frac{R^2}{2D^3} \right]$$

$$= \frac{1}{4\pi\epsilon_0} \frac{Q}{D^2} \quad \text{as expected}$$

We are doing the electrostatics - just do integrals
 Integrals - need set of tricks

Do $D \rightarrow 0$ means $D \ll R$



what about at 0 - $\vec{E} = 0$

What the point inside



Suppose off center



top view



How to find

Stu's Guess

2.3

8

Other ways to get $\vec{E}$

Consider a

measure of $\vec{E}$

$$\Phi_E = \int_S \vec{E} \cdot d\vec{a}$$

measures strength & direction of $\vec{E}$

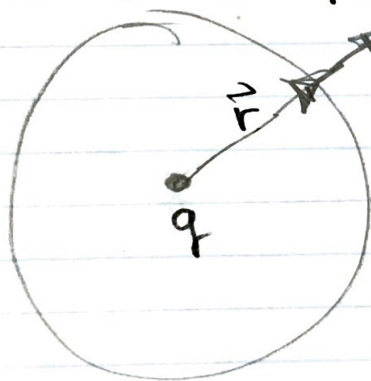
what does this mean

$$d\vec{a} = \hat{n} da$$

choose a 2 in general

Special $\vec{E} \cdot d\vec{a} = \vec{E} \cdot \hat{n} da$

Example q at center of sphere radius r



$$\oint d\vec{a} \cdot \vec{E}$$

closed
surface

For closed
surface

$\hat{n}$ points out

here $\hat{n} = \hat{r}$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} q \frac{\vec{r}}{r^3}$$

$$\oint \hat{r} \cdot d\vec{a} = \frac{1}{4\pi\epsilon_0} q \frac{\vec{r} \cdot \vec{r}}{r^3} = \frac{q}{4\pi\epsilon_0} \frac{1}{r^2} da$$

What coords Showed I do

$$da = r^2 \sin\theta d\theta d\phi$$

$$= r^2 d\Omega$$

$$d\Omega = \frac{da}{r^2}$$

$$\oint \vec{E} \cdot \hat{r} da = \frac{q}{4\pi\epsilon_0} \frac{1}{r^2} \int r^2 d\Omega$$

$$= \frac{q}{\epsilon_0}$$

Suppose q is not at center or
Surface not sphere

or q is not
Same answer?