

Formulating Linear Programming Models

Some Examples:

- Product Mix (Session #2)
- Cash Flow (Session #3)
- Diet / Blending
- Scheduling
- Transportation / Distribution
- Assignment

Steps for Developing an Algebraic LP Model

1. What decisions need to be made? Define each *decision variable*.
2. What is the goal of the problem? Write down the *objective function* as a function of the decision variables.
3. What resources are in short supply and/or what requirements must be met? Formulate the *constraints* as functions of the decision variables.

Steps for Developing an LP Model in a Spreadsheet

1. Enter all of the data for the model. Make consistent use of rows and columns.
2. Make a cell for each decision to be made (changing cells). Follow the same structure as the data. (Sometimes it is easier to do step 2 before step 1.)
3. What is the goal of the problem? Enter the equation that measures the objective in a single cell on the worksheet (target cell). Typically:

SUMPRODUCT (Cost Data, Changing Cells)

or

SUMPRODUCT (Profit Data, Changing Cells)

4. What resources are in short supply and/or what requirements must be met? Enter all constraints in three consecutive cells on the spreadsheet, e.g.,

amount of resource used , \leq , amount of resource available

or
amount attained , \geq , minimum requirement

LP Example #1 (Diet Problem)

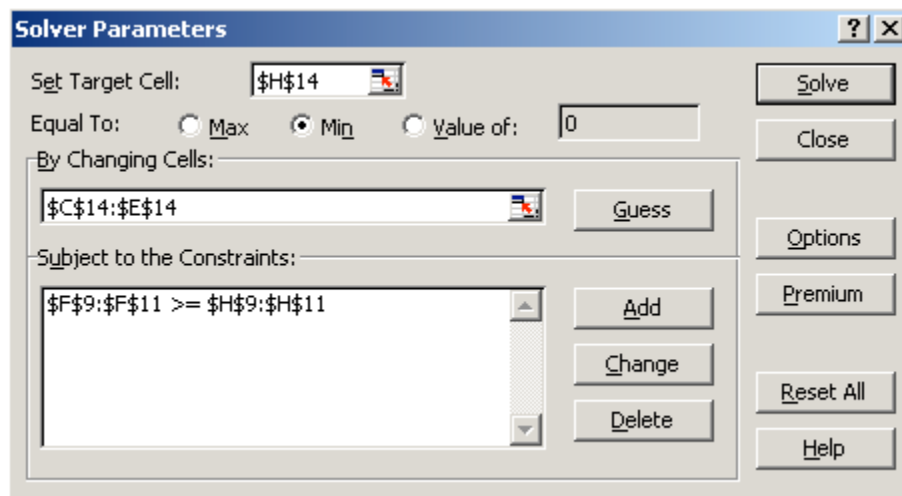
A prison is trying to decide what to feed its prisoners. They would like to offer some combination of milk, beans, and oranges. Their goal is to minimize cost, subject to meeting the minimum nutritional requirements imposed by law. The cost and nutritional content of each food, along with the minimum nutritional requirements are shown below.

	Milk (gallons)	Navy Beans (cups)	Oranges (large Calif. Valencia)	Minimum Daily Requirement
Niacin (mg)	3.2	4.9	0.8	13.0
Thiamin (mg)	1.12	1.3	0.19	1.5
Vitamin C (mg)	32.0	0.0	93.0	45.0
Cost (\$)	2.00	0.20	0.25	

A Spreadsheet Model for LP Example #1

	A	B	C	D	E	F	G	H
1	The Prison Diet Problem							
2								
3			Milk	Beans				
4			(gal.)	(cups)	Oranges			
5		Cost	\$2.00	\$0.20	\$0.25			
6								
7								Minimum
8			Nutritional Contents (mg)			Total		Requirement
9		Niacin (mg)	3.2	4.9	0.8	13	>=	13
10		Thiamin (mg)	1.12	1.3	0.19	3.438	>=	1.5
11		Vitamin C (mg)	32	0	93	45	>=	45
12								
13								Total Cost
14		Quantity	0	2.574	0.484			\$0.64
15		(per prisoner)						

	B	C	D	E	F	G	H
3		Milk	Beans				
4		(gal.)	(cups)	Oranges			
5		Cost	2	0.2	0.25		
6							
7							Minimum
8			Nutritional Content		Total		Requirement
9		Niacin (mg)	3.2	4.9	0.8	=SUMPRODUCT(C9:E9,\$C\$14:\$E\$14)	>= 13
10		Thiamin (mg)	1.12	1.3	0.19	=SUMPRODUCT(C10:E10,\$C\$14:\$E\$14)	>= 1.5
11		Vitamin C (mg)	32	0	93	=SUMPRODUCT(C11:E11,\$C\$14:\$E\$14)	>= 45
12							
13							Total Cost
14		Quantity	0	2.5740618	0.4838709		=SUMPRODUCT(C5:E5,C14:E14)
15		(per prisoner)					



Diet/Menu Planning Model in Practice

George Dantzig's Diet

- Stigler (1945) "The Cost of Subsistence"
- Dantzig invents the simplex method (1947)
- Stigler's problem "solved" in 120 man days (1947)
- Dantzig goes on a diet (early 1950's), applies diet model:
 - ≤ 1500 calories
 - objective: maximize (weight minus water content)
 - 500 foods
- Initial solutions had problems (500 gallons of vinegar, 200 bouillon cubes)

(for more details, see July-Aug 1990 *Interfaces* article "The Diet Problem", available for download at <http://www.interfaces.smeal.psu.edu> under "Published Issues")

Least-Cost Menu Planning Models in Food Systems Management

- Used in many institutions with feeding programs: hospitals, nursing homes, schools, prisons, etc.
- Menu planning often extends to a sequence of meals or cycle
- Variety important (separation constraints)
- Preference ratings (related to service frequency)
- Side constraints (color, categories, etc.)
- Generally models have reduced cost about 10%, meet nutritional requirements better, and increased customer satisfaction compared to traditional methods
- USDA uses these models to plan food stamp allotment

(for more details, see Sept-Oct 1992 *Interfaces* article "The Evolution of the Diet Model in Managing Food Systems", available for download at <http://www.interfaces.smeal.psu.edu> under "Published Issues")

LP Example #2 (Scheduling Problem)

Time Period	Number of Officers Needed
12 a.m. - 4 a.m.	5
4 a.m. - 8 a.m.	7
8 a.m. - 12 p.m.	15
12 p.m. - 4 p.m.	8
4 p.m. - 8 p.m.	12
8 p.m. - 12 a.m.	9

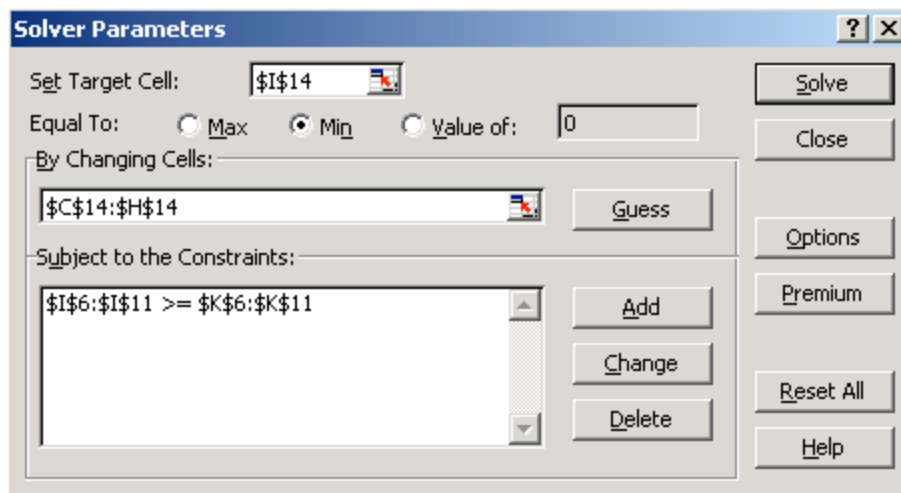
Union contract requires all employees to work 8 consecutive hours.

Goal: Hire the minimum number of officers needed to cover all shifts.

A Spreadsheet Model for LP Example #2

	A	B	C	D	E	F	G	H	I	J	K	
1	Security Force Scheduling Problem											
2												
3		Shift Works Time Period?										
4		Shift 1	Shift 2	Shift 3	Shift 4	Shift 5	Shift 6	Number		Minimum		
5		Time Period	12a-8a	4a-12p	8a-4p	12p-8p	4p-12a	8p-4a	Working		Required	
6		12am – 4am	1					1	5	>=	5	
7		4am – 8am	1	1					12	>=	7	
8		8am – 12pm		1	1				15	>=	15	
9		12pm – 4pm			1	1			8	>=	8	
10		4pm – 8pm				1	1		12	>=	12	
11		8pm – 12am					1	1	12	>=	9	
12												
13									Total			
14		Number Working	5	7	8	0	12	0	32			

	B	C	D	E	F	G	H	I	J	K	
3		Shift Works Time Period?									
4		Shift 1	Shift 2	Shift 3	Shift 4	Shift 5	Shift 6	Number		Minimum	
5		Time Period	12a-8a	4a-12p	8a-4p	12p-8p	4p-12a	8p-4a	Working		Required
6		12am – 4am	1					1	=SUMPRODUCT(C6:H6,\$C\$14:\$H\$14)	>=	5
7		4am – 8am	1	1					=SUMPRODUCT(C7:H7,\$C\$14:\$H\$14)	>=	7
8		8am – 12pm		1	1				=SUMPRODUCT(C8:H8,\$C\$14:\$H\$14)	>=	15
9		12pm – 4pm			1	1			=SUMPRODUCT(C9:H9,\$C\$14:\$H\$14)	>=	8
10		4pm – 8pm				1	1		=SUMPRODUCT(C10:H10,\$C\$14:\$H\$14)	>=	12
11		8pm – 12am					1	1	=SUMPRODUCT(C11:H11,\$C\$14:\$H\$14)	>=	9
12											
13									Total		
14		Number Working	5	7	8	0	12	0	=SUM(C14:H14)		



Workforce Scheduling Model in Practice

United Airlines in the 1980's

- Employ 5000 reservation and customer service agents
- Some part time (2-8 hr shifts), some full time (8-10 hour shifts)
- Workload varies greatly over day
- Modeled problem as LP:
 - Decision variables: how many employees of each shift length should begin at each potential start time (half-hour intervals).
 - Constraints: minimum required employees for each half-hour
 - Objective: minimize cost
- Saved UA about \$6 million annually, improved customer service, still in use today.

(for more details, see Jan-Feb 1986 *Interfaces* article "United Airlines Station Manpower Planning System", available for download at <http://www.interfaces.smeal.psu.edu> under "Published Issues")

LP Example #3 (Transportation Problem)

A company has two plants producing a certain product that is to be shipped to three distribution centers. The unit production costs are the same at the two plants, and the shipping cost per unit is shown below.

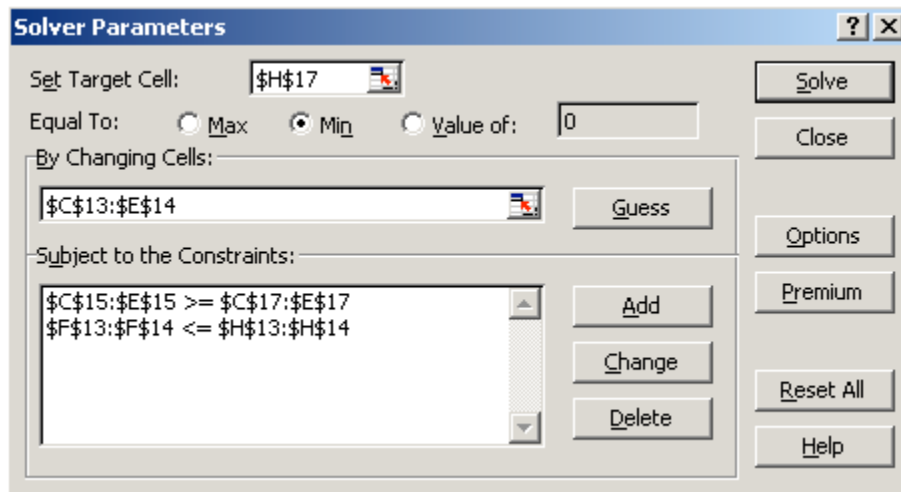
		Distribution Center		
		1	2	3
Plant	A	\$4	\$6	\$4
	B	\$6	\$5	\$2

Shipments are made once per week. During each week, each plant produces at most 60 units and each distribution center needs at least 40 units. How many units should be shipped from each plant to each distribution center, so as to minimize cost?

Spreadsheet Model for LP Example #3

	A	B	C	D	E	F	G	H
1	A Distribution Problem							
2								
3			Distribution	Distribution	Distribution			
4		Cost	Center 1	Center 2	Center 3			
5		Plant A	\$4	\$6	\$4			
6		Plant B	\$6	\$5	\$2			
7								
8								
9								
10								
11		Shipment	Distribution	Distribution	Distribution			
12		Quantities	Center 1	Center 2	Center 3	Shipped		Available
13		Plant A	40	20	0	60	<=	60
14		Plant B	0	20	40	60	<=	60
15		Shipped	40	40	40			
16			>=	>=	>=			Total Cost
17		Needed	40	40	40			\$460

	B	C	D	E	F	G	H
3		Distribution	Distribution	Distribution			
4	Cost	Center 1	Center 2	Center 3			
5	Plant A	4	6	4			
6	Plant B	6	5	2			
7							
8							
9							
10							
11	Shipment	Distribution	Distribution	Distribution			
12	Quantities	Center 1	Center 2	Center 3	Shipped		Available
13	Plant A	40	20	0	=SUM(C13:E13)	<=	60
14	Plant B	0	20	40	=SUM(C14:E14)	<=	60
15	Shipped	=SUM(C13:C14)	=SUM(D13:D14)	=SUM(E13:E14)			
16		>=	>=	>=			Total Cost
17	Needed	40	40	40			=SUMPRODUCT(C5:E6,C13:E14)



Designing a Distribution System for Entire Product Line (Proctor and Gamble)

Proctor and Gamble in early 90's

- Needed to consolidate and re-design their North American distribution system:
 - 50 product categories
 - 60 plants
 - 15 distribution centers
 - 1000 customer zones

- Solved many transportation problems (1 for each product category)

- Goal: find best distribution plan, which plants to keep open, etc.

- Saved \$200 million per year.

(for more details, see 1997 Jan-Feb *Interfaces* article, "Blending OR/MS, Judgement, and GIS: Restructuring P&G's Supply Chain", available for download at <http://www.interfaces.smeal.psu.edu> under "Published Issues")

LP Example #4 (Assignment Problem)

The coach of a swim team needs to assign swimmers to a 200-yard medley relay team (four swimmers, each swims 50 yards of one of the four strokes). Since most of the best swimmers are very fast in more than one stroke, it is not clear which swimmer should be assigned to each of the four strokes. The five fastest swimmers and their best times (in seconds) they have achieved in each of the strokes (for 50 yards) are

	Backstroke	Breaststroke	Butterfly	Freestyle
Carl	37.7	43.4	33.3	29.2
Chris	32.9	33.1	28.5	26.4
David	33.8	42.2	38.9	29.6
Tony	37.0	34.7	30.4	28.5
Ken	34.4	41.8	32.8	31.1

How should the swimmers be assigned to make the fastest relay team?

Spreadsheet Model for LP Example #4

	A	B	C	D	E	F	G	H	I
1	An Assignment Problem								
2									
3		Best Times	Backstroke	Breastroke	Butterfly	Freestyle			
4		Carl	37.7	43.4	33.3	29.2			
5		Chris	32.9	33.1	28.5	26.4			
6		David	33.8	42.2	38.9	29.6			
7		Tony	37.0	34.7	30.4	28.5			
8		Ken	34.4	41.8	33.6	31.1			
9									
10									
11		Assignment	Backstroke	Breastroke	Butterfly	Freestyle			
12		Carl	0	0	0	1	1	<=	1
13		Chris	0	0	1	0	1	<=	1
14		David	1	0	0	0	1	<=	1
15		Tony	0	1	0	0	1	<=	1
16		Ken	0	0	0	0	0	<=	1
17			1	1	1	1			
18			=	=	=	=			Time
19			1	1	1	1			126.2

	B	C	D	E	F	G	H	I
3	Best Times	Backstroke	Breastroke	Butterfly	Freestyle			
4	Carl	37.7	43.4	33.3	29.2			
5	Chris	32.9	33.1	28.5	26.4			
6	David	33.8	42.2	38.9	29.6			
7	Tony	37	34.7	30.4	28.5			
8	Ken	34.4	41.8	33.6	31.1			
9								
10								
11	Assignment	Backstroke	Breastroke	Butterfly	Freestyle			
12	Carl	0	0	0	1	=SUM(C12:F12)	<=	1
13	Chris	0	0	1	0	=SUM(C13:F13)	<=	1
14	David	1	0	0	0	=SUM(C14:F14)	<=	1
15	Tony	0	1	0	0	=SUM(C15:F15)	<=	1
16	Ken	0	0	0	0	=SUM(C16:F16)	<=	1
17		=SUM(C12:C16)	=SUM(D12:D16)	=SUM(E12:E16)	=SUM(F12:F16)			
18		=	=	=	=			Time
19		1	1	1	1			=SUMPRODUCT(C4:F8,C12:F16)

Solver Parameters [?] [X]

Set Target Cell: [fx]

Equal To: Max Min Value of:

By Changing Cells: [fx] [Guess]

Subject to the Constraints:

\$C\$17:\$F\$17 = \$C\$19:\$F\$19

\$G\$12:\$G\$16 <= \$I\$12:\$I\$16

[Add] [Change] [Delete]

[Solve] [Close] [Options] [Premium] [Reset All] [Help]