Socially Optimal Product Differentiation

By KELVIN LANCASTER*

This paper sets out to investigate the extent to which we can reach broad general conclusions concerning the social optimality of different degrees of product differentiation. The paper is concerned with consumer preferences and production conditions only, and it abstracts from such problems as search and information costs, and disutilities of uncertainty or consumer confusion in the face of variety. It examines the problem of optimal variety in a world in which every consumer knows exactly what he prefers and exactly how to achieve personal optimality in the face of the constraints upon his actions.

Product differentiation exists when, within a group of goods so similarly related to consumers that they can be considered to form a product class, there is a variety of similar but not identical goods. The theory of product differentiation has been historically associated with the theory of monopolistic competition,[1] and has been analyzed primarily from the point of view of the firm. Although some social policy conclusions have been drawn from this approach to the subject, there does not exist a firmly based analysis leading to an answer to the question, "how many different product variants should society provide?"

One cannot go very far in answering this question within the framework of conventional or "direct" consumer theory, in which preferences are assumed to be given directly in terms of goods. For this reason, I shall turn to the "characteristics" or "indirect" analysis of consumer behavior, based on my earlier work (1966, 1971), in which the consumer is assumed to derive his actual utility or satisfaction from characteristics which cannot in general be purchased directly, but are incorporated in goods. The consumer obtains his optimum bundle of characteristics by purchasing a collection of goods so chosen as to possess *in toto* the desired characteristics.

Use of the characteristics framework provides a clear definition of a product class (those goods possessing a particular set of characteristics) and permits quantitative definition and measurement of product differentiation (by comparing the proportions in which the various characteristics are possessed by different goods within the product class), both of which properties are necessary for the analysis of optimum product variety. The characteristics framework enables us to cast the problem into a spatial setting and proceed along lines reminiscent of the pioneer work by Harold Hotelling.

The structure of the paper is as follows. First, in Section I, I shall set up some special tools of analysis that will be basic to the remainder of the analysis. Then, in Section II, I shall state and prove two basic theorems concerning optimal product differentiation and returns to scale, after which Section III examines the conditions that must be satisfied by an optimal choice for the number and type of prod-

* Professor of economics, Columbia University. Work for this paper was supported by the National Science Foundation (DA-39496). I wish to thank the referee for his extremely valuable comments.

[1] Chamberlin's lifelong and often pathetic struggle to retain sole title to imperfect competition should not blind us to the fact that, however the honors are to be shared between Joan Robinson and himself on the general theory of the subject, there is no doubt that it was Chamberlin who first raised the problem of the degree of product differentiation.

ucts. In Section IV, I shall state and prove four theorems concerned with optimal pricing and the effects of imperfect competition and, finally, Section V summarizes the results with special reference to policy conclusions.

The analysis will be carried out for product classes defined on two characteristics only. It will be apparent from the analysis itself and the results obtained that generalization to any number of characteristics is simply a question of additional arithmetical complexity, a complexity that does not seem justified in a pioneer investigation of the problem.

I. Tools of Analysis

A. *The Production-Consumption Link*

Consumers derive their ultimate utility or welfare from characteristics which in turn are obtained from the specific product differentiates which are available. Each product is assumed to possess those characteristics in fixed proportions. The product differentiates are themselves no more than a *transfer mechanism* by which fixed bundles of characteristics are assembled at the production end and then made available to ultimate consumers, the goods playing the role of intermediaries rather than being either primary resources or ultimate objects of consumption. If we view the system as a whole, consumer welfare is determined by the characteristics available for consumption, while the ultimate constraints are those on resources, the two linked by the transfer through goods. The transfer mechanism depends on both the way in which the resources may be used to produce goods having characteristics in different proportions and on the way in which the specific bundles of characteristics so produced are related to consumers' preferences as between all possible characteristics bundles.

For a given level of resources, the level of welfare that can be attained by the various consumers will depend on:

1) The production conditions that determine how much of each characteristic can be supplied from given resources, when embodied in a good with specific characteristics proportions.

2) The preferences of the consumers, which determine the relative welfare levels associated with various bundles of characteristics.

3) The consumption process, which determines what characteristics combinations the consumer can actually obtain from different collections of goods.

4) The number and types of goods that determine the transfer link between production and consumption.

In this paper we shall take the production and consumption possibilities as given, and be concerned primarily with the transfer mechanism. It is obvious that transfer can be efficient or not, depending on the choice of the good used for the transfer. Suppose, for example, that existing resources can be used to produce a unit of either good G_1 (embodying 2 units of characteristic z_1 and 1 of z_2) or good G_2 (1 unit of z_1, 2 of z_2). If there is a single consumer whose preferences are for high z_1-content, a lower welfare level will be attained by producing G_2 than by producing G_1. In this case, G_2 represents inefficient transfer relative to G_1, but if the consumer's preferences are biased toward z_2, the relative efficiencies of the two transfer modes will be reversed. And what if there are some consumers with preferences biased towards z_1, some towards z_2, and others in between? That is the essence of the problem we shall be solving.

B. *Production Conditions*

1. *Differentiation Possibilities*

We assume that it is possible, in principle, to produce goods having all possible ratios of the two characteristics, so that the producer can plan to produce a good anywhere in the characteristics spectrum. Having chosen the proportions in which his particular product will contain the two characteristics, there will be a unique maximum quantity of those characteristics that can be produced with given resources when incorporated in that particular good. For a given level of resources, we can take any ratio of the two characteristics, determine the maximum output of the good with that specification from the given resources and thus the maximum quantities of the two characteristics that can be produced from the resources when embodied in the appropriate good. The set of all characteristics combinations producible from a given level of resources by incorporation in a good can be plotted as a curve in characteristics space. We shall refer to this curve as a *product differentiation curve* and abbreviate it to *PDC*. Of particular interest will be the *PDC* corresponding to unit resource use, the *unit PDC*.

Assume the *PDC* has geometric properties similar to those of the conventional transformation curve[2] or production possibility curve, namely: 1) it is continuous; 2) it slopes downward from left to right; 3) it is either a straight line or a curve which is concave toward the origin.

I shall make the following additional assumption: 4) *PDC*s for different resource levels are positive homothetic expansions or contractions of the *unit PDC*. Figure 1 illustrates the assumed properties of the *PDC*s.

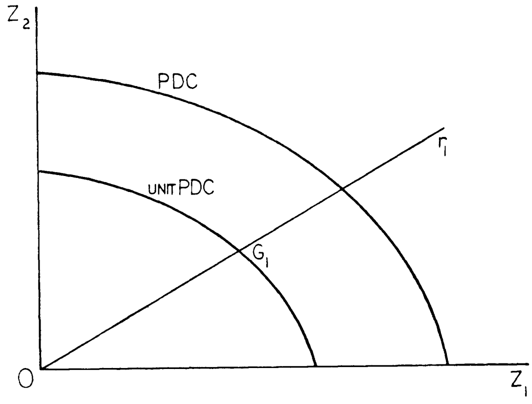

FIGURE 1

Note that I assume only a homothetic relationship between *PDC*s for different resource levels and particularly avoid any suggestion of linear homogeneity, which would imply constant returns to scale. Most of this paper is devoted to production under *nonconstant* returns to scale, but restricted to the homothetic case which implies the same returns to scale properties for all goods in the product class.[3]

2. *Measurement and Comparability*

The greatest single obstacle in the path of formal analysis of product differentiation is that of making quantitative comparisons between goods which are not identical. Since prices are endogenous, monetary measures cannot be used. One can only choose between comparing goods in terms of their final utility or their resource content. Since it is essential to the

[2] Although I referred to the *PDC* as a transformation curve in earlier versions of this paper, this is an inappropriate term. We would have a true transformation curve in characteristics space if the two characteristics were produced independently of each other, so that the curve would represent all combinations of the two separate characteristics that could be produced with given resources, each characteristic produced by its own production function with part of the total resources. Here we are concerned with the possible combinations of the two characteristics produced by incorporation in a single good. To use an analogy from high school chemistry, the regular transformation curve is concerned with *mixtures* of characteristics, the *PDC* with *compounds* made up of those characteristics.

[3] This is clearly a restrictive assumption, since it is reasonable enough to suppose that there are cases in which the returns to scale properties differ between goods in the same product class. Many of the observed differences in productivity between goods in the same product class, such as between mass-produced and custom-built automobiles, are differences in *scale* but not necessarily in the degree of *returns to scale*.

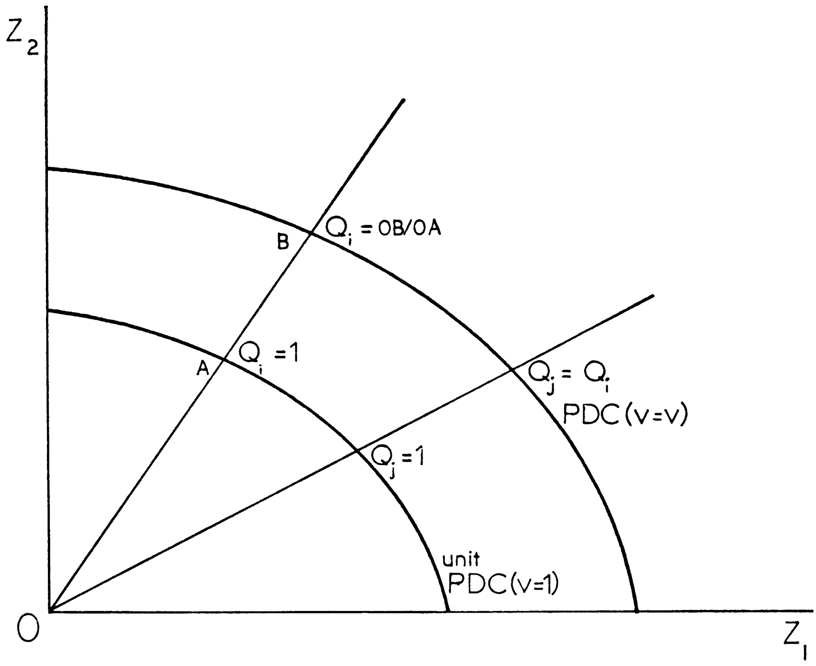

FIGURE 2

later analysis that there be many consumers with differing preferences, final utility measures are quite unsuitable. I shall, therefore, use a modified input measure that requires only that all firms face the same *PDC*s, an assumption that would be made in any case.

I shall therefore define and measure goods in the following way:

1) A good is defined by its characteristics ratio, goods with identical characteristics ratios being identical goods.

2) We bring *different* goods to the same measure by defining the unit quantity of any good to be that quantity which can be produced with unit resources.

3) Different quantities of the same good are scaled in proportion to the content of either characteristic (since the characteristics are in fixed proportions) relative to the content of that characteristic in a unit of the good as defined above.

4) As a consequence of 1)–3), plus the assumed homotheticity of the *PDC*s, quantities of different goods will receive the same measure if and only if those quantities require the same level of resources. Figure 2 illustrates the measurement system.

Note that a unit of a good is defined as requiring a unit of resources, but q units of a good do not necessarily require q units of resources since there may not be constant returns to scale. Resource content is used to relate quantities of *different* goods but quantities of the same good are scaled linearly from the resource content. Given the *unit PDC*, the characteristics content of unit quantities of different goods can be read straight off the curve and other quantities by linear scaling from these. Thus quantities of different goods are directly comparable, are in equivalent units, and can be arithmetically combined.

3. *The Input Function*

Once the choice has been made to produce a particular good with a specific characteristics ratio, there will be a defined functional relationship between the quantity of that good and the inputs required. It is convenient to use this production function in the inverse form

$$v_i = F(Q_i)$$

where v_i is the resource requirement[4] for the quantity Q_i of the good whose identity is defined by characteristics ratio $r_i(=z_2^i/z_1^i)$. I shall refer to $F(Q_i)$ as the *input function*, to stress its inverse nature.

Due to the way in which we have defined our quantity measure and to the assumed homotheticity of the *PDC*s, F has the following very important property: *The input function is the same for all goods. That is, the functional relationship between v_i and Q_i will be the same as the functional relationship between v_j and Q_j, for all i, j.* (We assume, of course, that i, j are in the same product class.)

Since the input function is the inverse of a production function, $F(Q)$ increases in proportion to Q for constant returns to scale, less than in proportion for increasing returns to scale, and more than in proportion for decreasing returns. Since we shall be interested in variations in the degree of returns to scale, we note that $F(Q')/F(Q) > 1$ for all $Q' > Q$ but that the ratio approaches unity as the degree of increasing returns to scale increases without limit and the ratio increases without limit as the degree of returns to scale becomes more and more decreasing.

C. *Consumption*

1. *Consumers*

Consumers have preferences defined on characteristics. Preferences with respect to goods are indirect and derived from preferences on characteristics. These are assumed to have the same general properties with respect to characteristics as conventional preferences with respect to goods. They may be represented by preference maps with indifference curves of the conventional kind, except that we do not rule out indifference curves which are linear or piecewise linear. We shall be more interested than usual in the elasticity of substitution (curvature of the indifference curves) and sometimes concern ourselves with indifference curves showing infinite elasticity of substitution (straight lines) or zero elasticity (fixed proportions). In the latter case excess amounts of either characteristic beyond the appropriate fixed proportion will have no effect, so that zero elasticity indifference curves can be considered to extend vertically and horizontally away from the point of optimum proportions.[5]

We shall generally assume that the population consists of a very large number of consumers with different preference patterns, so that there is a continuous spectrum of preferences. Later, we shall refer to a "uniform" distribution of preferences, a term that will be defined in the appropriate context. We shall also make a special assumption about the distribution of preferences, which we introduce at the appropriate point.

2. *The Consumption Process*

Consumption involves the extraction of the characteristics embodied in the goods at the production end. There are two major technical possibilities: we may have *combinable* consumption in which goods may be combined in the consumption

[4] We can take the "resource" to be a single input of the dollar value of a resource mix, resource prices being taken as exogenous to the sector under consideration.

[5] Implying free disposal of the surplus characteristic, or "open satiation" as discussed in the author (1971, ch. 9).

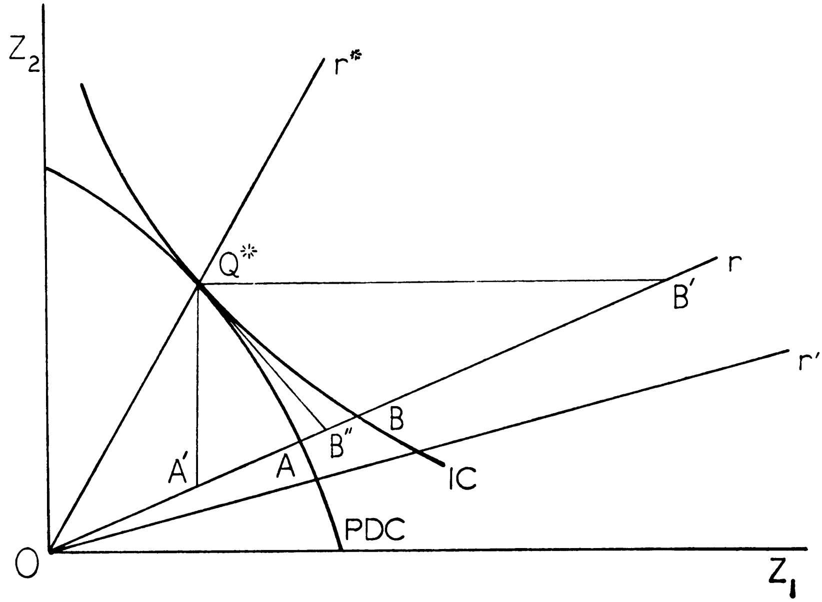

FIGURE 3

process to obtain some combination of the characteristics contents of the individual goods. Combinable consumption will be taken to be *linear*, the characteristics of the combination being the sum of the characteristics contents of the individual goods.[6] On the other hand, we may have *noncombinable* consumption in which only one good can be consumed at a time and characteristics can be obtained only in those proportions represented by an available good.

Both types of consumption are realistic in different contexts. Food nutrients fit the linear combinable case, but the services of many consumer durables fit the noncombinable pattern. The emphasis in this paper will be on noncombinable cases.

D. *Optimal and Sub-Optimal Transfer*

If we take a single individual in isolation and set out to minimize the resources required to attain a specified utility level, it is obvious which transfer good will achieve the optimum; it will be the good having a characteristics ratio r^* which corresponds to the point at which the relevant indifference curve for the consumer is tangent to a *PDC* (or at the corner of the indifference curve in the zero elasticity case). Figure 3 shows optimal transfer in diagrammatic form, and needs no further comment.

1. *Compensation for Sub-Optimal Transfer*

If the consumer's optimal good (that which would give him optimal transfer) is not available, there is some quantity of the "next best" good[7] that will enable him to achieve the utility level he would have attained with some specific quantity of the optimal good. If Q^* is the quantity of optimal good (characteristics ratio r^*) that

[6] This is the consumption process on which my original analysis (1966) was based.

[7] It is assumed throughout the analysis that the substitution of characteristics from other product classes can be ignored, so that the "next best" good is always within the same product class as the "best" good.

enables the consumer to attain a specified welfare level, and Q is the quantity of a good with characteristics ratio r that enables him to achieve the same level, we can regard Q as the *compensating* quantity corresponding to Q^*. Since Q, Q^* are in comparable units, we can take the ratio Q/Q^* and refer to it as the *compensating ratio* for good r as compared with optimal good r^*.

Figure 3 shows a consumer for whom the optimal good is r^* and another good r $(r<r^*)$ which is sub-optimal for this consumer. In order to bring the consumer to the same indifference level as attained with Q^*, he must be given an amount of good r which corresponds to point B on the diagram. From the properties of the quantity measure we are using, the compensating ratio is OB/OA.

It is obvious that the compensating ratio depends on three factors, the degree of curvature of the utility function, the degree of curvature of the *PDC*, and the difference between r and r^*. The ratio will increase with increased curvature of either curve and with an increase in the difference between r and r^*. The maximum possible value for the compensating ratio will be when B is at B' and A is at A' (corresponding to right-angled indifference and *PDC* curves with corners coincident at Q^*) and will be equal to r^*/r. In the case in which we have $r>r^*$, the maximum value of the compensating ratio will be r/r^*.

The compensating ratio will be one of the main tools of our subsequent analysis. Since it depends on r, r^* as well as on underlying preference and production conditions, it is useful to write the ratio as a function of r, r^*. We shall refer to the ratio in the form $h(r, r^*)$ as the *compensating function*, and it will have the following properties:

(1) $h(r^*, r^*)=1$

(2) $1\leq h(r, r^*)\leq \text{Max}\,(r^*/r, r/r^*)$, for all r

(3) $h_r(r, r^*)\geq 0$ for $r\geq r^*$, ≤ 0 for $r\leq r^*$

(4) $h_{r^*}(r, r^*)\leq 0$ for $r\geq r^*$, ≥ 0 for $r\leq r^*$

(5) $h_{rr}(r, r^*)>0$, unless both the indifference curve and *PDC* are linear,[8] at least for r within a "reasonable" distance of r^*.

II. Two Basic Theorems on Differentiation and Returns to Scale

Using the tools we have developed, it is possible to enunciate two basic theorems on the relationship between optimal product differentiation and returns to scale.

THEOREM 1: *If production is subject to constant or decreasing returns to scale, it will be socially optimal to produce every good which represents optimal transfer for any consumer. If and only if there are constant returns to scale in production, a linear product differentiation curve, and linearly combinable consumption processes, the social optimum can also be achieved with a number of goods in each product class not greater than the number of separate characteristics in the product class. Under decreasing returns to scale, it may be optimal to produce even more goods than under constant returns to scale with the same distribution of preferences, and certainly not less.*

Under constant returns to scale, the resources used to produce any quantity of any good are directly proportional to the quantity as measured, with the same constant of proportionality for all goods, because of the homotheticity assumption and the definition of the quantity measure. Any consumer who must consume a sub-

[8] This property may not hold for r sufficiently different from r^*, even in cases which would appear to be very well-behaved. For a linear *PDC* of the form $z_1+z_2=1$ and a Cobb-Douglas utility function of the form $u=z_1^{1/2}z_2^{1/2}$ (for which $r^*=1$), $h(r)$ has the form $h(r)=\frac{1}{2}(r^{1/2}+r^{-1/2})$, giving $h''=(3-r)/8r^{5/2}$. In this case $h''\geq 0$ only within the range $3r^*\geq r\geq 1/3r^*$.

optimal good is therefore using more resources than would be needed for him to attain the same welfare level with an optimal good, and it is necessarily optimal to provide every consumer with his optimal good.

If, in addition to constant returns to scale, there is linearly combinable consumption and a linear *PDC*, then production of the two extreme goods (each containing one of the characteristics only) will enable any consumer to attain the same welfare level by a combination of those goods as with his optimal good, for the same level of resource utilization.

Now consider the case in which there are decreasing returns to scale. In this case more goods, each produced in smaller quantities, will use less resources than fewer goods produced in larger quantities. It is obvious that it will never be optimal not to produce every consumer's optimal good. But now it may be optimal to have a consumer derive some of his welfare from a nonoptimal good, so as to reduce the average level of output of all goods.

Suppose that there is a linear *PDC* with equation $z_1+z_2=1$, and that there are two consumers whose preferences show zero elasticity of substitution and whose optimal goods are given by $r_1=3$, $r_2=1/3$. The input function has the form $v=Q^a$ ($a>1$ for decreasing returns to scale), and the consumers initially each receive one unit of their respective optimal goods for a total resource use of 2.

Now introduce a third good with characteristics ratio $r=1$, and suppose that each consumer receives some quantity of his optimal good, plus enough of the third good to bring him to his original welfare level. In this particular example the compensating ratios for the third good are equal at 1.5 and since the example is symmetrical we suppose each consumer receives an amount $(1-x)$ of his optimal good and an amount $1.5x$ of the third good, bringing each consumer to his original welfare level.[9]

Total resources used in the three-good case will then be given by

$$V = 2(1-x)^a + (3x)^a$$

If we choose x so as to minimize V, we obtain

$$x^* = [1 + 3(3/2)^b]^{-1}$$

where $b=1/(a-1)$ which gives $x^*>0$ for all $a>1$, and thus it is optimal to produce some of the third good for all degrees of decreasing returns to scale. For $a=2$, we have $x=2/11$ and $V=18/11$, a saving of 4/11 resource units. Thus it will be optimal to produce goods which are not the optimal goods of any consumers, if there are decreasing returns to scale.[10]

It is interesting to consider the pricing associated with the above case. Since the compensating ratio is 1.5, a consumer will be induced to buy both the third good and his optimal good if the price of the third good is 2/3 that of the optimal good. Since the cost function for any good has the form $C=AQ^a$, where A is the same for all goods, we can compute the marginal costs of the goods at the optimum. It turns out that the ratio of the marginal cost of the third good to that of either optimal good is 2/3 for all values of a, when x is given its optimum value. Thus competitive prices are appropriate.

[9] Rather than having a single consumer with split consumption, which is somewhat unrealistic, we can suppose each "consumer" is actually an aggregate of consumers with identical tastes, some of whom consume one good and some the other.

[10] We have proved here only what we set out to do, showing that optimality under decreasing returns to scale will call for more goods than the number of distinct preferences. If decreasing returns holds for all levels of output, however small, we can continue to improve the situation by adding more and more goods (with characteristics ratios between those of existing goods) until we have a continuum of goods. If the preferences form a continuum, we will necessarily have a continuum of goods in the *constant* as well as decreasing returns cases.

A. *Increasing Returns to Scale*

Economic intuition suggests that the effect of increasing returns to scale will be to reduce the socially optimal number of goods below what it would be under constant returns to scale, at least for a sufficient degree of increasing returns. For a sufficiently well behaved case, the socially optimal number of goods will be a function of the degree of increasing returns and will decrease as this degree increases. Finally, intuition suggests that a solution for the optimal number of goods will necessarily involve solving for the optimal choice of those goods in terms of characteristics ratios. A grand theorem incorporating all these possible results is not easy to produce, so we shall start with a simple but basic theorem.

THEOREM 2: *There is some degree of increasing returns to scale, sufficiently large, for which the socially optimal number of goods is one.*

Consider a situation in which the consumers are supplied with N goods, the amount consumed of the ith good being Q_i. We make no assumptions about the specification of these goods or the distribution of preferences except that there are nonzero quantities of at least two goods. We shall consider the effect of replacing these goods by a single good, arbitrarily chosen with respect to characteristics. The quantity of the single good is denoted by Q and, if it is just sufficient to enable all consumers to attain the same welfare levels as with the N goods, it is obvious that $Q \geq \sum_1^N Q_i$. Write $Q = k\sum_1^N Q_i$, where $k \geq 1$.

We need first to prove that k is bounded above. Now, among all the consumers in the initial (N-good) situation, there must be one who consumes the greatest amount of characteristic z_1 per unit of whatever good he consumes; denote this maximal amount by $\bar{z}_1$. There will be a maximal z_2, denoted by $\bar{z}_2$, derived in a comparable fashion. Suppose the amounts of the two characteristics in unit quantity of the single good are z_1^*, z_2^* (note we restrict the single good to having some of both characteristics), then the maximal value for the compensating ratio cannot exceed Max $(\bar{z}_1/z_1^*, \bar{z}_2/z_2^*)$ for any consumer. The value of k cannot exceed the maximum compensating ratio for a single consumer, hence k is bounded.

Resource use in the N-good case is given by

$$V_N = \sum_1^N F(Q_i)$$

Since there are increasing returns to scale, $\sum F(Q_i) > F(\sum Q_i)$, with the ratio $\sum F(Q_i)/F(\sum Q_i)$ increasing as the degree of increasing returns increases.

If the N goods are replaced by a single good, produced in a quantity just sufficient to enable consumers to attain original welfare levels, resource use will be given by

$$V = F(Q) = F(k\sum Q_i)$$

With increasing returns, the ratio $F(k\sum Q_i)/F(\sum Q_i)$ is greater than unity, but approaches unity as the degree of increasing returns is increased without limit.

Thus we have the following situation. V_N is greater than $F(\sum Q_i)$ and recedes from $F(\sum Q_i)$ as the degree of increasing returns increases, while V is also greater than $F(\sum Q_i)$ but approaches it more and more closely as the degree of increasing returns increases. Thus there is some degree of increasing returns sufficiently great to give $V \leq V_N$, and the single good is then optimal for increasing returns of this or any greater degree, as compared with the specific N goods of the initial situation. Since no restrictions were placed on N or on the specification of the N goods, it follows that there is some degree of increasing returns for which the single good is optimal as compared with any choice of N goods, for all $N \geq 2$.

This theorem gives only a part of the overall picture with respect to increasing returns to scale, but firmly establishes the basic proposition that increasing returns will be associated with an optimal number of goods that is less than with constant returns to scale, and does this with the minimum of assumptions about preferences and their distribution or about the *PDC*.

III. The Conditions for Optimal Differentiation

Under constant returns to scale, the optimal number of product differentiates will be finite only if the number of distinct preferences represented in the society is finite, and is essentially unbounded if preferences form a continuum. Under decreasing returns to scale, the optimal number of goods will not generally be bounded even if the number of distinct preferences is finite. These are the conclusions to be drawn from Theorem 1. For increasing returns to scale, on the other hand, Theorem 2 shows the optimal number to be bounded for a sufficient degree of increasing returns. Thus the study of optimal differentiation is confined to the cases in which there are increasing returns.

Determining the optimal number of goods under increasing returns to scale is not a simple matter. We cannot merely consider the N-good case, then add an $(N+1)$th good, because there will be a total structural change in passing from N goods to $N+1$ goods. It is obvious that, since we are seeking optimality, we must find the specifications, quantities, and distribution of N goods which provide given welfare levels for minimum use of resources, then find specifications, quantities, and distribution of $N+1$ goods that achieve the same welfare levels as before for minimum resource use. In general, the goods in the $N+1$ case will be entirely different goods from those in the N-good case. The only thing that will be comparable between them will be the minimum resources required to attain the given welfare level in the two cases, and it is direct comparison of this that will indicate whether $N+1$ goods are better or worse than N. Thus solving for the optimal number of goods is a two-part process, first solving the optimum configuration for each possible number of goods, then comparing the resources required between different numbers of goods to find the optimum number. Since the number of goods is an integer variable and the configuration changes between different numbers of goods are discrete, the second stage is simple once the first has been completed.

A. *The Optimum Configuration*

The most difficult and interesting part of the overall problem is that of finding the optimum configuration for a specified number of goods. This involves finding characteristics ratios for the goods and the distribution of those goods over the consumers, such that the given welfare levels for all consumers are achieved with minimum use of resources. It is obvious that opportunities for ill behavior abound in a general model, so we shall make the two following assumptions on the distribution of preferences which are designed to give a minimum level of good behavior:

Continuum Assumption. The distribution of preferences is such that the set of all points along the *PDC* that represent optimal transfer for some individual forms a continuum.

Anticrossover Assumption. The distribution and form of preferences is such that, if it is optimal to supply two consumers whose optimal transfer goods would have characteristics ratios r_i, $r_j (\leq r_i)$ with the same good Q_k, then it will be optimal to supply Q_k to all consumers whose optimal good would have a characteristics ratio r such that $r_j \leq r \leq r_i$.

The two assumptions between them ensure that the set of all consumers being supplied with the same good is compact. It is possible to visualize cases, such as two consumers with the same optimal good, one with zero elasticity of substitution and the other with infinite elasticity, in which it just might sometimes be optimal to supply them with different goods. We rule this out by the anticrossover assumption, which implies that the shape of preferences as between individuals with closely similar optimal transfer goods does not vary too much.

To determine the optimal configuration for N goods we proceed as follows. First we note that, as a consequence of the above assumptions, the continuum of consumers (identified by the characteristics ratios of their optimal transfer goods) will be divided into N segments, each segment supplied by one of the N goods. Denote by $R_1, \ldots, R_{N-1}$ the characteristics ratios which divide the segments from each other, and by $r_1, \ldots, r_N$ the characteristics ratios of the N goods. For given preference and population distributions and given production conditions, the quantity of the ith good (characteristic ratio r_i) required to bring all consumers in the segment bounded by R_i, R_{i-1} to specified welfare levels will be a function of r_i, R_i, R_{i-1} only, so that we can write Q_i as $Q_i(r_i, R_i, R_{i-1})$.

Let us take R as given, and consider the optimal choice for r_i. Since Q_i is not a function of r_j, nor Q_j of r_i, and since R is held constant, Q_i is independent of Q_j (all i, j) and thus the optimal choice for r_i must be that which minimizes Q_i. This gives us our first optimum condition

$$(6) \qquad \partial Q_i(r_i, R_i, R_{i-1})/\partial r_i = 0$$

If Q_i^* is Q_i optimized with respect to r_i, then $Q_i^* = Q_i^*(R_i, R_{i-1})$. We can now find the optimal value for R. The Q^*, considered as functions of R, are no longer independent of each other because each R_i appears as an argument in two of the Q^*s. To optimize we must minimize total resources, given by

$$V = \sum F_i(Q_i^*) = V(R_1, \ldots, R_{N-1})$$

Because R_j appears as an argument in Q_j^*, Q_{j+1}^* only, the optimum conditions for R takes the form

$$(7) \qquad \frac{\partial V}{\partial R_i} = \frac{\partial Q_{i+1}}{\partial R_i} F'_{i+1} + \frac{\partial Q_i}{\partial R_i} F'_i = 0$$

B. *Interpretation of the Optimum Conditions*

We can give the optimum conditions an interpretation with more direct economic appeal by expressing them in terms which involve the compensating ratios. To do this we shall make the additional assumption, that *all consumers having the same optimal transfer good have identical preferences.*

Consider the segment of consumers bounded by optimal transfer ratios R_i, R_{i-1}, all supplied by good G_i with characteristics ratio r_i. Consider those consumers whose optimal good would have characteristics ratio r $(R_i \geq r \geq R_{i-1})$, and let the quantity of that good that would bring all these consumers to the specified welfare levels be denoted by $s(r)$. Since the consumers are being supplied with a good having characteristics ratio r_i, not necessarily their optimal good, the total quantity of this good needed to achieve the welfare level is given by $h(r_i, r)s(r)$, where h is the compensating function as discussed in Section I. The total quantity of good G_i required to bring all consumers in this segment up to specified utility levels is then given by

$$Q_i = \int_{R_{i-1}}^{R_i} h(r_i, r)s(r)dr$$

Since R_i, R_{i-1} appear only as limits of integration, we thus have

$$\partial Q_i/\partial R_i = h(r_i, R_i)s(R_i)$$
$$\partial Q_i/\partial R_{i-1} = -h(r_i, R_{i-1})s(R_{i-1})$$

Inserting these values in the optimum conditions (7) gives

$$\frac{F'_i}{F'_{i+1}} = \frac{h(r_{i+1}, R_i)}{h(r_i, R_i)} \tag{8}$$

Now F'_i, F'_{j+1} are the marginal resource costs of producing goods G_i, G_{i+1}, respectively. The right-hand side of (8) is the ratio of the compensating ratios for the dividing consumer (whose optimal transfer good is on the boundary between the two adjacent segments) with respect to being supplied with G_{i+1} as compared with G_i.

Thus the second optimum condition, in the case in which consumers having identical optimal transfer goods have identical preferences, has the following easily interpretable form: *The ratio of the marginal resource costs of two adjacent goods will be the inverse of the ratio of the compensating ratios with respect to the two goods, for the dividing consumer.* No special insights are given by this formulation into optimum condition (6) or into the optimality of N goods relative to M goods.

C. *A Well-Behaved Example*

To show that intuitive notions about well-behaved relationships between the optimal number of goods and the degree of returns to scale are valid in appropriate cases, we shall sketch out and give the results for a simple example.

On the production side we assume a linear PDC[11] of the form $z_1+z_2=1$ and an input function of the form $V=Q^{1/b}$ where the degree of increasing returns increases with b. We assume that consumers show zero elasticity of substitution, have optimal goods distributed with uniform line density along the PDC,[12] are to be maintained on welfare levels corresponding to unit quantities of their optimal goods, and that the total population is unity.

It can be shown that, under these assumptions, the compensating function h and the density function s have the following forms:

$$h(r_i, r) = \frac{r(1+r_i)}{r_i(1+r)} \qquad r \geq r_i$$
$$= \frac{1+r_i}{1+r} \qquad r \leq r_i$$
$$s(r) = \frac{1}{(1+r)^2}$$

where r is the characteristics ratio of the optimal good and r_i the characteristics ratio of the good actually supplied to consumers with optimal good r.

Confining our investigations to the cases $N=3, 2, 1$, we obtain the following optimal configurations for the three cases:

$N=3$: $r_1=1/3$, $Q_1=7/18$
$r_2=1$, $Q_2=7/18$ $R_1=1/2$, $R_2=2$
$r_3=3$, $Q_3=7/18$
$N=2$: $r_1=1/2$, $Q_1=5/8$
$r_2=2$, $Q_2=5/8$ $R_1=1$
$N=1$: $r_1=1$, $Q_1=3/2$

Denoting by $V(N)$ the resources required to achieve the given utility levels with N goods of optimal configuration, we obtain $V(3)=3(7/18)^{1/b}$, $V(2)=2(5/8)^{1/b}$, $V(1)=(3/2)^{1/b}$. By solving for the values of b at which the various pairs are equal, we obtain our final result: The optimal number of goods is three for $b \leq 1.169$ (relative to the choice between 3 and a lesser num-

[11] With a linear PDC we shall assume consumption is noncombinable, to rule out the possibility of achieving optimality with no more than two goods.

[12] Uniform line density is one of several possible cases of "uniform" preference distribution.

ber of goods), two for $1.169 \leq b \leq 1.263$, and one for $b \geq 1.263$. Note that the total quantity of goods ($\sum Q_i$) increases from 7/6 with three goods to 5/4 with two goods to 3/2 with one good (it is 1 for $b=1$, the constant returns to scale case). If this were not so, the optimal number of goods would never be greater than unity, if there were any degree of increasing returns.

This example shows that, for a sufficiently well-behaved model, we do find the optimal number of goods decreasing steadily as the degree of increasing returns increases.[13]

IV. Four Theorems on Pricing and Imperfect Competition

The conditions for optimal configuration at the optimum number of goods lead directly to the following theorem:

THEOREM 3: *If the optimal number of goods is some number $N > 1$, then the optimal prices for those goods will be such that prices stand in the same ratio to marginal costs for all goods in the group. This can be shown to hold for any PDC, any forms of the preference functions,*[14] *and any forms for the input functions, provided there is a proper interior optimum at N goods.*

Consider the optimal distribution of the goods through a market mechanism (where each consumer has an income appropriate to the specified welfare levels), and consider the consumer whose optimal transfer good would have characteristics ratio R_i, the dividing point between the market segments supplied by G_i and G_{i+1}. If he is to be the dividing consumer, he must find that the dollar expenditures on G_i or G_{i+1} which are equivalent to a unit of his (unavailable) optimal good are the same. Thus the ratio P_i/P_{i+1} must be inverse to the ratio of the two compensating ratios $h(r_i, R_i)$, $h(r_{i+1}, R_i)$ for the dividing consumer. But we have already shown in equation (8) that the ratio of marginal costs, F_i'/F_{i+1}', must also equal the inverse of the ratio of the compensating ratios at the optimum configuration for N goods. Thus the ratio of prices between adjacent goods must equal the ratio of their marginal costs. By chain reasoning, the same relationship must then hold for all pairs of goods, proving the theorem.

Note that the theorem requires only that all prices bear the same ratio to marginal costs for goods within the group, and does not require equality of prices and marginal costs. However if we consider the group embedded in the larger economy then the usual arguments will lead to the requirement of equality in order to achieve optimality over the economy as a whole.

Equalization of the ratios of prices to marginal costs within the group is necessary but not sufficient for optimal configuration, and certainly not sufficient to guarantee that the number of goods is optimal (as we shall see in Theorem 6). It is, of course, easily possible to have the economy operating in an optimal configuration for N goods but be sub-optimal because the optimal *number* of goods is really M.

A. *Achieving the Optimum*

Since the interesting problems with respect to the optimum number of goods

[13] *A Conjecture*: The pattern of values for $\sum Q_i$ suggests that for N goods we would have $\sum^N Q_i = (2N+1)/2N$ with the optimal Q_i equal to each other at a value of $(2N+1)/2N^2$. If the conjecture were indeed true, we could then show that the value of b at which society would be indifferent between N goods and $N+1$ goods would be given by

$$b = 1 + \frac{\log[1 + 1/N(2N+3)]}{\log[1 + 1/N]}$$

This would give the lowest value of b for which the optimal number of goods would be three as 1.128 (the upper value for three goods is 1.169). The range of values for which the optimal number of goods would be ten would be $1.046 \leq b \leq 1.051$. I have not attempted to verify the truth of the conjecture.

[14] As proved, we require that all consumers with the same optimal goods have the same preferences. This can probably be relaxed.

arise with increasing returns to scale, and since the optimum pricing system is that of perfect competition, a problem arises in achieving that optimum. The competitive system will not work, first because obtaining the potential scale economies for each good requires a single producer for that good (unless the economies are all industry externalities) and, second, because all firms will make losses when price is equated to marginal cost. Furthermore, even marginal cost pricing cannot guarantee that the *number* of goods is optimal.

One possibility is a managed economy with single firm control of the production of each good and each firm constrained to adopt marginal cost pricing. This would then require a subsidy to cover the gap between total costs and total revenue for each firm. The subsidy itself would be a control variable however, since the optimum number of goods would require the minimum subsidy over the group. A subsidy set at this minimum level for the group as a whole should then induce the group to produce the optimum number of goods, given appropriate institutional rules.

B. *Imperfect Competition*

The remainder of the paper is concerned with imperfect competition and its effects on the optimal number, quantity, and specification of goods. We have different cases to consider, but in each case we shall commence from the optimum position and consider the changes introduced by the relevant type of imperfect competition. First we shall clear the air by disposing of a simple but important case.

THEOREM 4 (Constant Returns to Scale): *Under constant returns to scale, market imperfection will not cause the number or specification of goods to diverge from the optimum.*

If we commence at the optimum and there are constant returns to scale, no form of market imperfection will make it profitable for any firm to eliminate anyone's optimal transfer good because 1) there is no saving in costs from changing the number of goods produced, and 2) the maximum revenue that can be extracted from any one consumer for a given quantity of good supplied will be when he is sold his optimal transfer good. Thus the same goods will be sold, with the same characteristics ratios, as at the optimum. If there are any gains to be made by the monopolist, they will be by increasing prices and reducing quantities of the same goods which are produced at the optimum, not by changing the number of goods or their characteristics ratios. The remaining theorems are all presumed to refer to a context of increasing returns to scale.

THEOREM 5 (Single Good Monopoly): *Monopoly control of the production of any one good when firms producing other goods do not behave as monopolists will in general lead to a nonoptimal choice for the characteristics ratio of that good and, if other firms adopt marginal cost pricing, to a restriction of the output of the monopoly good.*

The second part of the theorem should surprise no one, but the first part represents a new kind of result for imperfect competition theory.

To prove the theorem, consider a particular good (characteristics ratio r) in the overall spectrum. Next to this good in the spectrum will be an "upper" good (characteristics ratio $r_U > r$) and a "lower" adjacent good (characteristics ratio $r_L < r$). The dividing consumers between the markets for the good in question and the upper and lower adjacent goods have optimal transfer goods with characteristics ratios R_U, R_L. We start from the optimal configuration and consider the changes that will be made by a monopolist taking control of the good.

We assume that over the range of variations being considered, there are no acceptable substitutes for the good from outside the group. If the price of the good increases, consumers in the center of the market will simply have to buy the same quantity of the good and spend more in order to remain at the same welfare level. Consumers at the market fringes, however, may switch to the purchase of adjacent goods if it is advantageous to do so. The market boundaries will adjust so that the dividing consumer is the one for whom the ratio of compensating ratios is inversely proportional to the price ratio. Thus the quantity of the good will be given by

$$(9)\qquad Q = \int_{R_L}^{R_U} h(r, x)s(x)dx = Q(r, R_U, R_L)$$

where $h(r, x)$ is the compensating ratio for the consumer whose optimal good has characteristics ratio x, and $s(x)$ is the market density at x. The effect of price changes operates through changes in R_U, R_L.

By definition of the dividing ratios, the following conditions must be satisfied everywhere:

$$(10)\qquad h(r, R_U)P = h(r_U, R_U)P_U$$

$$(11)\qquad h(r, R_L)P = h(r_L, R_L)P_L$$

where P, P_U, P_L are the prices of the good in question and the upper and lower adjacent goods, respectively.

Relationships (9), (10), (11), together with the profit definition

$$(12)\qquad \pi = PQ - C(Q)$$

(where $C(Q)$ is the cost function) form a set of four relationships that must be satisfied everywhere by the six variables π, P, Q, r, R_U, R_L. There are two degrees of freedom in the system, which we shall take to be the choice of r and the choice of Q.

To prove the first part of the theorem, hold Q constant and consider the effect of variations in r on the firm's profit. Using standard comparative static methods and noting that $\partial Q/\partial r=0$ at the optimum (optimum condition (6)), we obtain

$$(13)\qquad \frac{d\pi}{dr} = \frac{Q_L M_U h_r^L + Q_U M_L h_r^U}{Q_U M_L h^U + Q_L M_U h^L} PQ$$

at the optimum point, where

$$h^U = h(r, R_U), \qquad h^L = h(r, R_L)$$

$$h_r^U = \partial h^U/\partial r, \qquad h_r^L = \partial h^L/\partial r$$

$$M_U = \partial[Ph^U - P_U h(r_U, R_U]/\partial R_U$$

$$M_L = \partial[Ph^L - P_L h(r_L, R_L]/\partial R_L$$

$$Q_U = \partial Q/\partial R_U = h^U s(R_U)$$

$$Q_L = \partial Q/\partial R_L = - h^L s(R_L)$$

The h-functions always decrease as the two arguments move closer together and always increase as they diverge, so that M_U, h_r^L are positive and M_L, h_r^U are negative, while h^U, h^L are essentially nonnegative; Q_U is positive, and Q_L negative. Thus the denominator in (13) is always negative, but the numerator consists of two terms of opposite sign. Thus $d\pi/dr$ may have any sign (or be zero but only by coincidence), and the monopolist will in general find it profitable to change the characteristics ratio of his good away from the optimum.

The reason why the monopolist will not generally be satisfied with the socially optimum specification of the good is not difficult to see. From his point of view, the central consumers are locked into his market and it is only at the edges where he must compete with adjacent goods. Thus his actions in the market will be based on the properties at the market fringes, as shown by (13) where $d\pi/dr$ is seen to depend only on the properties at R_U, R_L. The socially optimal choice of r, on the other hand, is given by:

$$\frac{\partial Q}{\partial r} = \int_{R_L}^{R_U} \frac{\partial h(r, x)}{\partial r} s(x)dx = 0$$

a condition which gives weight to all consumers in the market segment.

To prove the second part of the theorem, we hold r constant and consider variations in Q. Proceeding as before, we obtain

$$(14) \qquad \frac{d\pi}{dQ} = (P - C') - \frac{QM_U M_L}{D}$$

where D is the same as the denominator in (13).

If, as assumed, we have marginal cost pricing at the optimum, then $P-C'=0$ and we have

$$(15) \qquad \frac{d\pi}{dQ} = - \frac{QM_U M_L}{D}$$

Since M_U, M_L have opposite signs and D has already been shown to be negative, $d\pi/dQ<0$ and the monopolist will find it profitable to reduce output (necessarily involving an increase in price and a shrinking of the market boundaries), completing proof of the theorem.

The next theorem is similar in its policy conclusions to the well-known "excess capacity" theorem of monopolistic competition, although it is based on a different process of reasoning and does not depend in any way on the ambiguous notion of "capacity."

THEOREM 6 (Monopolistic Competition): *Under increasing returns to scale and with a sufficiently uniform and evenly distributed market, "monopolistic competition" will lead to a greater degree of product differentiation than is socially optimal.*

Quotation marks have been placed around monopolistic competition here because one of the most basic assumptions of the Chamberlin model, that the effects of a behavior change by any one firm will be spread evenly over all other firms, most emphatically does not hold in this case. Here, the behavior of a firm affects very much those firms producing adjacent goods in the spectrum, and very little those producing goods remote from it in the spectrum. We make the remaining assumptions of monopolistic competition, that each firm produces one product, no other firm produces the same product, and there is free entry into the group that will continue so long as positive profits can be made. Thus, although the dynamics of the group cannot conform to the Chamberlin model because of the oligopolistic elements present, the equilibrium (if attained) will be of the Chamberlin kind—for a sufficiently evenly distributed total market, all firms will be of the same size, with marginal revenues equal to marginal costs and prices equal to average costs. Thus we shall consider the term *monopolistic competition* to be the most appropriate for the model being considered.

A "uniform" market is taken here to mean that, after a possible transformation of coordinates, the compensating ratio for a consumer whose optimal good has characteristics ratio corresponding to x, with respect to a good having characteristics ratio corresponding to x^*, depends only on the absolute value of the difference between x and x^*. That is, the compensating function has the particular form $h(x^*, x) = h(|x-x^*|)$. If the upper and lower dividing consumers for the market for the good represented by x^* have optimal goods with characteristics ratios x^*+c_2, x^*-c_1, and the market density is constant (taken to be unity), total demand for the good represented by x^* is given from:

$$(16) \qquad Q = \int_{x^*-c_1}^{x^*+c_2} h(|x - x^*|)dx = H(c_1) + H(c_2)$$

where $dH/dx=h$.

Given this uniformity and uniform

density, it is obvious that goods and firms will be arranged, both at the social optimum and at monopolistic competition equilibrium, so that $c_1 = c_2$ for every good, and c and Q are equal for all goods. Thus we can write $h(c) = H'(c)$ and $Q = 2H(c)$ for all goods.

Since the whole spectrum is to be covered by goods with equal market areas (in the coordinates being used), there is an inverse relationship between the number of goods n, and the market area measure c, which we can write as $n = A/c$ where A is an appropriate constant.

Now consider the optimum number of goods, and suppose it is sufficiently large to treat n as closely approximated by a continuous variable. For n goods, total resource use is given by

$$V = nF(Q) \tag{17}$$

where $F(Q)$ is the input function for the single good. Since both n and Q are functions of c only, we minimize V with respect to c to obtain the condition for the optimum

$$F' = F/2ch \tag{18}$$

where the second-order conditions are satisfied for increasing returns to scale.

Having established the optimum conditions, we turn to consider the monopolistic competition equilibrium, which will consist of firms facing identical conditions. Each firm will, since we assume no oligopolistic elements and thus that other prices are taken as given, face the same considerations as the firm of Theorem 5. Due to the special uniformity assumptions, the expressions that appear in Theorem 5 have the following simplified forms here

$$h^U = h^L = h \quad (= h(c))$$

$$M_U = -M_L = 2Ph'$$

$$Q_U = -Q_L = h$$

Inserting these values in (14) of Theorem 5, together with the value of $Q(=2H)$ from above, and noting that $h = H'$, $h' = H''$, we obtain

$$\frac{d\pi}{dQ} = P - C' - 2P\frac{HH''}{(H')^2} \tag{19}$$

We shall suppose prices to be measured in resource units, so that $C = F$, $C' = F'$. We commence at the optimum and thus assume that the condition (18), $F' = F/2cH'$ is satisfied.

There are two equilibrium conditions for the monopolistic competition group: 1) that all firms are at individual profit maximizing equilibria; and 2) that there are just sufficient firms to equate price and average cost everywhere. We shall proceed by supposing that we have the socially optimal number of goods and that price is equal to average cost for all firms, then investigate whether the firms are at profit maximizing equilibria.

Inserting the values $C' = F' = F/2cH'$, $P = F/2H$, in (19) we obtain, after some manipulation,

$$\frac{d\pi}{dQ} = \frac{F}{2H'}\left[\frac{H'}{H} - \frac{H''}{H'} - \frac{1}{c}\right] - \frac{FH''}{2(H')^2} \tag{20}$$

Since H'' is positive (the compensating ratio is an increasing function of c), the last term above is clearly negative. Now consider the expression in brackets. This is equal to $d(\log\, G)/dc = G'G$, where $G = H/cH'$ and is thus equal to the ratio of the average to the marginal compensating ratio over the half market from $x = x^*$ to $x = x^* + c$. Due to the assumed shape of the compensating function, G is a decreasing function of c so that G' is negative, the bracketed expression is negative, and $d\pi/dQ$ is negative.

Thus a configuration with the optimal number of goods and zero profits will not be profit maximizing for the individual firms. These can increase their profits by contracting output, leading to positive profits and the entry of more firms. The final monopolistic competition equilibrium will therefore result in a greater number of goods than at the social optimum.

Finally we shall consider the effect of monopolization of a market sector, that is control of the outputs of several goods (taken to be adjacent in the spectrum) by a single monopolist.

THEOREM 7 (Monopoly Control of a Market Sector): *Under increasing returns to scale, monopoly control of a market sector will lead to a lesser degree of product differentiation over that sector than is socially optimal.*

Consider an optimal configuration over an optimal number of goods, and consider the effect of monopoly control over several adjacent goods in the market. We shall suppose that the monopolist takes over control of the $(j-1)$th, jth and $(j+1)$th goods. Although not essential, it simplifies the argument to suppose that there is uniform distribution over this sector so that, at the social optimum, the prices of all three goods are the same.

Consider the effect of taking the jth good out of production. This will not affect those consumers who originally purchased the other two goods and, if prices are unchanged, will have no effect on consumers purchasing goods outside the monopolized sector. Since we assume throughout that there are no acceptably close substitutes for goods within the general product class from outside the class, those consumers who originally purchased the jth good will now purchase either the $(j-1)$th or the $(j+1)$th. If their incomes are fixed, they will obtain the same amount and the same quantity of the substitute as of their original good (since prices are the same), but will be somewhat worse off. If they are compensated, they will buy more of the substitute than of the original good. Thus the monopolist's average revenue will be unchanged on total sales no less than with all three goods.

Since there are increasing returns to scale, however, the increased output of the $(j-1)$th and $(j+1)$th goods will reduce average costs and thus the monopolist's profits will be increased as a result of closing down production of the jth good. Simply closing down the production of the jth good does not represent the monopolist's final equilibrium position, of course, but the profitability of this move is sufficient to prove the theorem.

V. Policy Conclusions

The policy conclusions to be drawn from the analysis can best be put in the form of answers to five questions which the policy maker must inevitably ask.

1. *Is there a socially optimal degree of product differentiation?* The answer to this basic question is yes, there is a socially optimal degree of product differentiation, divergence from which will increase the resources needed to enable consumers to attain specified levels of welfare.

2. *Can the optimum, or divergences from it, be easily recognized in the economy or in a particular industry?* The answer to this question seems, alas, to be no. Although there are clearly defined conditions that must be satisfied for the optimal configuration for a specific number of product differentiates, there are no easily recognizable conditions with respect to the actual number of goods.

3. *Would perfect competition throughout the economy result in attainment of the optimum?* The pricing system appropriate to the optimal configuration would be established under perfect competition. However, the problem of optimal differentiation is most important and most interesting under increasing returns to scale. Perfect competition, under these circum-

stances, could not take advantage of the scale economies and thus would not generate the optimum, which would require marginal cost pricing and single firm output for each good. Thus we cannot look to perfect competition to solve the optimum differentiation problem.

4. *Will market imperfections tend to give a nonoptimal degree of product differentiation?* Under constant returns to scale, no. Under increasing returns to scale, yes.

5. *Does market imperfection give a consistent bias in the degree of product differentiation, always too little or always too much?* Under increasing returns to scale, when imperfect competition tends to give nonoptimal differentiation, the direction of bias depends on the exact market structure. Monopolistic competition will lead to too much differentiation, monopolization of a market sector to too little. More complex, and thus more realistic, market structures may be expected to show effects of both kinds, leaving the direction of bias uncertain.

REFERENCES

H. Hotelling, "Stability in Competition," *Econ. J.*, 1929, *39*, 41–57.

K. Lancaster, "A New Approach to Consumer Theory," *J. Polit. Econ.*, Apr. 1966, *74*, 132–57.

———, *Consumer Demand: A New Approach*, New York 1971.