

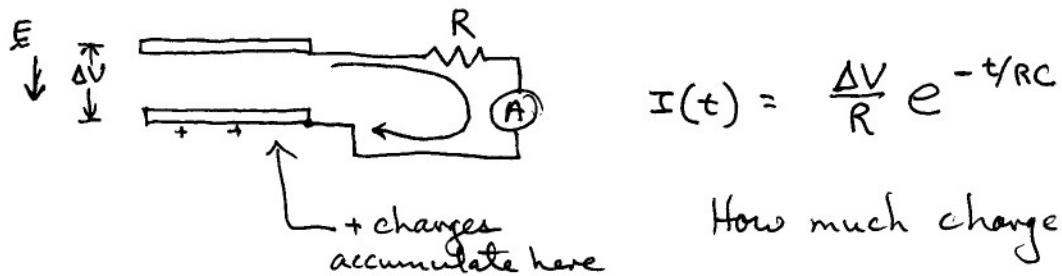
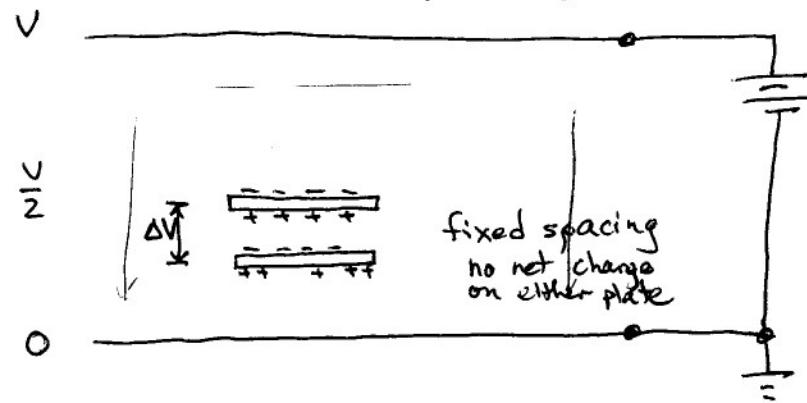
I. What is electric field? (Electrostatic field - slow)
 Measure of force exerted on a point charge (positive)
 Vector quantity

Assume constant in space, time so $\vec{E} = E\hat{z}$

$$\text{Units: } \frac{N}{C} \Leftrightarrow \frac{J/C}{m} = \frac{V}{m}$$

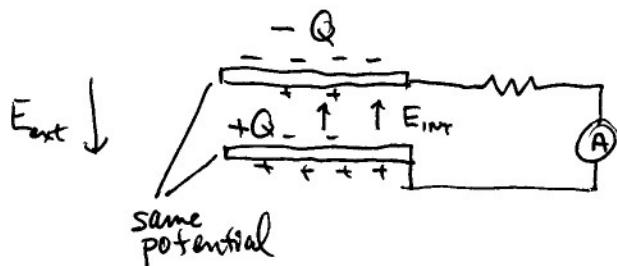
II. To measure electric field, allow the field to push around charges and measure this motion...

Set up an electric field geometry:



$$Q = \int_0^\infty I(t') dt' = \frac{\Delta V}{R} RC e^{-t/RC} \Big|_0^\infty$$

$$Q = C \Delta V$$



How much current is this?

How long does current flow last?

$$I_{\max} = \Delta V / R$$

$$\Delta V = 200 \frac{V}{m} \times 10^{-2} m \sim 2 V$$

$$R \sim \frac{1}{10} \Omega$$

$$I_{\max} \sim 20 A \quad (\text{big!})$$

$$RC = \frac{1}{10} \Omega \frac{\epsilon_0 (10^{-1} m)^2}{10^{-2} m}$$

$$\epsilon_0 \sim 10^{-11} F/m$$

$$\sim 10^{-12} s \quad \text{way too fast to see}$$

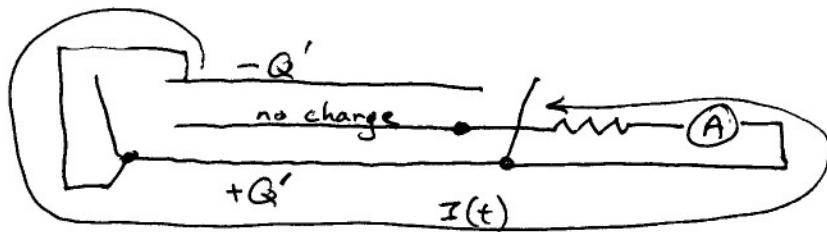
and our circuit analysis is too simple
as stray inductance would be
important.

Now modify the configuration as shown

$$\downarrow \quad \downarrow E$$

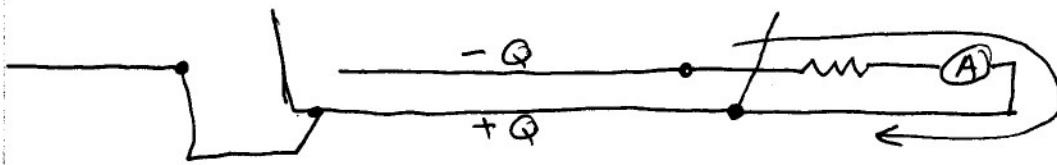


and add another electrode:



inner conductor is shielded from electric field
All its charge drains off

Move third conductor away



Capacitance between middle conductor and outer 2 conductors changes as one outer conductor is moved. When capacitance changes, current flows.

$$Q = C\Delta V$$

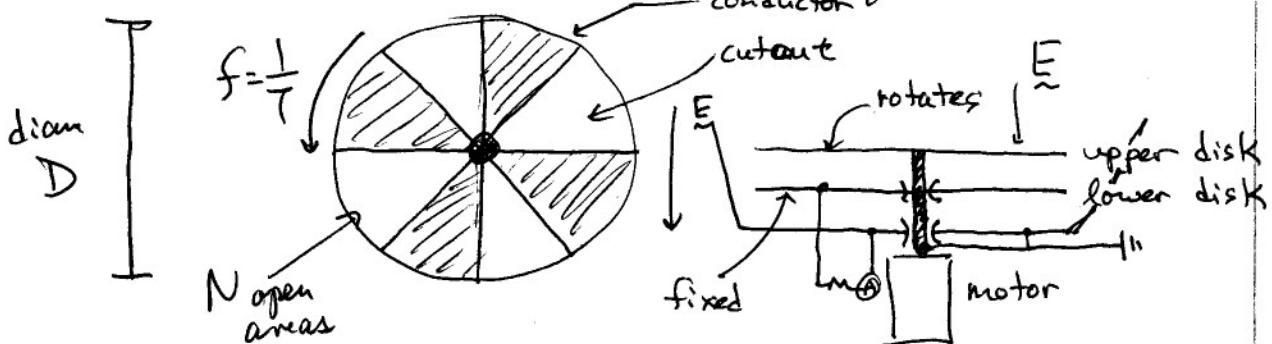
$$I = \dot{Q} = \dot{C}\Delta V + C\dot{\Delta V} = \dot{C}\Delta V$$

varies ↑ constant ↑

Field mills (electrostatic voltmeters) vary the capacitance and get currents to flow.

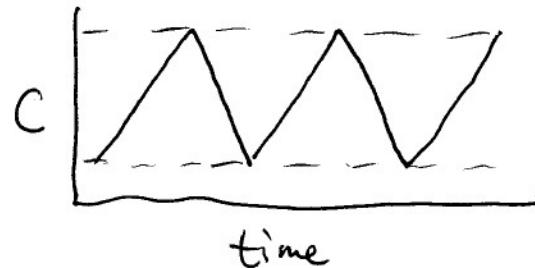
Here's a way to do this;

Make 2 disks with sectors of conductor removed



As upper disk rotates, it exposes and hides the lower disk to the field. The upper disk is grounded to the can. Measure current between fixed disk and can.

Capacitance



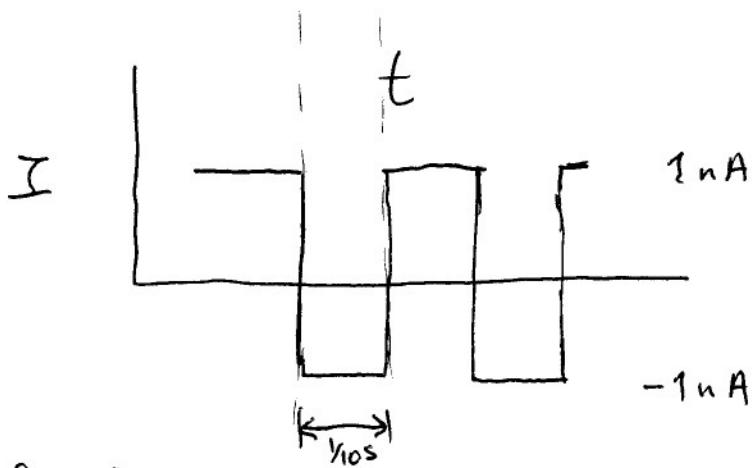
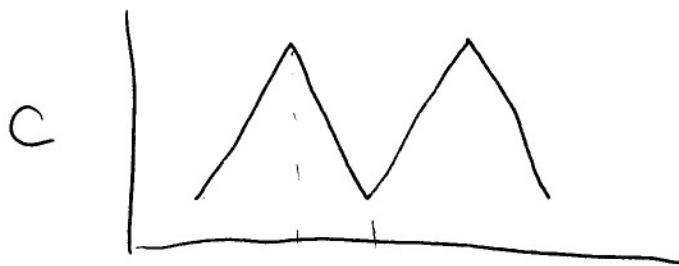
(4)

$$C_{\max} \text{ Capacitance} \approx \frac{\pi D^2}{4} \epsilon_0 \frac{1}{d} \frac{1}{2}$$

$$\frac{dC}{dt} = \frac{C_{\max} - C_{\min}}{T/(2N)} = \cancel{2} N f \frac{\pi D^2}{4} \frac{\epsilon_0}{2d}$$

$$I = \Delta V \dot{C} = \frac{\Delta V}{d} N f D^2 \frac{\pi \epsilon_0}{4}$$

$$I = E N f D^2 \frac{\pi \epsilon_0}{4}$$



$$f = 600 \text{ rpm} = 10 \text{ s}^{-1} \quad D = 0.1 \text{ m} \quad E = 2 \times 10^2 \text{ V/m}$$

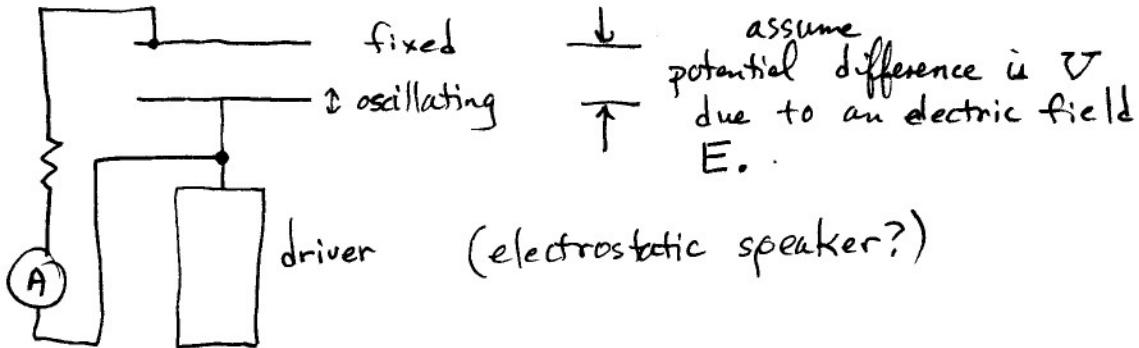
$$I = 2 \times 10^2 \times 1.2 \times (10 \times 10^{-10}) \frac{\pi^{2.2}}{4} 8.8 \times 10^{-12}$$

$$= 2.4 \times 2.2 \pi \times 10^{-10} \text{ A}$$

$$= 5.28 \pi \times 10^{-10} \text{ A}$$

$$= \underline{1.7 \text{ nA}}$$

Another similar scheme



$$\text{plate separation is } d(t) = d_0(1 + \epsilon \sin(2\pi ft))$$

$$\begin{aligned} I(t) &= CV + \dot{C}V = -\frac{\epsilon_0 A}{d^2} \dot{d}V \\ &= -\epsilon_0 A \frac{V}{d_0^2} \frac{d_0(f 2\pi \epsilon \cos(2\pi ft))}{(1 + \epsilon \sin(2\pi ft))^2} \end{aligned}$$

$$\text{amplitude of } I(t) \approx -\epsilon_0 A E 2\pi \epsilon f \quad \text{for } \epsilon \ll 1$$

$$\text{suppose: } E = 200 \text{ V/m}$$

$$A = 1 \text{ cm}^2 = 10^{-4} \text{ m}^2$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$$

$$\epsilon = 1/100$$

$$f = 10 \text{ kHz}$$

$$\rightarrow I \approx 8 \times 10^{-12} \times 10^{-4} \times 2 \times 10^2 \times 2\pi \times 10^{-2} \times 10^4$$

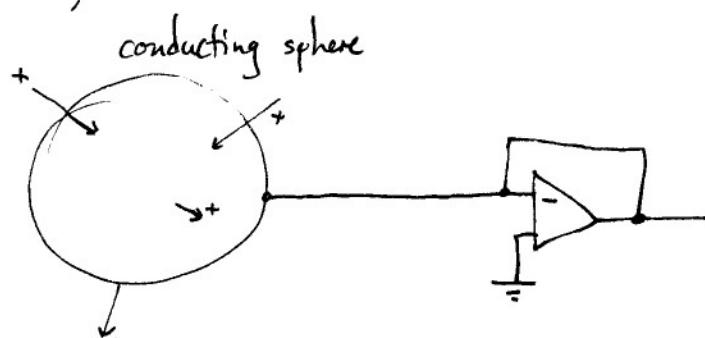
$$= 36\pi \times 10^{-12} \text{ A}$$

$$\approx 10^{-10} \text{ A} \quad (100 \mu\text{A})$$

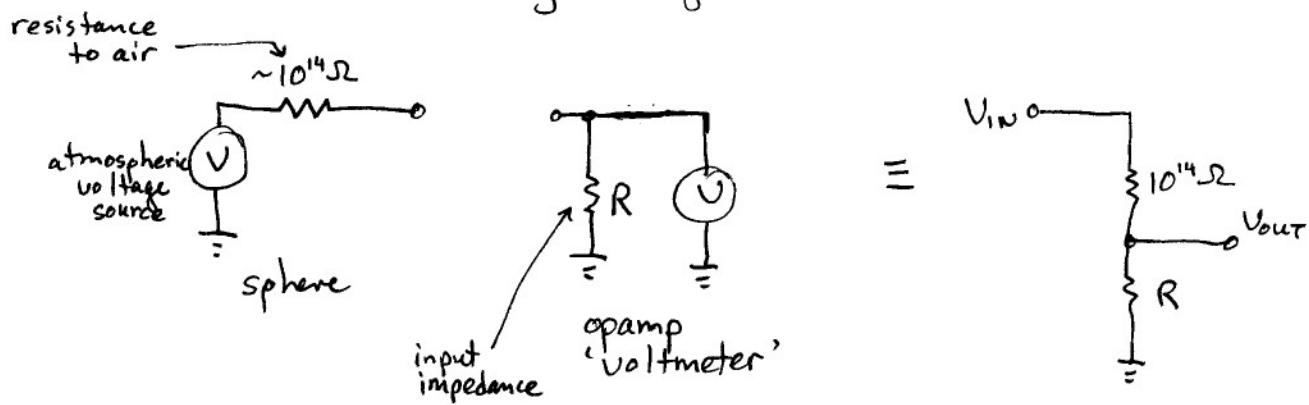
Increase plate area (A), amplitude of motion (ϵ), or oscillation frequency (f) to get more signal current.

Note: For fixed amplitude of oscillation, say $1/10 \text{ mm}$, moving the oscillating plate closer, say $1/3 \text{ mm}$ is helpful. But the $\epsilon \ll 1$ approximation is then violated, so the algebraic calculation must be redone.

Another technique works when the measurement is being done in a region in which current flows. This is the Langmuir probe technique, and it requires the sensor to collect charges — hence the need for currents to flow in the medium. This works in the atmosphere, better at higher altitudes where conductivity is larger.



sphere equilibrates to ambient potential by collecting or emitting charges.



$$\frac{V_{out}}{V_{in}} = \frac{R}{10^{14} + R}$$

typical Voltmeter has $R = 10M = 10^7 \Omega$ so $\frac{V_{out}}{V_{in}} \sim 10^{-7}$
too tiny!

special Voltmeter has $R \sim 10^{14} \Omega \Rightarrow \frac{V_{out}}{V_{in}} \sim \frac{1}{2}$, measurable!