OBJECT-BASED REASONING

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1. Introduction

Imagine a student who is asked to talk out loud while learning a worked-out solution to a mechanics problem (Chi, Bassok, Lewis, Reimann, & Glaser, 1989). The example problem, adapted from a popular physics textbook (Halliday & Resnick, 1981), presents a figure depicting a block hanging from a ceiling (see Fig. 1). The student looks at the block, which is held by three strings tied in a knot, and reads the first sentence of the solution text: “Consider the knot at the junction of the three strings to be the body.” She stops reading and asks, “Why is the knot the body . . . and I thought the block is the body?” This episode is representative of the way in which “good learners” in Chi et al. (1989) learned from examples. They noticed the gaps in their understanding of the worked-out solution and prompted themselves with questions that they then attempted to answer. Their answers, or “self-explanations,” were often correct. For instance, the student who asked herself “Why is the knot the body?” realized that, in the string example, the block exerts a force (i.e., the block’s weight) on the body (i.e., the knot) in the direction of gravity. From this she correctly inferred that a physical body is an arbitrary point of reference for which one has to perform a force analysis. This understanding, in turn, enabled her to solve successfully similar problems that were structurally isomorphic to the string example but that involved novel objects and novel spatial configurations (e.g., hockey sticks restraining a puck on a frictionless surface of ice). In general, Chi
et al. found a high positive correlation between successful transfer and the amount of spontaneous on-line inferences during learning (i.e., self-explanations).

In this chapter I describe results from several studies in which my collaborators and I examined how one type of spontaneous inferences, to which I refer here as object-based inferences, affect problem solving, analogical transfer, and similarity judgments. Object-based inferences reflect people’s knowledge about the entities that appear in word problems, figures, or statements. Such inferences were responsible for the students’ surprise that the knot rather than the block is the physical body—for raising the very question that initiated the self-explanation episode described earlier. Evidently, this student expected to find correspondence between the abstract physical concepts (i.e., body, forces), to which she was introduced while reading the physics chapter, and attributes of the specific objects (i.e., a block, strings) that instantiated these concepts in the example problem. In particular, because she understood that the block is upheld by the strings rather than vice versa, she expected that the asymmetric functional relation between the block and the strings will be preserved in the example problem: that the block will be the physical body and the strings the physical forces.

The studies I describe here are divided into two sets that show, respectively, effects of object-based inferences on the way people represent and process the stimuli they encounter. The first set of studies demonstrates that people include object-based inferences in the mental representations they construct for formally isomorphic word problems. For example, college students in Passos, Wu, and Osoth (1995) constructed different equations when solving mathematically isomorphic problems involving semantically symmetric versus semantically asymmetric object sets (e.g., two sets of
children vs children and prizes, respectively). The second set of studies demonstrates that object-based inferences lead to selective application of both basic processing mechanisms (e.g., comparison vs integration) and solution procedures acquired in formal training (e.g., addition vs division). For example, when asked to construct addition problems involving a given pair of object sets, college students readily added apples and oranges but refrained from adding apples and baskets (Bassok, Chase, & Martin, 1997).1 Because people's experience with the world is not arbitrary, they exploit semantic knowledge much like domain experts exploit nonarbitrary patterns of information in the domain of their expertise (e.g., Chi, Feltovich, & Glaser, 1981; Hinsley, Hayes, & Simon, 1977; Larkin, McDermott, Simon, & Simon, 1980). For example, when selecting an arithmetic operation, people are guided by knowledge that adding apples and oranges is both a more likely and a more meaningful operation than adding apples and baskets. In general, object-based inferences foster selection of stimuli-appropriate processing procedures and thereby ensure useful and reasonable conclusions. Unfortunately, this adaptive aspect of human cognition has been overlooked by most researchers who study learning, problem solving, and reasoning.

The studies reported here represent an attempt to incorporate object-based inferences into research on analogical transfer and similarity judgments. As I describe in the next section, researchers who study analogical transfer attempt to identify regularities in the process by which people retrieve analogous problems from memory (access) and the process by which people align the representations of analogous problems (mapping). In particular, they examine the impact of similarities and differences in the content covers of analogous training (base) and test (target) problems on access and mapping. Although researchers are concerned with effects of content on transfer performance, they treat content as irrelevant information that people fail to ignore. As a result, they fail to notice that content induces semantic knowledge that affects how people represent the base and the target stimuli, and that certain semantic distinctions implied by content lead to significant changes in processing. In order to situate the studies on object-based inferences within the conceptual framework that dominates research on analogical transfer, I begin with a brief description of the paradigmatic contrast between superficial content and solution-relevant structure of analogous problems.

1 The pair apples and oranges alludes to the common saying “It's like comparing apples and oranges.” As I explain later, the meaning of this saying (i.e., comparison is not independent of the things being compared) captures the gist of the argument I make in this chapter. Hence, I use the pair “apples—oranges” for the purpose of exposition. However, this pair of objects was not used in the experiments reported in this chapter (the closest object pair was peaches and plums).
II. Separation and Contrast between Content and Structure

A flexible application of knowledge, be it knowledge of Newton's laws, linear equations, or social rules, requires that people be able to recognize abstract structural similarities between situations that differ in appearance. This ability is measured on most intelligence tests. For example, people are credited with intelligence points on tests of analogical reasoning when in response to the pair "apples: baskets" they choose the pair "students: classrooms" rather than "apples: oranges." In other words, psychologists reward people for heeding similarities in the way different objects are interrelated (e.g., contain [container, content]) rather than similarities between attributes of the specific objects that serve as the arguments, or "shot filters," of different structural relations (e.g., apples).

Using a research paradigm similar to that of analogical-reasoning intelligence tests, researchers who study analogical transfer present subjects with a solution to one or more base problems and, after a short delay which may involve an intervening task, ask them to solve one or more target problems. In most studies the base and target problems are constructed such that they share a similar structure but differ in their content covers. Some studies also use base and target problems that share similar content covers but differ in structure. This orthogonal design enables researchers to separate and contrast the impact of similarities and differences in the content and structure of base and target problems on access and mapping.

To measure access, or spontaneous retrieval of a base problem from memory, researchers compare the frequencies of transfer solutions with and without informing subjects that the solution to one of the base problems can assist them in solving the target problem (i.e., compare informed and spontaneous transfer). To measure mapping, or a person's ability to successfully align the underlying structures of analogous problems, subjects are explicitly asked to solve the target problem by the method used in solving an analogous base problem.

The basic finding in this line of research is that analogical transfer is highly sensitive to similarities and differences in the context covers of base and target problems (see Reeves & Wiensberg, 1994, for a recent review). That is, content affects transfer even though it should not affect problem solutions. For example, many subjects fail to realize that a convergent solution learned in the context of a military problem can be applied to an analogous medical problem (Gick & Holyoak, 1980); that a statistics principle learned from an example about weather forecasting can be applied to a problem about arrangements of pizza toppings (Ross, 1987); or that a physics equation learned for motion can be applied to an analogous problem dealing with bushels of potatoes (Bussek & Holyoak, 1989).
Differences between content domains of analogous problems often impair access without affecting mapping (e.g., 60% spontaneous vs 90% informed transfer of the convergence solution between a military base and a medical target in Gick & Holyoak, 1980). However, differences between the specific objects that serve as arguments of analogous problems were found to mediate mapping performance (Gentner & Toupin, 1986; Ross, 1987, 1989). For example, children in Gentner and Toupin’s (1986) study learned a base story in which a squirrel visited a frog. Then they were explicitly asked to enact an analogous target scenario involving a chipmunk and a toad (i.e., mapping). Children’s mapping performance was much better when the chipmunk visited the toad than when the toad visited the chipmunk—when the guest and the host in the base and target stories were similar (e.g., squirrel—chipmunk, frog—toad) rather than different (e.g., squirrel—tadpole, frog—chipmunk).

Theoretical accounts of access and mapping mirror the experimental methodology by contrasting the impact of similarities and differences in content and structure on people’s performance (e.g., Falkenhainer, Forbus, & Gentner, 1989; Forbus, Gentner, & Law, 1995; Hofstadter, Mitchell, & French, 1987; Holyoak & Thagard, 1989; Hummel & Holyoak, in press; Thagard, Holyoak, Nelson, & Gaschler, 1990). Specifically, these accounts hold that access and mapping involve matches and mismatches in two distinct types of aspects that comprise the mental representations of the base and target stimuli: (1) attributes, one-place predicates that take objects as arguments (e.g., FURRY [x]), mapping [y]); and (2) relations, predicates that take as arguments two or more objects (e.g., VISIT [x, y]) or propositions (e.g., CAUSE [LIKE [x, y], VISIT [x, y]]). Despite some important differences in the processing assumptions of the proposed computational models, attributional and relational matches and mismatches receive different weights that capture their differential impact on access and mapping.

According to Gentner’s (1983) structure-mapping theory, what is important to analogical transfer is correct alignment of the relational structures in the base and target stimuli (e.g., VISIT [x, y]), while attributes of the specific objects that instantiate these structures (e.g., FURRY [x]) can or should be ignored. This is because the distinction between relations and attributes usually coincides with the distinction between what is and what is not relevant to problem solutions (but see Holyoak, 1985). If so, one may wonder why is it that people retain superficial content in problem representations and let similarities and differences in content affect their transfer performance. Medin and Ross (1989); see also Medin & Ortony, 1989) proposed that such behavior is justified by considerations of cognitive efficiency maximization. Specifically, they argued that because content and structure are often correlated in the world (e.g., chipmunks are more likely
than toads to behave like squirrels) people use similarities in content as a useful, albeit error-prone heuristic that can help them identify likely similarities in structure.

As with every heuristic, people rely on similarities and differences in content to conserve cognitive resources and speed access and mapping (Hinney et al., 1977; Novick, 1988). However, effects of similarities and differences in content are most pronounced when people do not know which aspects of the base and target problems are relevant to the problems’ solutions. That is, experts, good learners, and older children are more likely than novices, poor learners, and younger children to ignore superficial differences in content, context, and phrasing of analogous problems (e.g., Chi et al., 1981, 1989; Gentner & Rattermann, 1991; Gentner & Toupin, 1986; Novick, 1988, 1992; Schorr-Helft & Herrmann, 1982; Silver, 1981). Also, training conditions that encourage abstraction from content (e.g., helping people represent the squirrel as an abstract visitor) decrease the number of mismatches in content and therefore increase the probability of successful transfer (e.g., Basok & Holyoak, 1989; Brown, 1989; Catrambone & Holyoak, 1989; Gick & Holyoak, 1983; Gick & McGarry, 1992; Needham & Begg, 1991; Reed, 1993; Ross & Fenney, 1990).

To summarize, the conceptual framework that guides research on analogical transfer treats content (object attributes) and structure (relations between objects) as separate aspects of the stimuli and contrasts effects of similarities and differences in these two types of aspects on people’s performance. Within this view, transfer fails because people retain superficial aspects of content in problem representations, and the “undeleted” aspects of content lead to mismatches that impair access and mapping. Such effects are most likely to occur when people lack the ability, the knowledge, or the cognitive resources needed for abstracting the relevant structure from its superficial content.

The gist of this view (i.e., the difficulty involved in abstracting, or separating, structure from content) is captured in several metaphors that are implied by the labels with which researchers refer to content and structure (Lakoff & Turner, 1989). Equating content with “surface” implies that structure is like a hidden treasure, and that people must invest effort and ingenuity to discover it. Equating structure with “essence” implies that, like alchemists, people need special knowledge and skills to extract the valuable structural essence of an object hidden by content. Describing people’s performance as “content-bounded” implies that people are prisoners of content who strive to be “content-free.” Whatever their superficial differences, the hidden treasure, alchemists, and jail inmates share the underlying assumption that content effects arise from failures of an imperfect cognitive system to ignore content.
III. Semantic Knowledge Determines How People Represent Problems

In the previous section I pointed out that the paradigm by which researchers contrast people’s responses to relational and attributional similarities resembles that paradigm used in psychometric tests of analogical reasoning. Note, however, that by treating content as superficial information, researchers who study analogical transfer exclude from their theoretical accounts the very inferences for which people are credited with intelligent points on Analogies tests (e.g., the inference that applies and baskets imply contain-
ment). That is, despite the resemblance in labels and methodology, research on analogical transfer ignores the inferential process that proceeds and/or accompanies access and mapping—the interpretive process by which people understand, or abstract, the structures of the base and the target problems.

Researchers who study how content affects access and mapping acknowl-
edge that these processing mechanisms operate on mental representations people construct for the base and target problems (e.g., Holyoak, 1985). That is, they acknowledge that the current neglect of interpretation is only temporary. In the meantime, they test their processing assumptions using problem representations that they themselves construct as a working hy-
pothesis about representations that might be constructed by subjects. How-
ever, the hypothesized representations constructed by experimenters may not coincide with the representations constructed by subjects. In particular, although researchers assume that the base and the target problems share the same structure and differ only in content (i.e., mismatching object attributes), their subjects may infer from content that the problems differ in structure (i.e., mismatching inferential relations). Hence, in addition to neglecting potentially important interpretive effects of content, researchers are risking erroneous conclusions about processing.

In what follows I describe results from three studies in which the specific objects in analogous or mathematically isomorphic base and target prob-
lems induced inferences that led to abstraction of nonisomorphic interpreted structures. To foster comparison with previous work, the studies I describe here used base and target problems similar to those used in previous studies on analogical transfer.

A. MATERN OF CONVERGENCE

Holyoak and Koh (1987, Experiment 2) compared transfer of a convergence solution learned for two versions of a light bulb problem (base) to Duncker’s (1945) tumor problem (target). The two base problems described a research assistant who fused a broken filament of an expensive light bulb using,
either low-intensity lasers or low-intensity ultrasound waves. The research assistant applied the lasers (waves) simultaneously from different directions such that they converged on the broken filament. The combined intensity of the lasers (waves) at the point of convergence was sufficiently strong to fuse the broken filament. At the same time, because of their low intensity on the way to the filament, the lasers (waves) did not break the fragile glass surrounding the filament. The analogous tumor problem, which could be solved using the convergence solution, described a doctor who needed high-intensity X rays to destroy a stomach tumor, but was concerned that high-intensity rays may harm the healthy tissue surrounding the tumor.

Holyoak and Koh (1987) found that the frequency of convergence solutions to the target tumor problem (i.e., spontaneous transfer) was significantly higher in the lasers than in the waves base condition (69 vs. 38%, respectively). They explained this differential magnitude of spontaneous transfer by matches and mismatches between attributes of the converging forces (i.e., lasers are more similar to rays than are ultrasound waves). They also found that informing subjects about the relevance of the base solutions eliminated the effects of matches and mismatches in object attributes (75 vs. 81% informed transfer from the lasers and waves versions, respectively). From this they concluded that matches and mismatches in object attributes of structurally isomorphic problems (i.e., problems with matching structures) have a more pronounced impact on access than on mapping.

Tammy Smith and I examined the possibility that instead of or in addition to being mediated by direct matches and mismatches in object attributes, the results in Holyoak and Koh (1987) were mediated by matches and mismatches in the object-based interpreted structures of the base and the target problems. Specifically, we conjectured that people spontaneously infer the manner in which various forces converge, and treat the inferred manner of convergence as a structural constraint. That is, we conjectured that spontaneous transfer from lasers to X rays was supported by matches in manner of convergence (i.e., matching interpreted structures), while spontaneous transfer from waves to X rays was impaired by mismatches in the manner of convergence (i.e., mismatching interpreted structures).

The spatial nature of the convergence solution (Beveridge & Pankins, 1987) suggests that asking subjects to draw a diagram depicting the problem’s solution can capture important characteristics of their mental representations. We employed this methodology to examine how people represent converging forces in the three story problems used by Holyoak and Koh (1987). Specifically, after modifying the text of the tumor problem to include the convergence solution, we asked undergraduate students from the University of Chicago to read one of the three convergence problems.

2 This experiment was conducted by Tammy Smith as part of her B.A. honors project.
and draw a diagram depicting how the lasers, the ultrasound waves, or the X rays converged on the Blument (tumor).

Analysis of the diagrams revealed that, indeed, subjects included the manner of convergence in their representations of the problem solutions. We classified the diagrams into two categories: Lines or Waves (100% agreement between two independent judges). Diagrams were classified as Lines when the converging forces were represented by straight lines, arrows, or by tunnels filled with dots (representing particles). Diagrams were classified as Waves when the converging forces were represented by wavy lines (smooth or sharp) or by series of expanding arches. Figure 2 presents representative diagrams from the Line and Wave categories.

As we expected, subjects constructed different representations of convergence for the two light bulb problems. All 18 subjects in the light bulb–laser condition (100%) drew only 4 of the 22 subjects in the light bulb–waves condition (18%) drew Line diagrams. Moreover, 14 of the 17 subjects (82%) who received the tumor problem represented X rays with straight lines. That is, consistent with the pattern of spontaneous transfer in Holyoak and Koh (1987), the manner of convergence (rays) in the tumor target was similar to the manner of convergence in the lasers–light bulb base but differed from the manner of convergence in the waves–light bulb base. In order to examine whether and how matches and mismatches in manner of convergence affect access and mapping, we replicated Holyoak and Koh's (1987) transfer experiment with two modifications. The first modification was that subjects were asked to draw the solutions to all the problems they read or solved (base, filler, and target problems). The second modification was that we used two rather than one version of the tumor target problem: a rays version (the forces were labeled "gamma rays") and a waves version (the forces were labeled "omega waves"). The second subjects were 155 undergraduate students from the University of Chicago, who were randomly assigned to learn and solve one of the four combinations of two base and two target problems: two matching-manner conditions (lasers–rays, waves–waves) and two mismatching-manner conditions (lasers–waves, waves–rays). Subjects who did not draw diagrams for the base and/or the target problems were excluded from the relevant analyses.

The diagrams drawn for the light bulb base problems were similar for those drawn in the first experiment: most subjects drew Line diagrams to represent converging lasers and Wave diagrams to represent converging ultrasound waves (84 vs 31% Line diagrams, respectively). However, unlike in Holyoak and Koh's (1987) study, spontaneous transfer was uniformly high in all experimental conditions (above 80%). That is, matches and
mismatches in manner of convergence did not affect the frequency of spontaneous transfer. Whatever the reason for this uniformly high level of spontaneous transfer, subjects’ performance cannot be explained by the fact that they ignored the manner of convergence. On the contrary, analysis of the convergence diagrams that accompanied the transfer solutions to the waves and rays versions of the tumor target problems revealed that the manner of convergence had a very interesting and highly significant impact on application of the learned convergence solution.
Figure 3 depicts the frequency of Line diagrams drawn for the tumor target problems in the four experimental conditions, including diagrams drawn by subjects who applied the convergence solution only after being informed about its relevance (i.e., informed transfer). As can be seen in Fig. 3, subjects in the lasers base condition drew significantly more Line diagrams for the two tumor target problems than did subjects in the waves base condition (75 vs 39%, respectively). That is, subjects actually transferred the manner of convergence from the base to the target problems. Representations of convergence were also affected by the type of force in the tumor target problems: Subjects drew more Line diagrams for the rays (64%) than for the waves tumor problem (49%). However, transfer of manner did not interact with the type of force in the target problems. That is, subjects in the lasers base condition drew more Line diagrams than subjects in the waves base condition both for the rays (80 vs 44%, respectively) and for the waves (65 vs 30%, respectively) versions of the tumor problem.

This pattern of transfer indicates that, for many subjects, transfer occurred not because they realized that mismatches in manner of convergence can be ignored as irrelevant to the problem’s solution (i.e., abstraction in the sense of content deletion). Rather, transfer occurred because subjects interpreted the manner of convergence in the mismatching target problems.

Fig. 3. Frequency of Line diagrams drawn to represent the convergence solutions for the rays and waves tumor targets in the lasers and waves light bulb base training conditions.
as identical to the manner of convergence in the base problems (i.e., interpretation and accommodation). These results are consistent with the findings of Medin, Goldstone, and Gentner (1993) who found that similarity judgments may involve a constructive process of feature reorganization. Specifically, Medin et al. found that people who were asked to judge similarity between geometric figures interpreted ambiguous features of target stimuli such that these features matched the features of the base stimuli (e.g., they interpreted three and a half prongs in a target figure as either three or four prongs, depending on whether the base had three or four prongs).

Of course, mismatches between base and target stimuli cannot always be resolved by feature interpretation (or reinterpretation). The next study shows that discrepancies in manner of change, inferred from object attributes (e.g., ice melting is continuous but ice deliveries to a restaurant are discrete), can severely impair both access and mapping.

B. MANNER OF CHANGE

The experiments on manner of change follow up on previous work in which Klahn Holyoak and I compared transfer of the same solution procedure learned either in the context of algebra or in the context of physics (see Basok & Holyoak, 1993, for a summary and discussion of this earlier work). We expected that, because algebra training is more conducive to abstraction from content than is physics training, people will be more likely to transfer the learned solution from algebra to physics than vice versa. To test this hypothesis, Holyoak and I used analogous word problems from algebra and physics. All problems described situations of constant change. The algebra problems asked for the sum of an arithmetic series (e.g., money earned by a person whose yearly salary increased by a constant amount during a 5-year period). The physics problems asked for distance traveled by a moving body during a period of constant acceleration (e.g., distance traveled by a car whose speed constantly increased during a 5-min period).

Subjects received the algebra and physics problems either in training (base) or in transfer (target) tests. Consistent with our abstraction-com-content hypothesis, most subjects trained in algebra (90%) spontaneously used the arithmetic-series equations to solve the target physics problems. By contrast, only a minority of subjects trained in physics (10%) spontaneously used the distance equation to solve the target algebra problems (Basok & Holyoak, 1989). Our initial explanation for lack of transfer from physics to algebra was that the physics content (e.g., speed) is included in the representations of the base problems and interferes with people’s ability to recognize that the algebra problems have a similar structure (i.e., constant
change). However, subsequent experiments revealed that this explanation was incomplete because transfer depended on the nature of the changing entities.

In Banos (1990), subjects trained in physics spontaneously transferred the physics solution to algebra problems in which the changing entities were rates (e.g., typing rate in words per minute), but failed to transfer the physics solution to algebra problems in which the changing entities were amounts (e.g., yearly dollar amounts of salary). Because the base physics problems described constant change in a rate entity (e.g., speed in meters per second), the results of Bassok (1990) indicate that people readily ignore mismatches in object attributes (e.g., meters vs words), but they do not ignore mismatches in the internal structure of the changing entities. That is, rate is a two-place predicate (e.g., TYPING RATE (words, minute)), whereas amount is a one-place predicate (e.g., DOLLAR AMOUNT (salary)). In other words, these results suggest that mismatches in local, lower order relations (rate vs amount) impair recognition of matches in more global, higher order relations (constant change).

Karen Obert and I tested the possibility that the rate and amount entities in constant-change problems impaired transfer performance by creating mismatches in the global representations of constant change (Bassok & Obert, 1995). Specifically, we conjectured that, as in the convergence experiments described earlier, people inferred the manner of constant change, continuous or discrete, and treated differences in the inferred manner of change as differences in structure.

Using a categorization task and an analysis of gestures that accompanied verbal descriptions of constant change (Allbali, Bassok, Obert, Syc, & Goldin-Meadow, 1995), we first examined whether people spontaneously infer the manner in which various entities change with time. We found a pattern of matches and mismatches in manner of change that was consistent with the pattern of transfer in Bassok (1990). For example, both speed and typing rate were understood to be changing continuously (matching interpreted structures), but yearly changes in salary were understood to be changing discretely (mismatching interpreted structures).

We then proceeded to examine whether and how matches and mismatches in manner of change affect access and mapping. We used an experimental design that varied orthogonally the manner of change in the base and target problems. Moreover, to control for possible effects of other types of matches and mismatches between the base and the target problems, we compared transfer from continuous and discrete base problems to pairs of continuous and discrete target problems that were exposed in context and phrasing. Table 1 presents one continuous (speed) and one discrete
TABLE I

A Continuous and a Discrete Base Problem, with Matching Pairs of Continuous and Discrete Targets*  

| Speed base (Continuous): The speed of an airplane increases at a constant rate during a period of 12 min from 10 m/s to 34 m/s. What distance, in miles, will the plane travel during the 12-min period? |
| Population-rate target (Continuous): The rate of population growth by a certain country increased steadily during the last 12 years from 3000 people/year to 15,000 people/year. How many people total were added to the population during the 12-year period? |
| Attendance-rate target (Discrete): An annual arts and crafts fair is held on November 1 every year. The attendance rate at the annual fair increased steadily during the last 12 years from 3000 people/year to 15,000 people/year. How many people total attend the fair during the 12-year period? |
| Investment base (Discrete): During the last 16 months the monthly deposits into a certain savings account constantly increased from $200/month to $400/month. How much money total was deposited into the account during the 16-month period? |
| Melting-ice target (Continuous): The rate at which ice melts off a glacier steadily increases over an 8-week period from 50 ft/week to 106 ft/week. What is the total weight of the ice that will melt off the glacier over the 8-week period? |
| Ice-delivery target (Discrete): Ice is delivered to a restaurant once a week over an 8-week period. The weight of the deliveries steadily decreases from 50 lb the first week to 106 lb the 8th week. What is the total weight of the ice that will be delivered over the 8-week period? |


(investment) base problem, each with its matching pair of continuous and discrete targets. When the base problems described continuous change (speed), transfer to continuous targets (matching manner) was frequent, but transfer to discrete targets (mismatching manner) was rare. For example, significantly more subjects used the physics distance equation to find the number of people in the continuous-population than in the discrete-attendance target (71 vs. 27%, respectively). Because the continuous and discrete target problems were matched in all other aspects of content and phrasing (e.g., both referred to people/year), these results indicate that matches and mismatches in the inferred manner of change were responsible for this overwhelming difference in the frequency of spontaneous transfer. Interestingly, when the base problems described discrete change (investment), mismatches in manner of change did not impair spontaneous transfer. For example, following an economics training session in which subjects learned to solve invest-
ment problems (discrete), spontaneous transfer was similarly high to the discrete ice-delivery target (100%) and to the continuous melting-ice target (88%).

The asymmetric impact of mismatches in manner of change on the frequency of spontaneous transfer (i.e., high transfer in the discrete-to-continuous direction and low transfer in the continuous-to-discrete direction) was accompanied by an asymmetry in the relative difficulty of mapping (i.e., informed transfer). An analysis of the informed-transfer solutions revealed that subjects had no difficulty aligning the representations of discrete base and continuous target problems, but found it quite difficult to align continuous base and discrete target problems. Table II presents a representative excerpt of a verbal protocol from each of the two mismatching-transfer conditions. The top panel of Table II demonstrates a straightforward instantiation of the base sum-of-investments equation (discrete) with values of the target speed problem (continuous). The bottom panel demonstrates repeated attempts to apply the base distance equation (continuous).

<table>
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<th>TABLE II</th>
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<td>EXAMPLES OF VERBAL PROTOCOLS THAT ACCOMPANIED INFORMED TRANSFER IN THE DISCRETE-TO-CONTINUOUS AND CONTINUOUS-TO-DISCRETE CONDITIONS</td>
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**Discrete (investments) to continuous (speed):**

The sum would be equal to what you start with plus what you end with divided by 2.

65, start with 10 + 34 is 44, divided by 2 equals 22, times 12 is 264. . . miles.

**Continuous (speed) to discrete (investments):**

- **Alignment 1, mapping:**
  I guess I'm supposed to apply these equations to this somehow, or they're analogs. 12%, all right then. Initial payment would correspond to initial speed and final payment to final speed, and so, average payment would be their sum divided by . . .
  Alignment 2, repeat 12 times.
  But wait, I was thinking whether I should use each month as an initial payment, in other words, apply this 12 times. But I don't want to do that. It's too much work.
  Back to Alignment 1, solution:
  I'm going to see if I can come out with the right answer by just treating it with the first and last months' payments. So, I add 232 and 800 and get 332, then divide by 2 gives 166.
  Hesitation between Alignment 1 and 2.
  I guess . . . umm, that's my average payment. How much money total over a year. . . uh, 12 average payments? 166 times 12 is 1992.

to the target investments problem (discrete). These attempts involved shifts in alignment and explicit variable mappings.

The asymmetry in transfer between problems with mismatching manner of change cannot be explained by a difference in the level of abstraction from content. Rather, this asymmetry reflects a difference in the possibility of reinterpreting the target such that it fits the representation of the base. As in the convergence experiments, transfer in the discrete-to-continuous condition was possible because continuous change in the target problems could be transformed (pamed) into an arithmetic series of discrete values and therefore fit the discrete structure of the base. For example, if the rate at which ice is melting off a glacier constantly increases over a period of 8 weeks from 50 lb/week to 100 lb/week, the melting rate increases every week by a constant (8 lb/week), and the consecutive weekly values (e.g., 58 lb/week, 64 lb/week, . . . ) actually exist. This type of transfer was impossible in the continuous-to-discrete condition because, in order to transform discrete targets to fit a continuous base structure, people would have to hypothesize values that are unlikely to exist (e.g., hypothetize continuous deliveries of ice to a restaurant).

It is possible that the asymmetry in transfer between base and target problems with mismatching manner of change was responsible for the overwhelming asymmetry in transfer between algebra and physics in Bassok and Holyoak (1989). This is because the arithmetic-series algebra problems described discrete constant change, whereas the speed physics problems described continuous constant change. Of course, it is also possible that both transfer type (domain general vs domain specific) and the inferred manner of change (discrete vs continuous) contributed to the ease of transfer from algebra to physics and the difficulty of transfer from physics to algebra.

C. SEMANTIC SYMMETRY AND ASYMMETRY

Object-based inferences often refer to semantic relations between two or more entities. An important aspect of such inferences, to which I refer here as semantic symmetry and asymmetry, is whether the structural roles of arguments in a given semantic relation can or cannot be reversed. For example, PLAY (child A, child B) is a symmetric semantic relation, but PLAY (child A, ball B) is an asymmetric semantic relation. Ling-Ling Wu, Karen Olsho, and I found that inferences about semantic symmetry and asymmetry determined how people represented and solved probability problems (Bassok et al., 1995).

Our experiments were motivated by the work of Brian Ross (1987, 1989). Ross examined how college students used examples when solving novel
probability problems. He found that these adult subjects engaged in object
mapping much like the children who were aligning a chipmunk with a square
rather than with a frog in Gentner and Toupin’s (1986) study. For
instance, subjects in Ross (1989) received an example of a permutation
problem in which cars were randomly assigned to mechanics (base). Then
they were asked to apply the learned solution to an isomorphic permutation
problem in which scientists were randomly assigned to computers (target).
Subjects received the relevant equation \(1/n[n-1][n-2] \ldots [n-r+1]\)
and were asked to instantiate it with the appropriate values (i.e., \(n = \)
the number of scientists).

In the target problem, scientists had to be placed in the structural role
of the cars (because both were the randomly assigned sets) and computers
in the structural role of the mechanics (because both were the sets of
assignments). However, subjects erroneously solved the target problem by
placing scientists in the role of the mechanics and computers in the role
of the cars. That is, they solved the target problem as if computers were
assigned to scientists rather than scientists to computers. Ross explained
such erroneous solutions by the fact that subjects tend to place similar
objects in similar structural roles (i.e., object mapping). Specifically, he
argued that subjects placed scientists in the role of the mechanics because
both were animate objects, and placed computers in the role of the cars
because both were inanimate objects.

The object-mapping explanation accounts quite well for the pattern of
transfer results in Ross (1989). However, these results can be also explained
by object-based interpretation of the base and the target problems. For
example, because subjects know that mechanics receive cars and scientists
receive computers, it is possible that they included the asymmetric func-
tional relation car(receives, gives) in the mental representations of the
base and target permutation problems. From the worked-out solution to
the base problem they learned that the gives (cars) were assigned to
the receivers (mechanics). Accordingly, in trying to align the asymmetric
interpreted car structures of the base and target problems, they solved
the target problem by assigning the gives (computers) to the receivers
(scientists). In general, because animate objects were always the receivers
and inanimate objects were always the given in Ross’s problems, subjects’
performance cannot be explained definitively by either object-mapping or
alignment of interpreted structures.

In Basok et al. (1995), we modified the problems and the procedure used
by Ross (1989) such that we could examine whether transfer is mediated
by abstraction of object-based interpreted structures (e.g., car). All base
and target problems in our experiments described a person (e.g., a teacher)
who randomly assigned three elements from one set (n prizes) to three
elements from another set (in students) and asked for the probabilities of such random assignments. The assignment problems had the same mathematical structure (permutation), but differed in the elements that served as the assigned and assignee sets (s and m, respectively). Table III presents two representative problems from Experiment 1 in Bassok et al. In the first problem, computers are randomly assigned to secretaries; in the second problem, doctors from one hospital are randomly assigned to doctors from another hospital.

The paired element sets of objects (O) and people (P) were selected such that, if subjects engage in object-based problem interpretation, they should abstract asymmetric interpreted structures (e.g., cat) for problems involving objects and people (O-P, e.g., computers and secretaries) and symmetric interpreted structures (e.g., pair) for problems involving similar sets of people (P-P, e.g., doctors from two hospitals). To test this prediction, we asked undergraduate students to solve either the O-P or the P-P permutation problems. Because our subjects never learned how to solve such problems, their solutions were incorrect. Yet, consistent with our inter-

<table>
<thead>
<tr>
<th>TABLE III</th>
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<tbody>
<tr>
<td>SEMANTICALLY ASYMMETRIC AND SYMMETRIC PERMUTATION PROBLEMS^</td>
</tr>
</tbody>
</table>

**Computers and Secretaries (O-P, Objects assigned to People)**
In a big publishing company, some secretaries will get to work on new personal computers. The company received a shipment of 21 computers, with serial numbers in a running order, from 10075 through 10095. There are 25 secretaries in the company that would like to work on a new computer. The names of the secretaries are listed in order of their work experience, from the most experienced secretary to the least experienced one. The manager of the company randomly assigns computers to secretaries according to the work experience of the secretaries. What is the probability that the three most experienced secretaries will get to work on the first three computers (10075, 10089, and 10077), respectively?

**Doctors and Doctors (P-P, People assigned to People)**
In a medical meeting, doctors from a Minnesota hospital will get to work in pairs with doctors from a Chicago hospital. There is a list of 20 doctors from Chicago, arranged in alphabetical order. There are 15 doctors from Minnesota that would like to work with the doctors from Chicago. The names of the Minnesota doctors are listed in the order of their social security numbers, from highest to lowest. The chairman of the meeting randomly assigns doctors from Minnesota to doctors from Chicago according to the alphabetical order of the Chicago doctors. What is the probability that the first three doctors on the Minnesota hospital's social security number list will get to work with the first three doctors on the Chicago hospital's alphabetical list, respectively?

pretation hypothesis, these incorrect solutions captured the inferred semantic symmetry or asymmetry of the paired object sets. Specifically, most subjects (87%) in the O→P condition constructed equations in which the two sets of elements played asymmetric mathematical roles (e.g., \( m^2/n^2 \)). By contrast, most subjects (78%) in the P→P condition constructed equations in which the two sets of elements played symmetric mathematical roles (e.g., \( (n + m)(m + n)/2 \), \( \sqrt{mn} \)).

After they solved (incorrectly) the unfamiliar permutation problems (base), subjects read a chapter that explained the relevant probability concepts. The chapter included the correct equation for solving permutation problems and, for each subject, a worked-out solution to the base problem. The subject initially failed to solve correctly. Following this training session, subjects received a target problem together with the relevant equation (1/(n(n-1)(n-2))). They had to instantiate the equation for the target problem, that is, to choose which of the two sets was the randomly assigned set (n). The target either matched or did not match the interpreted symmetric or asymmetric structure of the base (e.g., O→P base and either O→P or P→P target). Moreover, as in Ross (1989), the direction of assignment in the base and target O→P problems was either the same (e.g., objects assigned to people) or reversed (e.g., people assigned to objects).

The main finding of interest was that subjects solving the asymmetric O→P targets had a very strong preference to assign objects to people (e.g., prizes to students) rather than people to objects (e.g., students to prizes). That is, subjects were acting as if the direction of assignment in the permutation problems was consistent with the semantically asymmetric outcome of assignment (e.g., students get prizes rather than vice versa). This interpretive preference mediated the magnitude of object mapping in the O→P training conditions in which, as in Ross (1989), animate objects were the receivers and inanimate objects were the givers.

In Experiment 2, we unconfounded matches in object attributes (e.g., animate) from matches in interpreted structures (e.g., receivers). For example, subjects received a base with two functionally asymmetric sets of people (i.e., caddies assigned to golfers) and an O→P target in which either carts were assigned to caddies or caddies were assigned to carts. Although caddies were the assigned set in the base problem, subjects did not assign caddies to carts in the target problems (i.e., object mapping). Rather, consistent with their knowledge about the asymmetric outcome of assignment, they assigned carts to caddies (94 vs 24% correct for the carts assigned to caddies vs caddies assigned to carts targets, respectively). That is, the impact of object-based interpretations was powerful enough to override matches in object attributes.
D. SUMMARY

Three main findings emerge from the studies described in this section. First, object-based inferences (e.g., lasers proceed in straight lines; speed changes continuously; students get priced) may lead to abstraction of nonsomorphic interpreted structures for formally isomorphic base and target problems. Second, transfer is sometimes mediated by accommodating an interpreted target to the interpreted structure of the base problem (e.g., treating rays in a target problem as if they were waves; treating continuous change in a target problem as if it were discrete) rather than by abstracting a structure that is common to distinct interpreted structures (e.g., converge; change). Third, object-based interpretive effects (e.g., caddies get carts rather than vice versa) can override effects caused by direct matches and mismatches of object attributes (e.g., caddies are more similar to caddies than are carts).

These findings have important implications for instruction in formal domains. They show that, just as the good learner in Chi et al. (1989) expected the block rather than the knot to serve in the role of the physical "body," students of formal domains expect a final correspondence between the mathematical structure of a word problem and the semantic structure of the situation described in the problems' cover story. This expectation guides their understanding of worked-out examples and affects transfer to novel problems. Interestingly, such interpretive effects of content are more likely to affect the performance of students with good rather than poor mathematical understanding (Bussouk, 1997; Hanley et al., 1977; Paige & Simon, 1966).

The studies described so far addressed a processing stage (i.e., representation) that, at present, remains outside the scope of research on analogical transfer. The interpretive effects found in these studies show that object-based inferences can affect how people represent base and target stimuli both before and during access and mapping. Note, however, that these studies have no direct bearing on the validity of the processing assumptions in extant models of access and mapping. For instance, it is possible that when people aligned the interpreted representations of the base and target stimuli (e.g., aligned receivers with receivers) they distinguished between relations and attributes in a way that is consonant with the assumptions of the alignment models. The studies I describe in the next section show that, in addition to their effects on representation, object-based inferences determine how people process the stimuli they encounter. Specifically, these studies show that semantic knowledge affects which processing strategy (e.g., comparison vs integration) is selected for a given pair of base and target stimuli. Unlike representation, the issue of stimuli-appropriate process selection was never on the research agenda of investigators who study analogical transfer or similarity.
IV. Semantic Knowledge Affects Selection of Processing Strategies

The common saying “It’s like comparing apples and oranges” is used to denote an inappropriate comparison. What is interesting about this saying is that apples and oranges, rather than, let’s say, apples and baskets, became idiomatic with a poor comparison. Wouldn’t apples and baskets, which obviously have less commonalities than apples and oranges, serve as a better example of a poor comparison? The answer to this question is that apples and baskets cannot exemplify a poor comparison because they do not exemplify a comparison. Rather, they exemplify a functional (thematic) relation between two entities. This answer emerges from a set of studies in which my colleagues and I found that, irrespective of task instructions, people tend to compare certain stimuli (e.g., apples and oranges) but tend to integrate other stimuli (e.g., apples and baskets). As a result, when people are faced with stimuli that do not fit the task requirements (e.g., asked to judge similarity between apples and baskets), they tend to replace a task-appropriate process (comparison) with a stimuli-appropriate process (integration)—performance to which I refer here as processing replacements.

Theoretical accounts of similarity seem to hold that the process of comparison is a general-purpose mechanism that can be applied to any arbitrary pair of stimuli (e.g., both apples and baskets and apples and oranges). Most probably, researchers who study similarity or analogical reasoning did not notice that people may integrate rather than compare certain stimuli because, much like their subjects, they typically select comparison-appropriate stimuli (but see Markman & Gentner, 1993). Of course, unintentionally, they may sometimes select stimuli that people tend to integrate rather than compare. However, being unaware of processing replacements, they would try to explain performance that was mediated by integration (e.g., baskets contain apples) in terms of the hypothesized process of feature comparison (e.g., both baskets and apples are objects found in orchards).

It was within this conceptual framework that Douglas Medin and I initially arrived at similarity judgments, not by comparing the representations of these stimuli, but rather by integrating them into joint thematic scenarios. Needless to say, these findings caught us by complete surprise. In what follows I first describe these unexpected processing replacements during similarity judgments (Bassok & Medin, 1997). Then, I describe results from a follow-up study that was explicitly designed to examine the impact of semantic knowledge on processing replacements (Wisniewski & Bassec, 1996). Finally, I describe results from a study in which the same
semantic distinction that determines which objects are compared and which are integrated led to selective application of arithmetic operations in reasoning about addition and division word problems (Bussok et al., 1997).

A. PROCESSING REPLACEMENTS IN SIMILARITY JUDGMENTS

Similarity, like analogy, involves comparison and alignment. This observation led several investigators to examine the possibility that the structural alignment approach to analogical mapping can be adapted to explain similarity judgments (e.g., Golstone, 1994; Goldstone & Medin, 1995; Goldstone, Medin, & Gentner, 1991; A. B. Markman & Gentner, 1993, 1996; Medin, Goldstone, & Gentner, 1990, 1993). It turned out that, indeed, when people judge similarity between various pairs of base and target stimuli they distinguish between relational and attributional matches and mismatches.

For example, Medin et al. (1990) presented subjects with stimuli consisting of interrelated geometric shapes. They asked subjects to choose which of two target alternatives, R (a star above a circle) or A (a triangle next to a circle), was more similar to or more different from a standard base (e.g., a triangle above a square). The stimuli were constrained such that R targets had an extra relational match (e.g., above) and A targets had an extra attributional match with the base (e.g., triangle). When the task was to judge similarity, subjects were more likely to choose R rather than A targets as being more similar to the base. However, when the task was to judge difference, this preference was less pronounced or even reversed. In an extreme case, subjects would choose the same R target as both more similar to and more different from the base. From these nonmirroring similarity and difference judgments, Medin et al. concluded that people distinguish between relational and attributional matches: assign higher weight to relational matches in similarity than in difference judgments and/or assign lower weights to attributional matches in similarity than in difference judgments.

As in most experiments that document differential effects of relational and attributional matches on similarity judgments, the stimuli used by Medin et al. (1990) were carefully chosen to enable a clear separation and contrast between attributes and relations. For example, the symmetry relation in OXO and $\triangle \nabla$ is independent of whether the symmetric shapes in the two figures happen to be circles or triangles, or whether a star is between two triangles rather than vice versa ($\triangledown \nabla$). However, in most cases it matters which objects serve as arguments in a particular relation. For example, carpenters are known to fix chairs rather than radios as part of their profession; cutting grass is a different type of cutting than cutting hair; and mockingbirds are believed to sing more nicely than crows. It
therefore remained unclear whether the processing regularities in similarity and difference judgments observed in the semantically independent case (e.g., “a triangle above a square”) would also hold for semantically interdependent combinations of objects and relations (e.g., “The carpenter fixed a chair”).

In Banosk and Medin (1997) we examined how semantic dependencies between attributes and relations affects similarity and difference judgments: whether such dependencies affect only the representations constructed for the base and target stimuli (i.e., interpreted representations) or, in addition, affect the weights assigned to attributional and relational matches (i.e., changes in processing). Our stimuli were simple base and target statements in which the nouns (denoting objects) and verbs (denoting relations) were semantically interdependent. The statements were designed in a way that allowed us to distinguish effects of separate matches in the explicitly stated nouns (A: attributes) and verbs (R: relations) from more abstract matches associated with the interpreted combined meaning of the statements. We assumed that matches in the combined meaning of the statements can be treated as matches in abstract higher order relations (R*R) that specify how separate attributes and relations are interrelated.

We used 12 quintuplets of statements from which we constructed three types of triplets. Each triplet consisted of a Base, its Attributional target, and one of its three Relational targets. Table IV presents one quintuplet of statements that exemplifies the design of our stimuli. As can be seen in Table IV, the combined meaning of RR* targets always matched the combined meaning of the base (e.g., a professional performing a job-related activity), whereas the combined meaning of R and RA targets did not (e.g., neither plumbers nor carpenters fix radios as part of their profession). As a result, although the three relational targets had the same separate relational

| TABLE IV |
|---|---|
| **One Quintuplet of Base and Target Statements**a | |
| Base | The carpenter fixed the chair |
| Attributional target (AA) | The carpenter sat on the chair |
| Relational target | |
| Relation + inferred Relation* (RR*) | The electrician fixed the radio |
| Relation (R) | The plumber fixed the radio |
| Relation + Attribute (RA) | The carpenter fixed the radio |

Note: Separate noun (A) and verb (R) matches are typed in boldface.
match with the bare (e.g., R = fix), RR* formed good analogies to the base, whereas R and RA formed poor analogies. The difference between the poor-analogy R and RA targets was that RA had an additional attributional match with the base (e.g., A = carpenter). The attributional target (AA) had a relational mismatch with the base (e.g., fix ≠ sit) and therefore a combined mismatch. Subjects received one triplet from each of the 12 different quartets, and for each triplet they rated the similarity (dissimilarity) between the base and its relational and attributional targets.

Applying Gentner’s (1983) “systematicity principle” to our stimuli, we predicted that matches and mismatches in the combined meanings of the statements—in the inferred higher order relations (R²)—will override explicitly stated relational (R) and attributional (A) matches and mismatches. Indeed, we found that subjects gave significantly higher similarity ratings to good than to poor analogy targets, both when these targets were equated in separate matches (RR* > R), and even when the poor-analogy targets had an additional separate match (RR* > RA). For example, subjects gave higher similarity ratings to the target “The electrician fixed the radio” (matching R*) than to the target “The carpenter fixed the radio” (mismatching R*) even though a carpenter is clearly more similar to a carpenter than an electrician. These findings are similar to the findings described in the previous section (i.e., interpreted structures). They show that semantic dependencices between attributes and relations affect similarity and difference ratings by affecting the representations people construct for the base and target stimuli.

To examine whether object-based inferences also affect processing, we compared the ratings in the similarity and difference conditions. However, we did not replicate the nonmirroring pattern of similarity and difference ratings found for arbitrary combinations of separable relations and attributes (Medin et al., 1990). That is, we did not find evidence that subjects gave higher weights to relational matches (R targets) in similarity judgments and/or gave higher weights to attributional matches (A targets) in difference judgments. These results indicate that in addition to affecting the representations of the base and the target stimuli, nonarbitrary dependencies between attributes and relations affect processing.

In order to better understand the process that mediates comparison of semantically rich stimuli, we asked subjects to explain in writing how they arrived at their similarity and difference ratings. We found that, indeed, subjects compared the inferred combined meanings of the base and the relational targets (RR*, RA, and R). For example, “A lawyer would never design a dance” (unlike engineers who design cars); “Most children do not

*We found nonmirroring effects in one experiment, but only for the good-analogy (RR*) targets.
enjoy jobs" (but they enjoy toys); or "The first sentence is a fact, whereas the second is an opinion" (for the base "Equation: are more accurate than words" and the target "Equations are more difficult than words"). However, to our great surprise, we found that subjects did not always compare the representations of the base and the attributional targets (AA). Rather, very often they integrated the combined meanings of the base and the attributional-target statements into joint thematic scenarios. For example: "A teacher may have listened to the lecture to prepare" (for the base "The teacher prepared a lecture" and the target "The teacher listened to the lecture"); "Something the child might do if he/she enjoyed the toy, out of selfishness" (for the base "The child enjoyed the toy" and the target "The child hid the toy"); or "A logical step: since equations are more accurate they are more difficult to use" (for the base "Equations are more accurate than words" and the target "Equations are more difficult than words").

Most combined explanations generated for the relational targets involved comparison (84%), whereas a substantial majority of combined explanations generated for the attributional targets involved integration (59%). This systematic difference in the distribution of comparison and integration explanations for the relational and attributional targets was found in the explanations generated by 73% of the subjects (N = 80 Northwestern University and University of Chicago undergraduates). Moreover, the pattern and magnitude of the ratings were consistent with both types of explanations. Subjects mentioned combined matches (e.g., "Both involve professionals doing their jobs") when explaining their high similarity ratings for the good-analogy relational targets (RR) and combined mismatches (e.g., "Not analogous because plumbers don't fix edgels as part of their job") when explaining their low ratings for the poor-analogy relational targets (R and RA). At the same time, a post hoc analysis of ratings for the attributional targets (AA) revealed that supporting scenarios (e.g., "Similar because the carpenter sat on the chair to see whether he fixed it well") were associated with significantly higher similarity ratings than impeding scenarios (e.g., "It does not necessarily follow that because he fixed the chair he had to sit in it").

These results strongly suggest that for both target types, the explanations captured two qualitatively different processes by which subjects arrived at their ratings. When the base and target statements shared a relational match (i.e., common verb), similarity and difference ratings were almost exclusively mediated by a process of comparison. By contrast, when the base and target statements shared only attributional matches (i.e., common nouns), similarity judgments were frequently mediated by a process of thematic integration. That is, the relational and attributional targets induced,
or highlighted, two qualitatively different processes. Given that the task (similarity or different ratings) demanded comparison, subjects' performance on the attributional targets indicates that they actually replaced a task-appropriate process with a stimulus-appropriate process.

Why is it that the attributional targets induced processing replacements? The nature of our stimuli precludes the possibility that subjects were forced to replace comparison with integration because they could not identify salient commonalities and differences between the base and the attributional statements. This is because the attributional targets were explicitly designed such that they had no matching nouns with the base (e.g., carpenter, chair) that could be readily used to explain similarity, and had a mismatching verb (e.g., fix vs sit) that could be used to explain difference. At the same time, our stimuli were neither designed nor obviously slanted toward familiar scripts or scenarios. For example, there is no obvious causal or temporal relation between catching a ball and selling it, or between accuracy of estimations and their difficulty. In fact, our subjects often used lack of thematic relatedness to explain why the paired statements were not similar (e.g., "It does not necessarily follow that because he fixed the chair he had to sit in it"). Thus, it is not the case that subjects were simply carried away by highly activated thematic scenarios. Rather, it appears that the attributional targets led subjects to actively search for, or construct, possible scenarios. Successful scenarios were taken as supporting similarity (high ratings) and unsuccessful scenarios as implying similarity (low ratings).

The systematic correspondence between processing (comparison vs integration) and target type (relational vs attributional) strongly suggests that processing replacements occurred in response to lack of relational matches between the base and the target stimuli. Interestingly, the pattern of processing replacements strongly supports the structural-alignment approach to analogy and similarity. It suggests that relational commonalities are crucial, if not absolutely necessary, for alignment of semantically structured stimuli—that matching relations (verbs) enable structural alignment, but matching attributes (nouns) do not. At present, it is unclear whether integration comes into play only when comparison fails to achieve some "satisfying" criteria, whether comparison and integration compete with each other as orthogonal alternatives, or whether these two processes operate simultaneously to support and impede similarity. What is quite certain, however, is that comparison is not the only processing mechanism that mediates similarity judgments for semantically rich stimuli. Hence, at the very least, the alignment models would have to be supplemented with a selection mechanism that specifies the conditions that may lead to processing replacements.

B. SEMANTIC COMPATIBILITY OF OBJECT PAIRS WITH COMPARISON AND INTEGRATION

The alignability account of processing replacements implies that when the base and target stimuli are objects, rather than semantically structured
combinations of objects, thematic integration should be more prominent when the paired objects are nonexistent (i.e., have few common attributes) than when they are alignable (i.e., have many common attributes). To test this prediction, Edward Winsiewski and I collected similarity ratings for object pairs that differed in alignability (Winsiewski & Bambakos, 1996). The stimuli were 12 quintuplets of nouns (denoting objects), each consisting of a base and four targets. Two targets shared many attributes with the base (A+, alignable targets) and the other two shared few attributes with the base (A−, nonalignable targets). The two alignable targets (A+) were selected from the taxonomic category of the base, whereas the nonalignable targets (A−) were selected from a different taxonomic category.

We also examined whether familiar thematic relations interact with alignability to induce processing replacements. Accordingly, one of the A+ and A− targets in each quintuplet was related to the base by a familiar thematic relation (T+) and the other was not (T−). The resulting structure of the four targets in each quintuplet was: A+T+, A+T−, A−T+, A−T−.

For example, the four targets for the base “milk” were, respectively, “coffee,” “lemonade,” “cow,” and “horse.” The 12 quintuplets used in our study appear in Table V.

We presented subjects with 12 base-target pairs, each pair from a different quintuplet. Subjects read instructions telling them that they would see some pairs of common, everyday things, and that they would have to rate

<table>
<thead>
<tr>
<th>Base</th>
<th>A+T+ target</th>
<th>A+T− target</th>
<th>A−T+ target</th>
<th>A−T− target</th>
</tr>
</thead>
<tbody>
<tr>
<td>milk</td>
<td>coffee</td>
<td>lemonade</td>
<td>cow</td>
<td>horse</td>
</tr>
<tr>
<td>ship</td>
<td>lifeguard</td>
<td>canoe</td>
<td>sailor</td>
<td>sailor</td>
</tr>
<tr>
<td>car</td>
<td>tow truck</td>
<td>pickup truck</td>
<td>mechanic</td>
<td>plumber</td>
</tr>
<tr>
<td>chair</td>
<td>table</td>
<td>bud</td>
<td>carpenter</td>
<td>electrician</td>
</tr>
<tr>
<td>telephone</td>
<td>stereo</td>
<td>tape recorder</td>
<td>receptionist</td>
<td>waitress</td>
</tr>
<tr>
<td>tie</td>
<td>suit</td>
<td>dress</td>
<td>man</td>
<td>woman</td>
</tr>
<tr>
<td>chisel</td>
<td>hammer</td>
<td>screwdriver</td>
<td>sculptor</td>
<td>painting</td>
</tr>
<tr>
<td>cut</td>
<td>mouse</td>
<td>hammer</td>
<td>specification</td>
<td>musician</td>
</tr>
<tr>
<td>cup</td>
<td>kettle</td>
<td>pan</td>
<td>sea</td>
<td>wine</td>
</tr>
<tr>
<td>pen</td>
<td>pencil</td>
<td>beetle</td>
<td>screen</td>
<td>curtain</td>
</tr>
<tr>
<td>peanut butter</td>
<td>jelly</td>
<td>cream cheese</td>
<td>knife</td>
<td>fork</td>
</tr>
<tr>
<td>apple pie</td>
<td>ice cream</td>
<td>jelly</td>
<td>baker</td>
<td>tailor</td>
</tr>
</tbody>
</table>

how similar the two things are on a 7-point rating scale. Also, to ensure that thematic replacements do not occur only when subjects explain their ratings, half of the subjects explained in writing why they thought the two things had the degree of similarity that they did and the other half did not.

First, consistent with the definition of similarity as a monotonic function of common and distinctive attributes (e.g., Tversky, 1977), A+ targets received higher similarity ratings than A− targets (e.g., coffee and lemonade were rated as more similar to milk than cow and horse). Moreover, familiar thematic relations affected similarity ratings, such that T− targets received higher ratings than T+ targets (e.g., coffee and cow were rated as more similar to milk than lemonade and horse). Because preexisting thematic relations can be treated as commonalities (e.g., milk and coffee appear in the same cup), these results do not prove that similarity ratings were mediated by integration rather than by comparison. However, consistent with the alignability account of processing replacements, the impact of preexisting thematic relations or similarity ratings was much more pronounced for the nonalignable than for the alignable targets. That is, the difference between similarity ratings for A− T+ and A− T− targets (e.g., cow and horse) was larger than that for A+ T+ and A+ T− targets (e.g., coffee and lemonade).

Asking subjects to explain their ratings did not affect the magnitude of ratings. At the same time, as in Basseg and Medlin (1997), explanations provided direct evidence for processing replacements. We examined the relative proportion of explanations, both supporting (+) and impeding (−) similarity, that involved either comparison (e.g., "people drink both milk and lemonade," "a horse can be white like milk") or thematic integration (e.g., "people often add milk to coffee," "you do not milk a horse"). As the alignability account would predict, and consistent with the pattern of ratings, the tendency toward thematic integration was much higher for nonalignable than for alignable targets (52 vs 19% for A− and A+ targets, respectively).

Importantly, the difference in frequency of thematic replacements between the A− and A+ items was larger, but not limited to the T+ targets. That is, preexisting thematic scenarios (e.g., milk and cow vs milk and horse) simply magnified people's spontaneous tendency to integrate nonalignable stimuli (e.g., "you do not milk a horse"). Obviously, most thematic explanations for the T+ targets supported similarity (e.g., "carpenters fix chairs"). By contrast, most thematic explanations for the T− items were impeding scenarios, that is, counterfactual arguments that explained why the paired objects were not similar, for example, "Cat and mouse aren't similar because they are enemies," or "Women usually do not wear ties."
To the extent that alignability determines the likelihood of processing replacements in similarity judgments, one would expect a reversed pattern of processing replacements when the task is to judge thematic relatedness. We tested this hypothesis using the same materials (Table V) and asking subjects to rate the degree of functional or causal relatedness between the paired entities. Indeed, we found a pattern of processing replacements that mirrored the pattern observed in similarity judgments. First, attributional replacements (i.e., comparison instead of integration) were much more frequent in $A_+$ than in $A_-$ targets (10 vs 8%). For example, subjects were more likely to "erroneously" compare milk and lemonade than to "erroneously" compare milk and horse. Importantly, the difference in frequency of attributional replacements between the $A_+$ and $A_-$ targets was larger, but not limited to the $T_-$ targets (a difference of 62 vs 25% for the $T_+$ and $T_+$ targets, respectively). That is, lack of preexisting thematic scenarios (e.g., milk and lemonade vs milk and coffee) simply magnified people's spontaneous tendency to compare alignable stimuli (e.g., "Both are beverages").

To summarize, stimuli differ in their compatibility with processing strategies. When the stimuli are incompatible with a task-appropriate process, or when they are more compatible with an alternative process, people may replace the task-appropriate process with a stimulus-appropriate process (e.g., replace comparison with integration or vice versa). Of course, thematic integration is not a viable processing option for stimuli consisting of arbitrary combinations of interrelated objects (e.g., a triangle above a square; a triangle next to a square). In other words, the type of processing replacements demonstrated in our studies is specific to semantically rich stimuli.5

C. Semantic Compatibility of Object Pairs with Addition and Division

Processing replacements of the sort documented by Basso and Medin (1997) and Winstenicki and Basso (1996) have been previously found only in young children (see E. M. Markman, 1989, for a review) and in adults from illiterate cultures (Luria, 1976). In one study, Luria (1975) asked illiterate adults from Uzbekistan to compare pairs of objects—to explain what the objects had in common and in what way they were alike. These adults often responded by noting possible thematic relations between the

5 The difference in processing options available for arbitrary versus semantically meaningful stimuli may have been responsible for the fact that certain processing regularities found for interrelated geometric shapes (e.g., matching similarity and difference ratings in Medin et al., 1996) were not replicated with semantically interdependent objects (Basso & Medin, 1997).

6 In his recent dissertation, Emanuel Litt (1996) reports similar results for American adults who were performing a categorization task.
objects. For example, one illiterate adult who was asked by Luria "What do water and blood have in common?" responded: "What's alike about them is that water washes off all sorts of dirt, so it can wash off blood too" (p. 82). That is, much like other content effects (e.g., failures of analogical transfer), processing replacements are associated with errors that indicate lack of maturity, poor intelligence, or insufficient knowledge.

Because the subjects in our studies were neither young nor illiterate, their performance cannot be explained away by insufficient cognitive or cultural development. Of course, one could maintain that processing replacements are errors and reflect confusion. In particular, one could argue that it is relatively easy to confuse integration and comparison because relations between objects (e.g., putting milk in coffee lead to commonalities (e.g., both milk and coffee are in a cup). However, treating processing replacements as confusion errors does not explain the systematicity in which they occur. Moreover, by treating processing replacements as failures of the cognitive system, the confusion argument marginalizes the importance of this phenomenon and therefore detracts attention from the regularities with which processing is attuned to the processed stimuli.

The last study I describe in this review (Bassok et al., 1997) replicated the pattern of processing replacements described earlier in a task that associated object-based process selection from confusion errors. In fact, the task was designed such that object-based process selection demanded investment of extra cognitive effort. Specifically, Valerie Chase, Shirley Martin, and I asked undergraduate students to construct simple addition or division word problems for pairs of object sets that we provided (e.g., tulips-daffodils; tulips-vases). Subjects were free to invent the number of elements for each set (e.g., 6 tulips and 6 daffodils). The minimal mathematical solution that meets the requirements of the construction task is to relate the given pair of object sets directly by the required arithmetic operation: direct addition (m + n) when the task is to construct addition problems, and direct division (m/n) when the task is to construct division problems. However, as I describe below, the object pairs were selected such that half were semantically incompatible and the other half were semantically incompatible with direct addition and direct division.

1. Direct Addition (m + n)

In the additive mathematical structure, two or more distinct sets of elements (e.g., m and n) are combined to form a union set (u). The combined sets play symmetric structural roles in the formation of the union set (m+n = n+m) and elements from the constituent sets become equivalent members in the union set (u). The best semantic match to the mathematical con-
strains of symmetry and membership equivalence is obtained when the added sets consist of identical elements (e.g., m tulips + n tulips = m tulips). When the elements in m and n are not identical, adding elements from the same taxonomic category (e.g., m tulips + n daffodils = m flowers) leads to a more meaningful combined set than adding elements from distinct taxonomic categories (e.g., m tulips + n vases = m things).

2. **Direct Division (m/n)**

In the multiplicative mathematical structure, elements from two distinct sets (m, n) are related by a function (k) that maps elements from the m set onto elements from the n set. The mapping function, whose value equals the outcome of m divided by n, is directional and usually asymmetric \((m/n ≠ n/m)\). The multiplicative structure is therefore most compatible with situations in which the m and n elements are known to be related by an asymmetric functional relation. For example, the asymmetric functional relation contain (vases, tulips) naturally corresponds to the asymmetric mapping function and therefore can be directly aligned with the operation of division \((k = m \text{ tulips}/n \text{ vases})\). Moreover, and unlike in the symmetric additive structure, considerations of semantic compatibility constrain the structural roles of the paired sets in the asymmetric mapping function (e.g., m tulips are contained in and therefore divided by n vases rather than vice versa).

Functionally asymmetric relations between elements that belong to the same taxonomic category exist (e.g., CATCH spiders, flies) and other examples in the A=T+ column of Table V). However, most pairs of elements from the same taxonomic category lack a natural functional relation. In this case, the only semantic meaning that corresponds to the mapping function \((k)\) is a proportional comparison of set sizes (i.e., the number of tulips is \(k\) times the number of daffodils). Unless such proportional asymmetry is supported by semantic knowledge (e.g., there are more robins than bluebirds), the proportional meaning of division will be arbitrary and therefore a less compelling match to the asymmetric mapping function than a functionally asymmetric relation.

The difference in compatibility of direct addition and direct division with semantically symmetric and asymmetric relations between the paired object sets afforded us the opportunity to test for object-based process selection that does not lead to mathematical errors. If people ignore considerations of semantic compatibility in the construction task, then direct addition \((m+n)\) and direct division \((m/n)\) of members in the given sets should be the least effortful strategy in the addition and division construction tasks, respectively. If, however, people are guided by considerations of semantic
compatibility, then they should avoid direct addition of semantically asymmetric sets and direct division of semantically symmetric sets. Instead, they should employ a variety of semantic-escape strategies. For example, they might invent another variable \( p \) that can be added or divided in a semantically compatible way, and fulfill the task requirements by constructing problems with a more complex mathematical structure (e.g., \( m + p/p/6 \)).

As predicted, subjects had a very strong preference to construct problems in which the arithmetic operations were compatible with the semantic relations implied by the paired object sets. First, the great majority of problems constructed for semantically compatible object pairs in the addition and the division conditions (70–88%) involved direct addition and direct division of the paired sets, respectively. An example of a direct-division problem constructed for the semantically compatible pair boys–teachers in the division condition is (men/they/“Three teachers want to evenly divide a class of 60 boys. How many boys should go with each teacher?”). At the same time, a significant proportion of problems constructed for semantically incompatible object pairs (27–30%) were semantic escapes. For example, an escape problem constructed for the semantically incompatible pair tulips–daffodils in the division condition also involved addition of the two target sets of flowers \((m+n)/p\): “Wilmu planted 250 tulips and 250 daffodils and it took 20 days to plant them. How many flowers did she plant per day?”

Most semantic-escape problems involved the requested arithmetic operation, although this operation did not relate the incompatible object sets. However, in some semantic escapes subjects actually failed to fulfill the task requirements. Instead of constructing complex problems, they replaced the requested but semantically incompatible arithmetic operation with a task-inappropriate but semantically compatible arithmetic operation (i.e., division instead of addition or addition instead of division). An example of such operation replacements is a division escape problem constructed in the addition condition for the semantically incompatible pair peaches–baskets: “Two baskets hold 30 peaches, how many peaches does 1 basket hold?”; or an addition escape problem constructed in the division condition for the semantically incompatible pair peaches–plums: “If there is a basket with peaches and plums in it, and we know that the total number of pieces of fruit is 20, and that there are 5 peaches, how many plums must there be in the basket?”

The operation-replacement problems subjects constructed for semantically incompatible object pairs mirror the processing replacements exhibited by subjects who rated similarity and thematic relatedness in Wistow and Bassok (1996). That is, subjects both added and compared apples and oranges (alignable stimuli), and they both divided and integrated apples
and baskets (nonalignable stimuli). Importantly, such semantically induced operation replacements occurred even though addition and division are much less confusable than comparison and integration. In other words, the pressure to relate the paired object sets in a semantically meaningful way was powerful enough to override clear task instructions.

The correspondence between processing replacements and performance in the construction task highlights the practical implications and the reason- ableness of object-based process selection. There is little doubt that the undergraduate students in Bassok et al. (1997) had sufficiently abstract knowledge of arithmetic—that they could readily add and divide abstract Xs and Ys. Nonetheless, they did not treat object sets as abstract variables and did not treat arithmetic operations as general-purpose procedures. Rather, they used addition and division as semantically sensitive modeling tools. By analogy, people do not use comparison and integration as general-purpose processing mechanisms, but rather, use them as semantically sensitive reasoning tools.

While mirroring processing replacements, semantic-escape problems also mirror the pattern of semantic alignments exhibited by subjects who solved the permutation problems in Bassok et al. (1995). Whether solving novel probability problems or constructing familiar arithmetic problems, subjects were trying to align the semantic and the mathematical structures of word problems. By mirroring both types of object-based effects described in this chapter, the Bassok et al. (1997) study ties together the impact of semantic knowledge on representation and process selection. It therefore highlights the inherent dependency between these two aspects of reasoning and implies that they should be studied jointly.

V. Discussion

Semantic knowledge is organized such that it affords meaningful and adaptive inferences (e.g., apples and oranges are fruit and therefore can play similar functional roles). The studies reported in this chapter show that such object-based inferences affect how people solve problems, transfer previously learned solutions to novel problems, or judge similarity. Put in this way, the present findings are not very surprising—it is not very surpris-
ing that people prefer to compare or combine apples and oranges rather than apples and baskets. What is quite surprising, however, is that such effects have been overlooked by researchers who study higher order cognition.

It is not that researchers failed to notice that object-based inferences affect reasoning. A classic example of such effects would be Duncker’s (1945) work on “functional fixedness,” whereby the functional role of a box as a container prevented people from using the box as a platform on which they could mount a candle. In a more recent example, Kotsur, Hayes, and Simons’ (1985) study compared people’s solutions to two versions of the Tower of Hanoi problem. They found that subjects had little difficulty placing a large disk on top of a small disk, but when the disks were labeled “in-between,” subjects refrained from letting a large acrobat jump on the shoulders of a small acrobat. Unfortunately, such demonstrations were merely added to the list of studies showing that superficial aspects of content and phrasing lead to errors, or make some problem isomorphs more difficult than others. They did not initiate research that looks for regularities in the way people select, or adjust, their reasoning tools (e.g., comparison vs. integration) to semantic distinctions they deem important (e.g., functional symmetry vs. asymmetry).

A line of research that, in its gist, is probably the closest to the studies reported here looks for regularities in the way semantic knowledge affects reasoning about conditional syllogisms (e.g., Cheng & Holyoak, 1985, 1989; Cheng, Holyoak, Nisbett, & Oliver, 1986; Cosmides, 1989; Cosmides & Tooby, 1994; Cummins, 1995; Gigerenzer & Hug, 1992). This work has shown that different content instantiations of the material implication (“if p then q”) induce reasoning rules of distinct pragmatic and/or social schemas. According to Cheng and Holyoak (1985), who initiated this line of research, people apply different reasoning rules to formally isomorphic statements such as “if there are clouds, then is rain” or “if you drink beer, then you must be at least 21 years old.” In the first case, people are guided by knowledge that clouds are a necessary albeit insufficient cause for rain (e.g., causation schema), whereas in the second they are guided by knowledge that drinking age is established by a law that might be disobeyed (i.e., permission schema).

As in the case of arithmetic operations (Rossok et al., 1997), the rules of formal logic are semantically compatible with the rules of some schemas but not others. For example, Modus Tollens (“if not q then not p”) is compatible with the rules of the permission schema (e.g., if you are under 21 [not q], then you are not permitted to drink alcohol [not p]). However, this rule sometimes conflicts with the rules of the causation schema (e.g.,
if it doesn’t rain [not q], it does not necessarily follow that there are no clouds [not p]. Cheng et al. (1996) found that, much as the mathematically sophisticated subjects in Bassok et al. who sometimes “failed” to construct addition or division word problems for semantically incompatible object sets, subjects who were trained in evaluating the validity of conditional syllogisms committed “logical errors” on test problems that induced the rules of incompatible pragmatic schemas.

Cheng et al. (1996) argued convincingly that adherence to semantic and pragmatic constraints (i.e., content effects) protects people from arbitrary and anomalous conclusions. In fact, the results of Bassok et al. (1997) strongly suggest that, when application of formal rules conflicts with people’s semantic and pragmatic knowledge, they may prefer arriving at reasonable and logically invalid conclusions to arriving at logically valid but anomalous conclusions. Unfortunately, the notion that abstraction of structure from content is a mark of intellectual achievement appears to be so appealing that even sensible responses to semantic constraints may be classified by researchers as reasoning errors. The explanatory parsimony that is implied by this notion may also explain why researchers prefer to construct and test content-independent accounts of reasoning. While testing such accounts, they typically average people’s responses to stimuli that differ in content, viewing such responses as measurement errors that obscure basic processing regularities (see Goldstein & Weber, 1995, for an insightful discussion and historical analysis of this view). This practice would most probably average out the very effects that were the focus of the studies described in this chapter.

Object-based inferences that reflect adherence to semantic and pragmatic distinctions implied by content are inherent to rather than a deviation from normal processing. Also, such inferences do not indicate poor understanding, lack of maturity, or insufficient cognitive resources. Hence, effects of object-based inferences on reasoning do not fit well the “hidden-treasure” or any other variant of the failure-of-abstraction-from-content metaphor. Instead, such effects seem to suggest a “toolbox” metaphor, whereby people attempt to find the best fit between their processing tools and the constraints implied by the stimuli they encounter. I strongly believe that, by adopting the toolbox metaphor, researchers are likely to discover interesting regularities in the way people adjust processing to their highly organized semantic and pragmatic knowledge.

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References


