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STATISTICAL POWER IN NONRECURSIVE LINEAR MODELS

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In nonrecursive models, estimates of simultaneous relationships are often subject to high sampling variability. In this paper, we apply classical procedures for computing statistical power to the issue of sampling variability in estimates of reciprocal causal effects. Using a model of married women's attitudes regarding work and family size as an example, we show how the power to detect nonrecursive relationships depends on the model's parametric structure. Specifically, we show how the power of statistical tests depends on the strength of instrumental variables, the number of overidentifying restrictions, and the covariation among disturbances. We conclude by discussing the implications of our results for applications of nonrecursive models in the social sciences.

1. INTRODUCTION

Social scientists are often interested in estimating reciprocal causal relationships among variables measured contemporaneously.

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For example, economists attempt to estimate simultaneous relationships among sets of supply-and-demand equations (Liu 1963), sociologists seek to disentangle the reciprocal influence of one peer on another (Duncan, Haller, and Portes 1968), and demographers try to determine whether childbearing determines labor force participation, or vice versa (Waite and Stolzenberg 1976; Smith-Lovin and Tickamyer 1978; Cramer 1980). In principle, such reciprocal effects can be routinely estimated using nonrecursive estimators such as two-stage least squares (2SLS), three-stage least squares (3SLS), and maximum likelihood (ML). In practice, however, researchers often find that nonrecursive models provide estimates of simultaneous relationships that are subject to high sampling variability, making it difficult to rule out chance in drawing inferences. Thus, researchers are unable to draw definitive conclusions about crucial relationships.

In single-equation linear models and in recursive multiple-equation models, the problem of high sampling variability typically arises because of multicollinearity or small sample size. However, sampling variability can be a more serious problem in nonrecursive models. Even with relatively large samples and exogenous variables that are only modestly correlated, estimates of relationships among endogenous variables can be quite unstable; i.e., they can have large amounts of sampling variability. The problem, sometimes called weak empirical identification or poor instrumental variables, is usually handled informally, using rules of thumb and *ad hoc* indexes. In this paper, we argue that the problem can be viewed as one of statistical power and can be addressed by classical methods for protecting against type II error, the error of failing to reject a false null hypothesis.

We proceed in four steps. First, we review estimation and testing within nonrecursive models and provide an intuitive explanation of the problem of high sampling variability in estimates of reciprocal effects. We focus on full-information estimation, using 3SLS to present analytical results. Second, we show how statistical power can be calculated using a power function for a test of general linear constraints. Third, we present calculations that show how the power to detect nonrecursive relationships depends on the parametric structure of the model. As an example, we use a model of married women's attitudes regarding work and family size. We conclude by discussing the implications of our results for applications of nonrecursive models in the social sciences.

2. NONRECURSIVE MODELS: ESTIMATION

To set up the analyses presented below, we first briefly review estimation of nonrecursive models by the method of 3SLS. We focus on 3SLS for three reasons. First, power is a function of the estimator's asymptotic covariance matrix, and for the 3SLS estimator, that matrix can be expressed in terms of moments among exogenous and endogenous variables. Since moments can be expressed in terms of parameters of the model, 3SLS estimation allows us to explore how parametric structure influences power. Second, the asymptotic covariance matrix for the 3SLS estimator is identical to that for the full-information maximum likelihood (FIML) estimator (Theil 1971, p. 526); therefore, all of our results apply to nonrecursive models estimated by FIML methods.[1] Third, conceptualizing estimation as a three-stage process provides insights into the sources of sampling variability that are not as apparent when estimation is approached from the principle of maximum likelihood.

Consider the following system of simultaneous equations:

$$\mathbf{y}_i = \mathbf{B}\mathbf{y}_i + \mathbf{\Gamma}\mathbf{x}_i + \boldsymbol{\epsilon}_i, \tag{1}$$

where $\mathbf{y}_i$ is a vector consisting of the ith observation on p jointly determined endogenous variables, $\mathbf{x}_i$ is the ith observation on g exogenous variables, $\boldsymbol{\epsilon}_i$ is a vector of disturbances for the p equations, and $\mathbf{B}$ and $\mathbf{\Gamma}$ are coefficient matrices of order $p \times p$ and $p \times g$, respectively. The model assumes that $\mathrm{E}(\boldsymbol{\epsilon}_i) = \mathbf{0}$, $\mathrm{E}(\mathbf{x}_i\boldsymbol{\epsilon}_i') = \mathbf{0}$, and $\mathrm{E}(\boldsymbol{\epsilon}_i\boldsymbol{\epsilon}_i') = \mathbf{\Sigma}$. We assume that formal conditions for identification hold in all models discussed below (Theil 1971, pp. 489–95). In addition, we assume that the structural disturbances are multinormally distributed.

[1]The equivalence of FIML and 3SLS holds only for simultaneous equation models in observable variables and not for the more general covariance structure model with latent variables. In general, the FIML asymptotic covariance matrix cannot be expressed directly in terms of observable moments. Thus, the relationship between power and parametric structure in such models cannot be explored with closed expressions relating asymptotic covariances to observable moments and parametric structure (see Matsueda and Bielby 1986 for details).

For the jth equation in the set of p equations, all n observations can be represented as

$$\mathbf{y}_j = \mathbf{Y}_j\boldsymbol{\beta}_j + \mathbf{X}_j\boldsymbol{\gamma}_j + \boldsymbol{\epsilon}_j \tag{2}$$

or as

$$\mathbf{y}_j = \mathbf{Z}_j\boldsymbol{\delta}_j + \boldsymbol{\epsilon}_j, \tag{3}$$

where $\mathbf{Z}_j = [\mathbf{Y}_j\,\mathbf{X}_j]$ and $\boldsymbol{\delta}_j' = [\boldsymbol{\beta}_j'\,\boldsymbol{\gamma}_j']$. In these expressions, $\mathbf{y}_j$ is an $n \times 1$ vector of observations of the jth dependent variable; $\mathbf{Y}_j$ is an $n \times p_j$ matrix of observations of the p_j endogenous variables in equation j; $\mathbf{X}_j$ is an $n \times g_j$ matrix of observations of the g_j exogenous variables in the equation; and $\boldsymbol{\beta}_j$ and $\boldsymbol{\gamma}_j$ are coefficient vectors of order $p_j \times 1$ and $g_j \times 1$, respectively. Given the disturbance specification for the system, it follows that $\mathrm{E}(\boldsymbol{\epsilon}_j) = \mathbf{0}$, $\mathrm{E}(\mathbf{X}'\boldsymbol{\epsilon}_j) = \mathbf{0}$, and $\mathrm{E}(\boldsymbol{\epsilon}_j\boldsymbol{\epsilon}_j') = \sigma_{jj}\mathbf{I}$.

Given expression (3), the p structural equations for all n observations combined can be expressed as

$$\mathbf{y} = \mathbf{Z}\boldsymbol{\delta} + \boldsymbol{\epsilon}, \tag{4}$$

where

$$\mathbf{y} = \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_p \end{bmatrix} \mathbf{Z} = \begin{bmatrix} \mathbf{Z}_1 & 0 & \ldots & 0 \\ 0 & \mathbf{Z}_2 & \ldots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \ldots & \mathbf{Z}_p \end{bmatrix} \boldsymbol{\delta} = \begin{bmatrix} \boldsymbol{\delta}_1 \\ \boldsymbol{\delta}_2 \\ \vdots \\ \boldsymbol{\delta}_p \end{bmatrix} \boldsymbol{\epsilon} = \begin{bmatrix} \boldsymbol{\epsilon}_1 \\ \boldsymbol{\epsilon}_2 \\ \vdots \\ \boldsymbol{\epsilon}_p \end{bmatrix}$$

In the above expressions, $\mathbf{y}$ and $\boldsymbol{\epsilon}$ are each $np \times 1$ matrices, $\mathbf{Z}$ is $np \times q$, and $\boldsymbol{\delta}$ is $q \times 1$, where q is the total number of coefficients in the system ($[p_j + g_j]$, summed over all p equations).

2.1. *The 2SLS Estimator*

The coefficient vector for the jth equation, $\boldsymbol{\delta}_j$, can be estimated consistently with the 2SLS estimator:

$$\mathbf{d}_j = (\mathbf{A}_j'\mathbf{A}_j)^{-1}\mathbf{A}_j'\mathbf{y}_j, \tag{5}$$

where $\mathbf{A}_j = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Z}_j$ (Theil 1971, p. 451). Matrix $\mathbf{A}_j$ can also be expressed as $[\hat{\mathbf{Y}}_j\,\mathbf{X}_j]$, where $\hat{\mathbf{Y}}_j$ is $\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}_j$ or the predicted value of right-hand-side endogenous variables obtained from the reduced-form regression.[2] Thus, equation (5) is equivalent to OLS estimation for the second-stage regression of $\mathbf{y}_j$ on $\hat{\mathbf{Y}}_j$ and $\mathbf{X}_j$. The asymptotic covariance matrix of the 2SLS estimator is

$$\mathbf{V}(\mathbf{d}_j) = \sigma_{jj}(\mathbf{A}_j{}'\mathbf{A}_j)^{-1}, \tag{6}$$

which is simply the OLS computation for the covariance matrix of the second-stage estimates.

2.2. *The 3SLS Estimator*

The 3SLS estimator for $\boldsymbol{\delta}$ (with known $\boldsymbol{\Sigma}$) is

$$\hat{\boldsymbol{\delta}} = (\mathbf{Z}'[\boldsymbol{\Sigma}^{-1} \otimes \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Z})^{-1}\mathbf{Z}'[\boldsymbol{\Sigma}^{-1} \otimes \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}']\mathbf{y}, \tag{7}$$

were $\otimes$ is the Kronecker product operator. The asymptotic covariance matrix for the 3SLS estimator (Theil 1971, pp. 510–12)[3] is

$$\mathbf{V}(\hat{\boldsymbol{\delta}}) = (\mathbf{Z}'[\boldsymbol{\Sigma}^{-1} \otimes \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}']\mathbf{Z})^{-1}. \tag{8}$$

For any two-equation system (i.e., when $p = 2$), the asymptotic covariance matrix can be expressed as follows (Theil 1971, p. 515):

$$\mathbf{V}(\hat{\boldsymbol{\delta}}) = \begin{bmatrix} \sigma^{11}\mathrm{A}_1'\mathrm{A}_1 & \sigma^{12}\mathrm{A}_1'\mathrm{A}_2 \\ \sigma^{21}\mathrm{A}_2'\mathrm{A}_1 & \sigma^{22}\mathrm{A}_2'\mathrm{A}_2 \end{bmatrix}^{-1}, \tag{9}$$

where σ^{jk} is the (j,k) element of $\boldsymbol{\Sigma}^{-1}$. Computationally, 3SLS is equivalent to joint GLS estimation of the p second-stage equations

[2]Since $\mathbf{Z}_j = [\mathbf{Y}_j\ \mathbf{X}_j]$, $\mathbf{A}_j$ can be expressed as

$$\mathbf{A}_j = [\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}_j\ \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{X}_j].$$

However, the first term in $\mathbf{A}_j$, $\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}_j$, is equivalent to $\mathbf{XP}_j$, where $\mathbf{P}_j$ is the OLS estimate of the reduced-form coefficient vector for the right-hand-side endogenous variables. Thus, $\mathbf{XP}_j = \hat{\mathbf{Y}}_j$ or the predicted values for $\mathbf{Y}_j$ from the first-stage regression. The second term in $\mathbf{A}_j$, $\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{X}_j$, is equal to $\mathbf{X}_j$, since $\mathbf{X}_j$ is perfectly predicted from the full set of exogenous variables in $\mathbf{X}$. Therefore, $\mathbf{A}_j = [\hat{\mathbf{Y}}_j\ \mathbf{X}_j]$.

[3]When $\boldsymbol{\Sigma}$ is unknown, estimators of $\boldsymbol{\delta}$ and $\mathbf{V}(\hat{\boldsymbol{\delta}})$ are obtained by replacing $\boldsymbol{\Sigma}$ with $\mathbf{S}$, the sample disturbance covariances computed from 2SLS residuals. In this paper we are interested in calculations of statistical power given specific parameter values under the null hypothesis. In calculating power, the investigator assumes specific values for all parameters, including $\boldsymbol{\Sigma}$. Consequently, our results are based on expressions for known $\boldsymbol{\Sigma}$.

as a seemingly unrelated regression system (Zellner 1962; Theil 1971, p. 510).

In a two-equation system, the asymptotic covariance matrix for $\hat{\boldsymbol{\delta}}_1$, the estimator of the coefficients of the first equation, is

$$\mathbf{V}(\hat{\boldsymbol{\delta}}_1) = \sigma_{11}[\mathbf{A}_1'\mathbf{A}_1 + \{\rho^2/(1 - \rho^2)\}\mathbf{A}^*]^{-1}, \tag{10}$$

where $\mathbf{A}^* = \mathbf{A}_1' (\mathbf{I} - \mathrm{A}_2(\mathbf{A}_2' \mathrm{A}_2)^{-1}\mathbf{A}_2')\mathbf{A}_1$ and ρ is $\sigma_{12}/(\sigma_{11}\sigma_{22})^{1/2}$, the correlation between $\boldsymbol{\epsilon}_1$ and $\boldsymbol{\epsilon}_2$ (Theil 1971, p. 515). The second term in brackets in equation (10) vanishes if either $\rho = 0$ or the model is just-identified (Theil 1971, p. 511). In either case, the 3SLS and 2SLS estimators are identical, with covariance matrix $\sigma_{11}(\mathbf{A}_1' \mathrm{A}_1)^{-1}$.

Insight into sources of sampling variability in the two-equation case can be gained by viewing $\mathbf{A}_1$ and $\mathbf{A}_2$ as the right-hand-side variables of second-stage estimation equations. If the model is just-identified or if $\rho = 0$, then $\mathbf{V}(\hat{\boldsymbol{\delta}}_1) = \sigma_{11}([\hat{\mathbf{Y}}_1\, \mathbf{X}_1]'[\hat{\mathbf{Y}}_1 \mathbf{X}_1])^{-1}$ is simply the OLS variance-covariance matrix computed from the second-stage regression. In this situation, all the results obtained by Bielby and Kluegel (1977) for the general linear model apply to the second-stage regression. In particular, the sampling variability of $\hat{\beta}_1$ will increase as $\hat{\mathbf{Y}}_1$ becomes increasingly collinear with $\mathbf{X}_1$. Below, we explore how that collinearity varies as a function of the model's parameters.

When ρ differs from zero and the model is over-identified, the sampling variability of $\hat{\beta}_1$ decreases as both ρ^2 and the generalized variance of $\mathbf{A}^*$ increase. But $\mathbf{A}^*$ is equivalent to the sum-of-squares and cross-products matrix of the second-stage right-hand-side variables in the first equation after they have been residualized on the second-stage right-hand-side variables of the second equation. In other words, for an over-identified model with correlated disturbances, sampling variability in estimates of the coefficients of equation (1) will increase with (a) collinearity between the exogenous variables unique to the $\mathbf{y}_1$ equation and the remaining exogenous variables in the model and (b) collinearity between $\hat{\mathbf{Y}}_1$ and the $\mathbf{y}_2$ equation second-stage right-hand-side variables, $\hat{\mathbf{Y}}_2$ and $\mathbf{X}_2$.[4] Below,

[4]In a two-equation seemingly unrelated regression model, the relative efficiency of GLS over equation-by-equation OLS is a decreasing function of the canonical correlations between the exogenous variables in the two equations (Theil 1971, pp. 322–23). Thus, the relative efficiency of 3SLS over 2SLS can be expressed as a decreasing function of the canonical correlations between $\mathbf{A}_1$ and $\mathbf{A}_2$.

we explore how these conditions vary as a function of the model's parameters.

3. NONRECURSIVE MODELS: THE GENERAL LINEAR HYPOTHESIS AND THE POWER OF STATISTICAL TESTS

Any linear hypothesis within a nonrecursive system of equations can be expressed as H_o: $\mathbf{R\delta} = \mathbf{c}$, where $\mathbf{R}$ is a $t \times q$ matrix (of rank t) composed of coefficients for t constraints among the q parameters, and $\mathbf{c}$ is a $t \times 1$ matrix of constants. The test statistic is

$$\nu = (\mathbf{c} - \mathbf{R}\hat{\boldsymbol{\delta}})'[\mathbf{RVR}']^{-1}(\mathbf{c} - \mathbf{R}\hat{\boldsymbol{\delta}}), \tag{11}$$

where $\mathbf{V}$ is short for $\mathbf{V}(\hat{\boldsymbol{\delta}})$. Under the null hypothesis, the test statistic, ν, is asymptotically distributed as a chi-square variate with t degrees of freedom (Judge et al. 1985, p. 614). Following Gallant and Jorgenson (1979), it can be shown that under the alternative hypothesis H_A: $\mathbf{R\delta} \neq \mathbf{c}$, ν is asymptotically distributed noncentral chi square with noncentrality parameter[5]

$$\tau = (\mathbf{c} - \mathbf{R\delta})'[\mathbf{RVR}']^{-1}(\mathbf{c} - \mathbf{R\delta}). \tag{12}$$

Using equations (8) and (12), we can compute the statistical power of the test of t constraints by specifying the model under the alternative hypothesis and calculating values for $\mathbf{V}$ and τ. Given τ, we can obtain power from tables for the noncentral chi-square distribution (Hayman, Govindarajulu, and Leone 1970). Those tables were used to construct Figure 1, which shows the relationship between statistical power and the noncentrality parameter τ for both one- and two-degrees-of-freedom tests, given type I error rates of .05 and .001. The figure indicates how large the noncentrality parameter must be to achieve a certain level of protection against type I and type II errors. For example, for a one-degree-of-freedom test, to achieve a type II error of .90, the noncentrality parameter must be

[5]For a derivation of the noncentrality parameter of the likelihood-ratio test statistic for nonlinear simultaneous equation systems estimated by maximum likelihood, see Gallant and Holly (1980). More generally, this result was independently applied to linear covariance structure models by Satorra and Saris (1985) and Matsueda and Bielby (1986). As noted above, the results for 3SLS presented here are asymptotically equivalent to maximum likelihood results (Theil 1971, pp. 525–26).

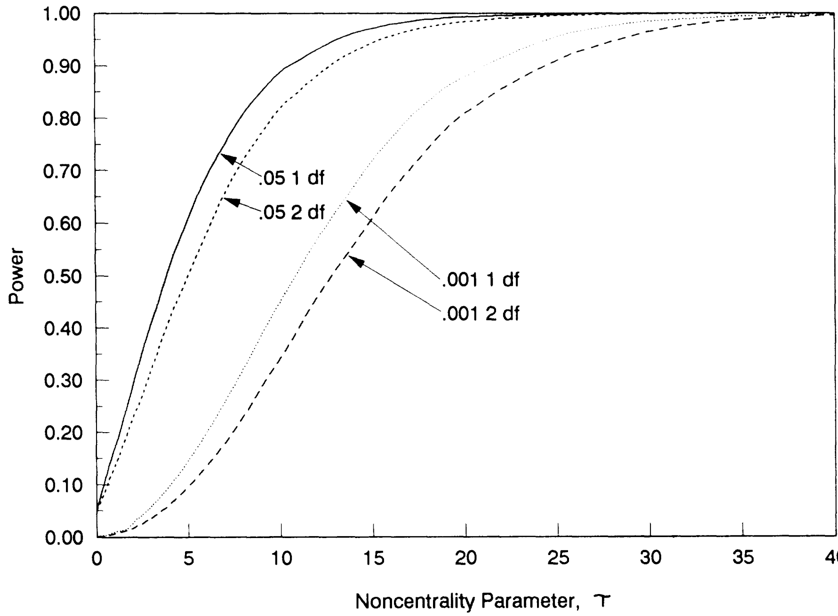

FIGURE 1. Power as a function of the noncentrality parameter, τ.

just over 10 at α = .05 and just over 20 at α = .001. For a two-degrees-of-freedom test, the corresponding noncentrality parameters must be 13 and 24. Furthermore, to achieve type I and type II error rates of .05 (α = .05 and power = .95), the noncentrality parameter must be at least 13 for the one-degree-of-freedom test and 16 for the two-degrees-of-freedom test.

For a given nonrecursive model, once sample size, null and alternative hypotheses, and level of protection against type I error have been specified, the noncentrality parameter—and therefore the power of the test—is a function of the variance-covariance matrix **V**, which in turn is a function of the model's parameters. Accordingly, power of tests within nonrecursive models can be computed in four steps:

1. Given the values of a model's parameters under the alternative hypothesis, the implied moments among endogenous and exogenous variables are computed.

2. Those moments are used to compute **V**, according to equation (8) or (for a two-equation model) equation (9).
3. The noncentrality parameter τ is computed using equation (12).[6]
4. Given τ, power is obtained from tables of the noncentral chi-square distribution. Equivalently, power can be obtained from computerized representations of those tables, such as LISPOWER under LISREL VII (Jöreskog and Sörbom 1989).

In the analyses below, we examine the impact of parametric structure on power and vary a parameter of the model across a range of values. Steps 1–4 are repeated for each specific value of the parameter of interest.

4. PARAMETRIC STRUCTURE AND STATISTICAL POWER

The power to detect parameters of a simultaneous equation model is influenced by the overall parametric structure of the model. In the single-equation linear model, $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$, the noncentrality parameter for the general linear hypothesis $\mathbf{R}\boldsymbol{\beta} = \mathbf{c}$ is

$$\tau = (\mathbf{c} - \mathbf{R}\boldsymbol{\beta})'[\mathbf{RVR}']^{-1}(\mathbf{c} - \mathbf{R}\boldsymbol{\beta}), \tag{13}$$

where $\mathbf{V} = \sigma_{\epsilon\epsilon}(\mathbf{X}'\mathbf{X})^{-1}$. Thus, for the classical linear model, power is a function of the disturbance variance, $\sigma_{\epsilon\epsilon}$, the degree to which parameters depart from the hypothesized linear relationship, $\mathbf{c} - \mathbf{R}\boldsymbol{\beta}$, and

[6] In analyses below, computations for the first three steps were obtained from matrix expressions using GAUSS statistical software (Edlefsen and Jones 1986) described in the appendices. Alternatively, the noncentrality parameter can be computed in LISREL using the procedure described by Matsueda and Bielby (1986, pp. 132–33): The *alternative* model is fit to the implied moments; the asymptotic covariance matrix **V** is computed from the correlation of estimates and standard errors produced by LISREL; and the noncentrality parameter is computed from equation (12). Our procedure for computing the noncentrality parameter differs from the approximation suggested by Satorra and Saris (1985) and Jöreskog and Sörbom (1989), which is biased asymptotically. Their approximation to the noncentrality parameter is the chi-square statistic obtained by fitting the model under the *null* hypothesis to the moments implied by the alternative model. For models that can be represented by equation (1), our procedure is tractable and asymptotically unbiased. Our approach can be intractable for more complex covariance structure models with latent variables, but the biased approximation is always tractable. On this point, see Satorra and Saris (1985, pp. 85–89) and Matsueda and Bielby (1986, pp. 148–52).

the moments among the exogenous variables, $\mathbf{X}'\mathbf{X}$ (Bielby and Kluegel 1977).

Despite the similarity of equations (12) and (13), the impact of parametric structure on power is considerably more complicated in a nonrecursive model than in a single-equation model, because in the former case, the variance-covariance matrix, $\mathbf{V}$, is a function of moments involving endogenous right-hand-side variables (see equations (8)–(10)). These moments are not exogenous to the model and are therefore functions of the model's parameters. Thus, the power of a test regarding the parameters of one equation is typically a function of parameters of other equations in the model.

In this section we present results from simulations that show how the power of selected tests varies as a function of several features of the parametric structure of the model. First is the strength of instrumental variables. Specifically we examine power as a function of the strength of the effect in the second equation of exogenous variables excluded from the first equation. Second, we examine statistical power as a function of the number of instrumental variables (or over-identifying restrictions). Specifically, we compare power calculations for a just-identified model (with one exogenous variable excluded from each of two equations) with calculations for an over-identified model (with two exclusions in each equation). Finally, we compute statistical power as a function of the strength of the reciprocal relationship between two endogenous variables and the degree of covariation between the structural disturbances.

4.1. *The Hypothetical Model*

Our example is a hypothetical nonrecursive model of married women's attitudes regarding (a) the desirability of working outside the home while one's children are young and (b) the desirability of having a large family. Below, we refer to these as *work attitude* and *family attitude,* respectively. We assume that they are measured on the same metric and that they are negatively related to one another. The model, diagrammed in Figure 2, has two endogenous variables (work attitude and family attitude) and five exogenous variables (woman's years of schooling, woman's work experience, husband's occupational status, husband's educational status, and number of siblings). Hypothetical baseline values for a just-identified model are

$$\mathbf{B} = \begin{bmatrix} 0 & -.20 \\ -.40 & 0 \end{bmatrix},$$

$$\mathbf{\Gamma} = \begin{bmatrix} .20 & .15 & -.40 & .10 & 0 \\ -.30 & 0 & -.15 & -.20 & .20 \end{bmatrix},$$

$$\mathbf{\Sigma}_{\epsilon\epsilon} = \begin{bmatrix} .60 & -.10 \\ -.10 & .50 \end{bmatrix}, \text{ and}$$

$$\mathbf{\Sigma}_{xx} = \begin{bmatrix} 1 & -.10 & .34 & .55 & -.25 \\ -.10 & 1 & -.02 & -.08 & -.30 \\ .34 & -.02 & 1 & .62 & -.03 \\ .55 & -.08 & .62 & 1 & -.04 \\ -.25 & -.30 & -.03 & -.04 & 1 \end{bmatrix}.$$

To simplify the selection of baseline parameter values, we have rescaled the exogenous variables to standard-deviation units (variances of one). However, results presented below do not depend on scaling of the measured variables.[7]

The model is just-identified and assumes that γ_{15} and γ_{22} are zero, i.e., that number of siblings has no effect on work attitude and that work experience has no effect on family attitude.[8] Values for a baseline over-identified model are identical to those above except that γ_{14} and γ_{23} are assumed to equal zero.

Given these baseline parameter values, the just-identified model is "weakly" identified, in the sense that estimates of β_{12} and β_{21} are subject to substantial sampling variability. This is reflected in the low statistical power of tests of each of these parameters. For example, for a sample size of 1,000, a type I error rate (α) of .05, and a just-identified model, the power to detect $\beta_{12} = -0.20$ with a one-degree-of-freedom (nondirectional) t test is only .39. For a type I error rate of $\alpha = .001$, the power to detect $\beta_{12} = -0.20$ is just .05.[9]

[7]The coefficients of the models reported below are not fully standardized, since the variance of each endogenous variable is a function of the parameters of the model and cannot be fixed at one.

[8]The model is just-identified if parameters to be estimated include all elements of $\mathbf{\Sigma}$ (the covariance matrix among disturbances), all elements of $\mathbf{\Sigma}_{xx}$ (the covariance matrix among exogenous variables), the off-diagonal elements of $\mathbf{B}$, and the nonzero elements of $\mathbf{\Gamma}$.

[9]We obtained these figures as follows. First, we generated implied moments from the baseline parameter values and sample size. Second, we used these moments to compute $\mathbf{V}$ from equation (9). Third, given $\mathbf{V}$, we used equation (12) to compute the noncentrality parameter τ of 2.581. Fourth, we referred to power tables for one degree of freedom and $\tau = 2.581$ to obtain power at $\alpha =$ .05 and at $\alpha = .001$.

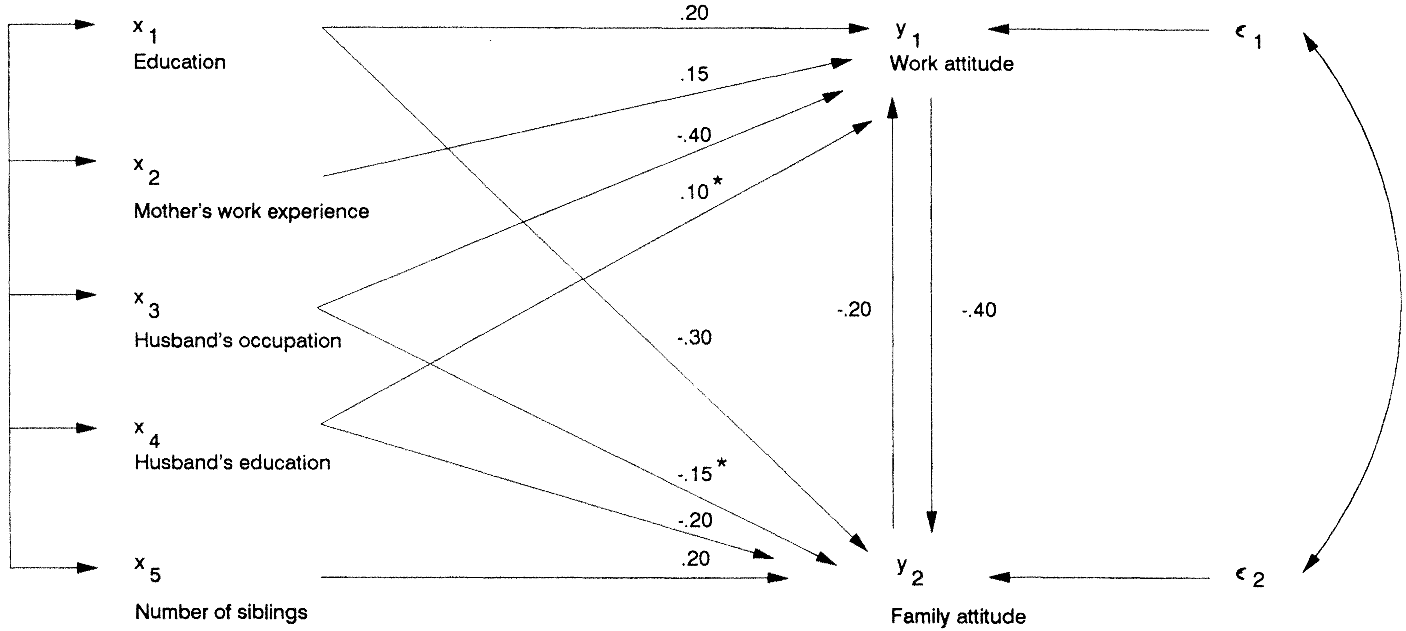

FIGURE 2. Hypothetical nonrecursive model. An asterisk indicates that the coefficient is set to zero for the over-identified model.

The corresponding power to detect $\beta_{21} = -0.40$ is 0.82 when $\alpha = .05$ and 0.34 when $\alpha = .001$.

We examine three tests of the reciprocal causal relationship between y_1 and y_2. Each represents a test typically conducted in the evaluation of nonrecursive models. First, we examine the one-degree-of-freedom test of a single coefficient relating the two endogenous variables ($\beta_{12} = 0$). This test determines whether one endogenous variable has any effect on the other. Second, we examine the two-degrees-of-freedom test that the reciprocal effects between the two variables are jointly zero ($\beta_{12} = \beta_{21} = 0$). This evaluates whether there is any relationship in either direction between the endogenous variables. Third, we examine the one-degree-of-freedom test that the difference between the coefficients is zero ($\beta_{12} - \beta_{21} = 0$). This test of whether the causal effect is larger in one direction or the other is of considerable interest in research on fertility and labor force participation (Lehrer and Nerlove 1986).

4.2. *Power and the Strength of Instrumental Variables*

We noted above that in a just-identified model, the sampling variability of estimates of reciprocal effects increases with the collinearity between predicted endogenous and exogenous variables on the right-hand side of the second-stage regression. Thus, in our example of a just-identified model, the sampling variability of the estimate of β_{12} increases as $\hat{y}_2$ becomes collinear with x_1 through x_4. Since $\hat{y}_2$ is a linear function of all five exogenous variables, the reduced-form effect of x_5 on y_2 is the only source of nonredundant variation in $\hat{y}_2$. For given values of β_{12} and β_{21}, the degree to which $\hat{y}_2$ varies independently of x_1 through x_4 is determined by the structural coefficient γ_{25}. As γ_{25} approaches zero, $\hat{y}_2$ approaches perfect collinearity with x_1 through x_4. Conversely, as the magnitude of γ_{25} increases, the sampling variability of the estimate of β_{12} decreases. Thus, in our hypothetical example, we interpret γ_{25} as an index of the strength of x_5 as an instrumental variable for the first equation.

To index the degree of collinearity in the second-stage regression, we can use either the proportion of variance explained in $\hat{y}_2$ by x_1 through x_4 (Cramer 1980) or the corresponding variance-inflation factor, $1/(1 - R^2)$ (Chaterjee and Price 1977). However, these are merely descriptive indices. Neither takes into account how sample size influences sampling variability, nor are these R^2 measures sensitive to all of the parameters that influence sampling variability (Maddala 1988, pp. 228–29). Thus, these measures are of limited use in addressing the problem of sampling variability within the context of formal statistical inference. By defining the issue as protection against type I and type II error, we can systematically analyze how sample size, parametric structure, and type I error rates affect inference in nonrecursive models.

Figure 3 shows the power of a one-degree-of-freedom chi-square test of β_{12} as a function of the strength of the instrumental variable x_5. The null hypothesis, $\beta_{12} = 0$, is contrasted with the alternative hypothesis, $\beta_{12} = -0.20$. For the just-identified model, the solid line shows that the noncentrality parameter, τ, increases curvilinearly with γ_{25}, the structural effect of x_5 on y_2.[10] (See Appendix A for a description of the GAUSS program that produced the computations upon which Figure 3 is based.) According to Figure 3, γ_{25} has to approach 0.4 before the noncentrality parameter exceeds 10, roughly the value at which power reaches .90 for $\alpha = .05$. Given the parameter values in the hypothetical just-identified model, when $\alpha = .001$, γ_{25} must approach 0.6 before τ exceeds 24, roughly the value at which power reaches .99. Thus, for a sample size of 1,000, the effect of x_5, the instrumental variable, on y_2, the right-hand-side endogenous variable, must be considerable if we are to have a reasonable probability of detecting a value of $\beta_{12} = -0.20$. At the baseline value of $\gamma_{25} = .20$,

[10]For the special case of the one-degree-of-freedom test of β_{12} in the just-identified model, the relationship between γ_{25} and τ can be expressed analytically. The sampling variance of the estimate of β_{12} computed from the second-stage regression is

$$\mathrm{Var}(\hat{\beta}_{12}) = \sigma_{11}(\hat{\mathbf{y}}_2'\mathbf{M}_1\hat{\mathbf{y}}_2)^{-1},$$

where $\mathbf{M}_1 = (\mathbf{I} - \mathbf{X}_1(\mathbf{X}_1'\,\mathbf{X}_1)^{-1}\mathbf{X}_1')$, so the noncentrality parameter is

$$\tau = (\beta_{12}^2/\sigma_{11})(\hat{\mathbf{y}}_2'\mathbf{M}_1\hat{\mathbf{y}}_2).$$

Further, $\hat{\mathbf{y}}_2'\,\mathbf{M}_1\hat{\mathbf{y}}_2 = \boldsymbol{\pi}_2\mathbf{X}'\mathbf{M}_1\mathbf{X}\boldsymbol{\pi}_2' + \mathbf{u}_2'\,\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{M}_1\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{u}_2 + 2\boldsymbol{\pi}_2\mathbf{X}'\mathbf{u}_2$, where $\boldsymbol{\pi}_2$ is the second row of the matrix of reduced-form coefficients, $(\mathbf{I} - \mathbf{B})^{-1}\boldsymbol{\Gamma}$. The term $2\boldsymbol{\pi}_2\mathbf{X}'\mathbf{u}_2$ is linear in γ_{25}, while the term $\boldsymbol{\pi}_2\mathbf{X}'\mathbf{M}_1\mathbf{X}\boldsymbol{\pi}_2'$ is quadratic in γ_{25}.

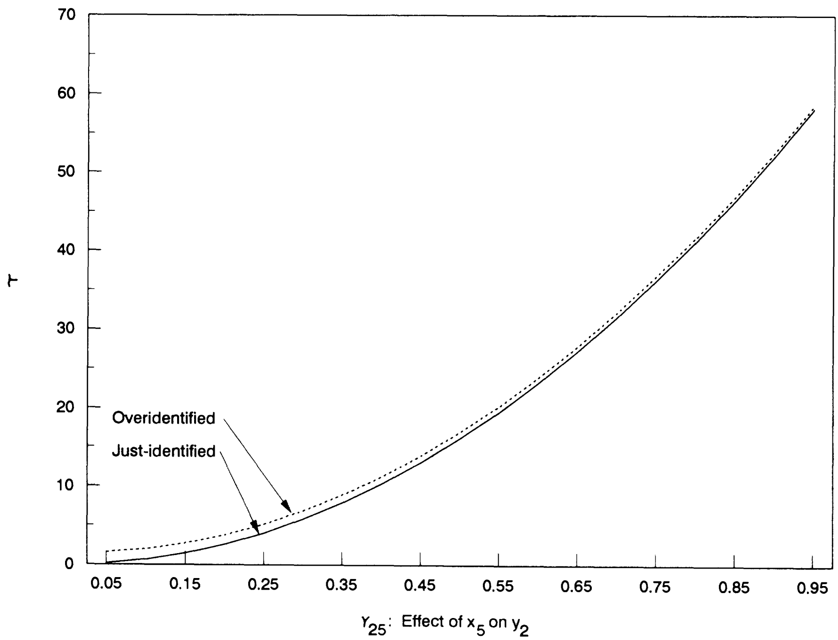

FIGURE 3. Noncentrality parameter τ as a function of γ_{25}: One-*df* test, $\beta_{12} = 0$.

a sample size of 3,874 would yield a noncentrality parameter of 10 for a power of approximately .9 at $\alpha = .05$.[11]

Figure 3 shows that the one-degree-of-freedom test is slightly more powerful for the over-identified model than for the just-identified model. The gain in power is relatively small because x_4 is also a relatively weak instrumental variable ($\gamma_{24} = -0.2$). Thus, we can conclude that for the hypothetical model posed in Figure 2, the probability of detecting an effect of family attitude on work attitude is weak even when two exogenous variables can be excluded from each structural equation.

Figure 4 shows the noncentrality parameter as a function of the strength of the instrumental variable x_5 for the two-degrees-of-freedom test that β_{12} and β_{21} are jointly zero (i.e., $\beta_{12} = 0$ *and* $\beta_{21} = 0$). The noncentrality parameter is evaluated at baseline values of the reciprocal effects of -0.20 for β_{12} and -0.40 for β_{21}.

[11]The noncentrality parameter, τ, is proportional to sample size n. For $n = 1{,}000$, $\tau = 2.581$ at $\gamma_{25} = 0.20$. Therefore, τ is 10 when $n = (10/2.581) \times 1{,}000 = 3{,}874$.

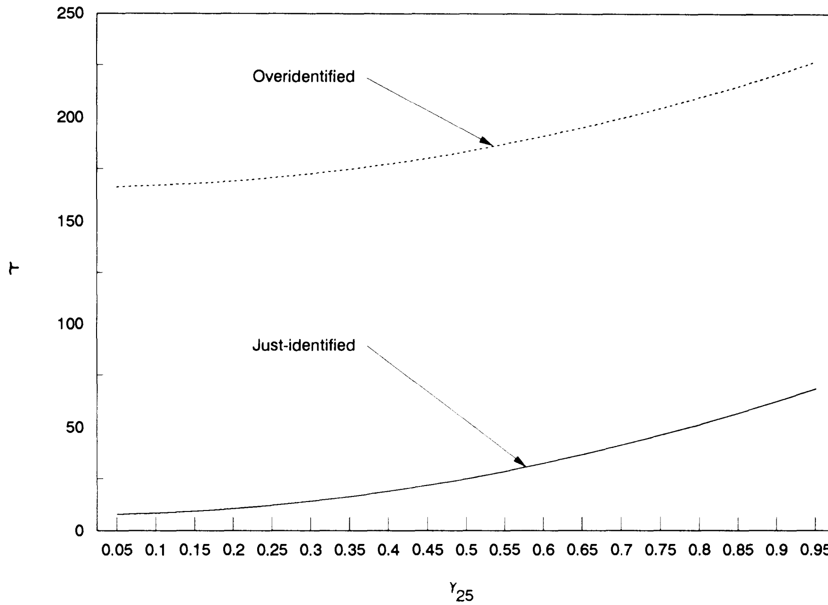

FIGURE 4. Noncentrality parameter τ as a function of γ_{25}: Two-*df* test, $\beta_{12} = \beta_{21} = 0$.

According to Figure 4, the power to reject the hypothesis of no reciprocal causation depends decisively on whether the model is over- or just-identified. For a two-degrees-of-freedom test at α = .05, power reaches .90 when the noncentrality parameter is approximately 12.5. At α = .001, power does not reach .90 until the noncentrality parameter is 24. For the just-identified model, the noncentrality parameter approaches 12.5 at $\gamma_{25} = 0.44$ and 24 at $\gamma_{25} = 0.61$. Therefore, for the just-identified model, reciprocal causation will not be detected at a reasonable level of type II error unless x_5 is a strong instrument for the first equation. This is because x_2, the instrumental variable for the second equation, is a weak instrument ($\gamma_{12} = 0.15$). Thus, estimates of *both* β_{12} and β_{21} are subject to substantial sampling variability when γ_{25} is small.

In contrast, for the over-identified model at α = .001, the power to reject the null hypothesis of no reciprocal causation exceeds .999 regardless of the strength of x_5 as an instrumental variable.[12] We

[12]The power of a two-degrees-of-freedom test at α = .001 exceeds .999 when the noncentrality parameter reaches 46.

have already noted in the over-identified model, both x_4 and x_5 are weak instruments for the first equation. However, x_3 is a strong instrument for the second equation ($\gamma_{13} = -0.4$). The standard error of the estimate of β_{21} is reduced by nearly 80 percent when x_3 is added as an instrument for the y_2 equation. Thus, the strong protection against type II error in the over-identified model is due to the precision with which we can estimate β_{21}, even when β_{12} cannot be estimated precisely.

We conclude from these results that statistical power varies in complicated ways with the hypothesis being tested, with the strength of the instrumental variables, and with the identification of the model. In the hypothetical model posed here, the likelihood of detecting the causal impact of y_2 on y_1 is low, regardless of whether the model is just- or over-identified, unless the instrument for the y_1 equation is quite strong (indeed, implausibly strong for the substantive example considered here). In contrast, the likelihood of rejecting the hypothesis of no reciprocal causation between y_1 and y_2 is high for the over-identified model, regardless of the strength of the instrument for the y_1 equation.

This conclusion is reinforced when we examine the probability of detecting whether the causal relationship among the endogenous variables is stronger in one direction than the other. The null hypothesis is $\beta_{12} - \beta_{21} = 0$, and the alternative is $\beta_{12} = -0.2$ and $\beta_{21} = -0.4$. Figure 5 shows that for the just-identified model, the noncentrality parameter is less than 2.0, regardless of the strength of the instrument (x_5) for the y_1 equation. In other words, with a just-identified model, we must have a sample of over 5,000 to have a reasonable likelihood of detecting a difference between β_{12} and β_{21} (when the actual population values are -0.2 and -0.4, respectively).

In contrast, in the over-identified model, the probability of detecting the difference between β_{12} and β_{21} rises sharply with γ_{25}, the strength of x_5 as an instrument for the first equation. Nevertheless, even in the over-identified model, a noncentrality parameter of 10 is not reached until γ_{25} exceeds 0.45. Thus, given a sample size of 1,000, two over-identifying restrictions, the parameter values in our baseline model, and weak to modest instruments for the y_1 equation, we have little chance of detecting whether causal effects are stronger in one direction than in the other.

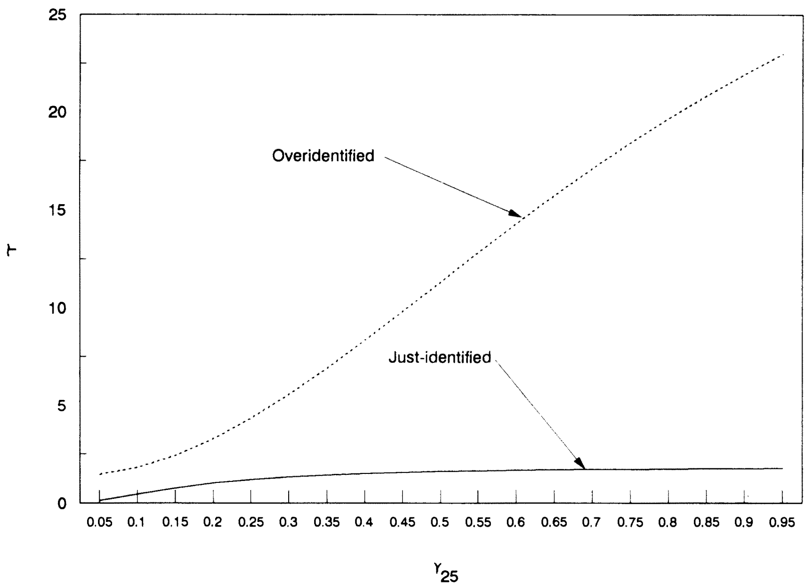

FIGURE 5. Noncentrality parameter τ as a function of γ_{25}: One-*df* test, $\beta_{12} - \beta_{21} = 0$.

4.3. *Power and the Size of Reciprocal Effects*

To examine how the size of reciprocal effects influences the power of various tests, we assume an alternative hypothesis, $\beta_{12} = 0.5\beta_{21}$, and compute the noncentrality parameter as a function of β_{12}. All other parameters (except β_{21}) are set to their baseline values for the just- and over-identified models. Figure 6 shows the influence of reciprocal effects on power for the one-degree-of-freedom test of the null hypothesis that $\beta_{12} = 0$ and for the hypothesis that $\beta_{21} = 0$.

Again, because of the differential strength of the instruments for the two equations, over-identification is much more consequential for the test of β_{21} than for the test of β_{12}. For a type I error rate of .05, the power to reject $\beta_{21} = 0$ exceeds .90 (i.e., the noncentrality parameter exceeds 10) when β_{12} and β_{21} are as small as -0.05 and -0.10, respectively, for the over-identified model. For the just-identified model, the same level of power is not reached until β_{12} and β_{21} are -0.23 and -0.46, respectively.

In contrast, the probability of detecting departures from the null hypothesis, $\beta_{12} = 0$, is weak regardless of whether the model is

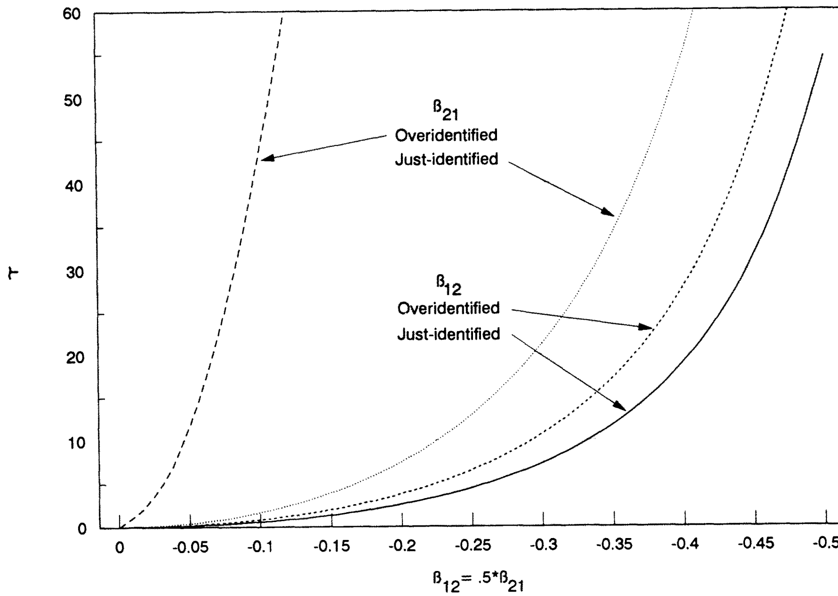

FIGURE 6. Noncentrality parameter τ as a function of $\beta_{12} = 0.5^*\beta_{21}$: One-*df* tests, $\beta_{12} = 0$ and $\beta_{21} = 0$.

over-identified. For the over-identified model and a type I error rate of .05, the power to reject $\beta_{12} = 0$ does not exceed .90 until β_{12} and β_{21} are -0.30 and -0.60, respectively. Thus, for the hypothetical model posed here, we would have a difficult time detecting an effect of family attitude on work attitude even when there is a sizeable effect in the population. In contrast, there is a high probability that we would detect even a small effect in the other direction (work attitude on family attitude) in the over-identified model.

Figure 7 reveals that the chances of detecting whether the reciprocal effect is larger in one direction than in the other are low unless the difference between the effects is quite large. The null hypothesis is the one-degree-of-freedom test $\beta_{12} - \beta_{21} = 0$, and again the alternative is $\beta_{12} = 0.5\beta_{21}$, with the noncentrality parameter computed across a range of values for β_{12}. For the just-identified model, the noncentrality parameter does not exceed 10 until $\beta_{12} = -0.43$ and $\beta_{21} = -0.86$. Even in the over-identified model, the noncentrality parameter does not exceed 10 until the reciprocal effects are -0.31 and -0.62, despite the precision with which β_{21} is estimated.

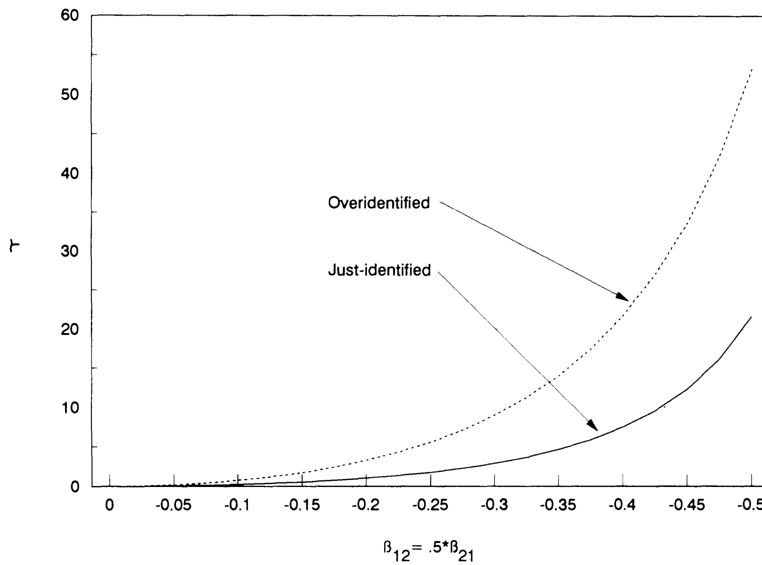

FIGURE 7. Noncentrality parameter τ as a function of $\beta_{12} = 0.5^*\beta_{21}$: One-*df* test, $\beta_{12} - b_{21} = 0$.

In sum, given the baseline values of our hypothetical model and a sample size of 1,000, our ability to detect reciprocal causal effects between work attitude and family attitude is limited. The only effect that we are likely to detect with a minimally acceptable level of certainty is the effect from attitude about working mothers (y_1) to attitude about large families (y_2), and then only if the model is over-identified. Detecting whether the causal effect is greater in one direction than in the other is especially problematic unless the effects are quite large. This is because of weak instruments for the y_1 equation.

4.4. *Power and the Size of the Disturbance Correlation*

Equation (10) above shows that for an over-identified model, the sampling variability of 3SLS coefficient estimates decreases as the absolute value of ρ, the correlation among structural disturbances, increases. Thus, the power of a one-degree-of-freedom test of a hypothesis about a single coefficient increases as ρ departs from zero. This is illustrated by the solid line in Figure 8 for the test of the

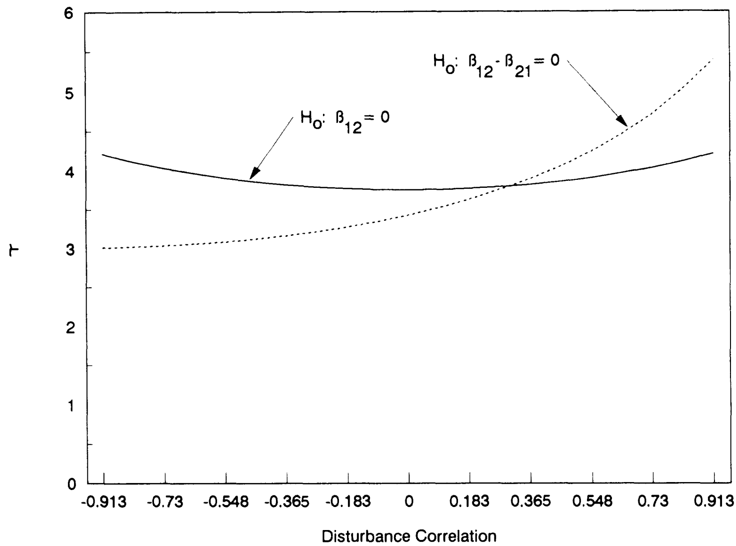

FIGURE 8. Noncentrality parameter τ as a function of correlation between disturbances: One-*df* tests, $\beta_{12} = 0$ and $\beta_{12} - \beta_{21} = 0$.

null hypothesis $\beta_{12} = 0$. The noncentrality parameter for the over-identified baseline model was computed as a function of the disturbance covariance over the range -0.5 to $+0.5$, which corresponds to a range of -0.913 to $+0.913$ for the correlation between the structural disturbances.

Given the particular baseline parameters we chose, the disturbance covariation has little effect on the power to detect β_{12} under the alternative hypothesis $\beta_{12} = -0.20$. The noncentrality parameter computed at $\rho = 0$ ($\tau = 3.75$) is 89 percent as large as that computed at $|\rho| = 0.916$ ($\tau = 4.212$). According to Figure 1, at a type I error rate of .05, the power of the test is close to .50, regardless of the size of the disturbance correlation.

The disturbance correlation is more consequential for the one-degree-of-freedom test of the difference between the coefficients, which corresponds to the null hypothesis $\beta_{12} - \beta_{21} = 0$. For that test (under the over-identified baseline model), the noncentrality parameter increases with ρ at an increasing rate. At $\rho = -0.913$ the noncentrality parameter ($\tau = 3.01$) is only 56 percent as large as it is at $\rho = +0.913$ ($\tau = 5.39$). At a type I error rate of .05, the corresponding probabilities of rejecting the null hypothesis are .42 and .64, respectively.

The power of a test on a single coefficient will always be lowest when $\rho = 0$, given any specific values of the other parameters in an over-identified model. However, the power to detect departures from $\beta_{12} - \beta_{21} = 0$ need not necessarily increase with ρ. The sampling variability of $\hat{\beta}_{12} - \hat{\beta}_{21}$ is

$$\mathrm{Var}(\hat{\beta}_{12}) + \mathrm{Var}(\hat{\beta}_{21}) - 2\mathrm{Cov}(\hat{\beta}_{12},\hat{\beta}_{21}).$$

In our particular example, the sampling covariance increases with ρ, thereby decreasing the sampling variability of the difference between the coefficient estimates (and increasing the power of the test). Moreover, in our example, the sampling covariance dominates the sampling variances in the above expression. That is, the rate at which the sampling covariance increases as ρ ranges from -0.916 to 0 (which reduces the sampling variability of the difference) more than offsets the increases in the sampling variability of $\hat{\beta}_{12}$ and $\hat{\beta}_{21}$ over the same range.

Finally, equation (10) implies that the impact of the disturbance correlation on sampling variability (and therefore power) is contingent upon $\mathbf{A}^*$, which in turn depends on the strength of instrumental variables. Consequently, the sensitivity of τ to chnages in ρ should be greater for the test of β_{21} than for the test of β_{12}, since the y_2 equation has stronger instruments than the y_1 equation. This is apparent when we compare Figure 9 with Figure 8. The proportionate change in the noncentrality parameter is indeed greater for the test of $\beta_{21} = 0$ (against the alternative $\beta_{21} = -0.40$). For that test, the noncentrality parameter evaluated at $\rho = 0$ ($\tau = 158.51$) is 84 percent as large as that computed at $|\rho| = 0.916$ ($\tau = 189.05$).

In sum, in an over-identified model, the sampling variability of 3SLS estimates of individual coefficients decreases with the magnitude of the covariance between structural disturbances, thereby increasing the power of tests on individual coefficients.[13] However, for a test of coefficients from more than one equation, the impact of the error covariance on statistical power is contingent upon the overall parametric structure of the model.

[13]The relative efficiency of 3SLS compared with 2SLS also increases with the magnitude of the covariance between structural disturbances.

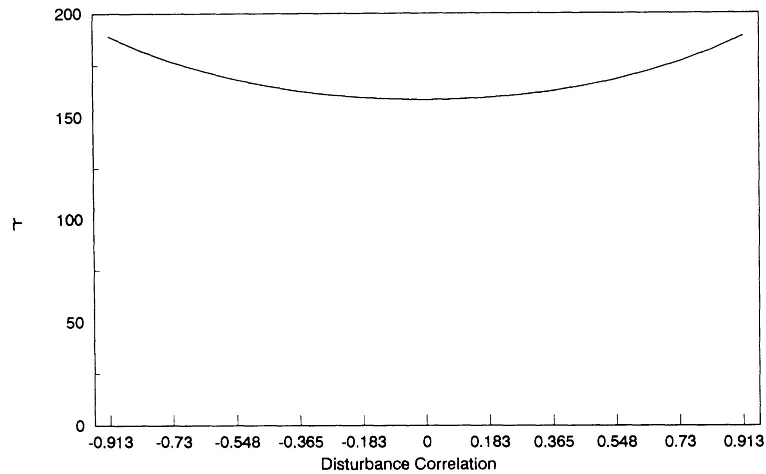

FIGURE 9. Noncentrality parameter τ as a function of correlation between disturbances: One-*df* test, $\beta_{21} = 0$.

5. SUMMARY AND CONCLUSIONS

In nonrecursive models, sampling variability and the probability of detecting causal effects among endogenous variables can depend on the parametric structure of the model in ways quite different from what sociologists encounter in classical regression and recursive structural equation models. Depending on the hypothesis tested, the power of the test can vary in complicated ways with the strength of the instrumental variables, with the number of over-identifying restrictions, and with the covariation among disturbances. Consequently, rules of thumb regarding appropriate sample sizes, magnitudes of coefficients that are substantively significant, and so on, can be grossly misleading. The likelihood of detecting asymmetric causal relationships, a central issue in research on fertility behavior (Smith-Lovin and Tickamyer 1978; Rindfuss, Bumpass, and St. John 1980) and on work and family interaction (Berk and Berk 1978; Bielby and Bielby 1989), can be especially problematic.

Our results suggest several ways in which sociological applications of nonrecursive models can be improved. For example, sociologists should be more sensitive to the issue of type II error when interpreting results from nonrecursive models. As our example illustrates, having a viable substantive rationale for restrictions that identify a model's parameters does not guarantee that valid inferences will be drawn from *estimates* of those parameters. An effect of an endogenous variable that is formally identified can often be difficult to detect, even when the sample size is large by sociological standards. Without explicitly assessing the type II error rate, sociologists are likely to conclude incorrectly that effects are absent (or symmetric) more often than they realize.

Sociological applications of nonrecursive models should systematically address the issue of type II error in tests of hypotheses concerning the effects of endogenous variables. For example, a researcher conducting such an analysis with our hypothetical model would be confronted with several issues. First, she or he would probably conclude that a sample of 1,000 is too small to assess (a) the effect of family attitude on work attitude and (b) asymmetry in the reciprocal relationship between the endogenous variables. Second, the researcher would note that the probability of detecting a causal effect of work attitude on family attitude depends decisively on whether the additional restriction that renders the y_2 equation over-identified ($\gamma_{23} = 0$) can be justified substantively. Indeed, depending on the researcher's loss function regarding the trade-off between bias and efficiency, she or he might conclude that the substantial gain in power obtained by imposing the over-identifying restriction more than offsets the bias introduced by small departures from the restriction in the population. As Figures 5–7 illustrate, depending upon the hypothesis, the potential gain in power due to the addition of a strong instrument can be comparable to that of a very substantial increase in sample size.

Of course, the specific findings presented in Figures 3–9 depend upon the values of the baseline parameters we have chosen. Had we chosen lower correlations among exogenous variables, smaller disturbance variances, or more over-identifying restrictions, the power of the statistical tests we examined would have been greater. The analysis of statistical power is always problem-dependent. To specify null and alternative hypotheses completely requires the specification of plausible values of the model's parameters under both sets of circumstances. If, for example, an analyst were to choose values for exogenous moments based on consistent sample estimates of those moments, then power calculations would be contingent upon those sample values. In a substantive area different from the one we have chosen, sample sizes, collinearity among exogenous variables, and the magnitudes of effects could be quite different, yielding very different calculations of statistical power.

Nevertheless, the *procedures* we have presented are widely applicable. Indeed, they apply to any model that can be specified in terms of the classical textbook econometric model described in equation (1). These include nonrecursive models with more than one equation, fully recursive models, and seemingly unrelated regression models, among others. The same four steps that we have used here to compute power can be applied to any of these models, and the GAUSS program described in the appendices can be modified to accommodate any of these specific instances of the classical simultaneous equation model. Moreover, equations (8) and (12) allow the researcher to express the mathematical relationship between a model's parameters and the noncentrality parameter for a statistical test.

Our analyses open several areas for future research. One is to explore more formally the trade-offs between bias and efficiency according to different loss functions when an exogenous variable is known to have a very strong effect on one endogenous variable and (at most) a small effect on another endogenous variable. Another is to examine the impact of multiple indicators on the probability of detecting effects in nonrecursive models. We have shown elsewhere that for recursive models, additional indicators can have an impact on power that is comparable to a substantial increase in sample size (Matsueda and Bielby 1986). The extension of these results to nonrecursive models would allow researchers to evaluate the relative costs and benefits of increased sample size versus additional indicators during the design stage of a research project.

APPENDIX A
A GAUSS PROGRAM FOR COMPUTING NONCENTRALITY PARAMETERS

For any test of parameters in a linear structural equation model, the associated noncentrality parameter can be expressed as a function of the asymptotic covariance matrix, **V**, of the estimator for the model's coefficients, as in equation (12) above. In this paper we exploit the fact that for a 3SLS estimator of a simultaneous equation model, **V** can be expressed in terms of moments among the observable variables, which in turn can be expressed in terms of the model's parameters. The GAUSS program in Appendix B shows how we have computed the relationship between a model's parameters and the noncentrality parameters associated with various statistical tests.

The program applies to the just-identified model in Figure 2 and the analysis of power as a function of the strength of the instrumental variables. It computes the noncentrality parameters associated with the three tests, $\beta_{12} = 0$, $\beta_{12} = \beta_{21} = 0$, and $\beta_{12} - \beta_{21} = 0$, as a function of parameter γ_{25}. Specifically, all parameters of the model other than γ_{25} are set to values that correspond to the alternative hypothesis, while γ_{25} is varied from 0.05 to 0.95 in increments of 0.05. For each value of γ_{25} (i.e., for each iteration of the loop), the program computes the implied moments, the 3SLS asymptotic covariance matrix, and the three noncentrality parameters of interest. The four columns of numbers in Appendix C are the output of the program. They are plotted as the solid lines in Figures 3, 4, and 5. We computed the over-identified model by changing the *gamm=* expression to let $\gamma_{14} = \gamma_{23} = 0$ and by changing the *let rowsx1* and *let rowsx2* expressions to reflect the two additional exclusions of exogenous variables.

We computed power as a function of the size of reciprocal effects by modifying the program to iterate on β_{21} and setting β_{12} equal to $0.5\beta_{21}$. Finally, we computed power as a function of the size of the disturbance correlation by modifying the program to iterate on σ_{12}.

Although the program has been set up for a system with five exogenous variables, it can easily be modified to accommodate any two-equation system by changing the dimensions of the appropriate matrices. Moreover, the program can be generalized to more than two equations by using the more general expression in equation (8) instead of equation (9) to compute the asymptotic covariance matrix. Further, the effects of other parameters on power can be assessed by modifying the program to iterate on the parameter of interest. Finally, the effect on power of different baseline parameter values can be assessed by changing the entries in step 1, part 1.

APPENDIX B
GAUSS PROGRAM FOR COMPUTING NONCENTRALITY PARAMETERS AS A FUNCTION OF γ_{25}, JUST-IDENTIFIED MODEL

```
/* program to compute noncentrality parameter for test of */
/* general linear hypothesis in a simultaneous equations */
/*context. William T. Bielby and Ross L. Matsueda, 7/90 */
/* this version computes tau as a function of gamma25 */
/* for the just-identified model—see Figure 3, 4, 5 */
format 6,3;
output file = c:\power\generic.out reset;
outwidth 250;
n=1000;

/* Step 1, part 1: SPECIFY PARAMETER VALUES */
/* this is set up to iterate on gamma25 */
let sdx[5,1]=1 1 1 1 1;
dx=diagrv(eye(5),sdx);
/* above is s.d. of x's */
let rx[5,5]=
  1    -.10    .34    .55   -.25
-.10     1    -.02   -.08   -.30
 .34   -.02     1     .62   -.03
 .55    .08    .62     1    -.04
-.25   -.30   -.03   -.04     1   ;
/* above is correlation of x's */
sigxx=dx*rx*dx;
gamm25=.05;
b21 = .400;
b12 = .200;
do while gamm25 <1.0;
/* above iterates on parameter of interest */
gamm=( .200~ .150~ -.400~ .100~ .000) |
        (-.300~ .000~ -.150~ -.200~ gamm25);
/* above is initial gamma matrix, we'll iterate on gamma25 */
beta = (1~b12) | (b21 ~1);
let see[2,2]=.600 -.100 -.100 .500;
```

```
/* above are beta and psi matrices (LISREL IV notation) */
seinv=invpd(see);
seinv11=seinv[1,1];
seinv22=seinv[2,2];
seinv12=seinv[1,2];

/* Step 1, part 2: COMPUTE ENDOGENOUS MOMENTS */
sigyx=inv(beta)*gamm*sigxx;
sigyy=inv(beta)*gamm*sigxx*gamm'*(inv(beta))' +
inv(beta)*see*(inv(beta))';
sigxy=sigyx';
vary=diag(sigyy);
dy=eye(2).*sqrt(vary);
ryx=inv(invpd(dy)*beta*dy)*invpd(dy)*gamm*dx*rx;
ryy=invpd(dy)*sigyy*inv(dy);
let rowsx1= 1 2 3 4 ; /* x variables included in eq 1 */
let rowsy1=2; /* y vars included in equation 1 */
sxyl=submat(sigxy,0,rowsy1); /* cov matrix of eq l x vars w */
/* eq 1 y vars */
sy1x=sxy1';
sx1x=submat(sigxx,rowsx1,0); /* cov matrix of eq 1 x vars w */
/* all x vars */
xxinv=invpd(sigxx);
/* similar computations for equation 2 follow */
let rowsx2 = 1 3 4 5;
let rowsy2=1;
sxy2=submat(sigxy,0,rowsy2);
sy2x=sxy2';
sx2x=submat(sigxx,rowsx2,0);

r=(ryy~ryx) | (ryx'~rx);
std=(diag(dy) | diag(dx))';
/* above are correlations and standard deviations */
/* can be output to LISREL

/* Step 2: COMPUTE VARIANCE-COVARIANCE */
/* MATRIX OF 3SLS ESTIMATOR */
/* what follows is 3sls asymptotic covariances following Theil */
```

```
/* z1 includes eq 1 rght hnd vars, z2 includes eq 2 rght hnd vars */
sz1x=sy1x | sx1x; /* var-cov matrix of z1 vars with all x vars */
sz2x=sy2x | sx2x; /* var-cov matrix of z2 vars with all x vars */
a11=n*sz1x*xxinv*sz1x';
a12=n*sz1x*xxinv*sz2x';
a22=n*sz2x*xxinv*sz2x';
covest=invpd((seinv11*a11~seinv12*a12)
| (seinv12*a12'~seinv22*a22)); /* see Theil, p. 515, top */
stderr=sqrt(diag(covest));
varb12=covest[1,1];
varb21=covest[6,6];
b12=beta[1,2];
b21=beta[2,1];

/* Step 3: COMPUTE NONCENTRALITY PARAMETERS */
/* FOR VARIOUS TESTS */
tau1=b12*b12/varb12;
tau2=b21*b21/varb21;

/* create submatrix for tests on b21 and b12 */
rowscv=1~6;
vr=submat(covest,rowscv,rowscv);

/* compute 2 df tau for b12 = b21 = 0 */
tau2df=(b12~b21)*invpd(vr)*(b12 | b21);
h=(1~-1);

/* compute 1df tau for b12 - b21 = 0 */
taudiff=(h*(b12 | b21))'*invpd(h*vr*h')*(h*(b12 | b21));

/* OUTPUT */
/* create output vector of all parameters to plot */
outpar=gamm25~tau1~tau2df~taudiff;
;
print /m0 /rd /m1 outpar;
gamm25=gamm25+.05;
endo;
```

APPENDIX C
OUTPUT OF GAUSS PROGRAM

	Noncentrality Parameter, τ		
γ_{25}	(1) $\beta_{12}=0$	(2) $\beta_{12}=\beta_{21}=0$	(3) $\beta_{12}-\beta_{21}=0$
0.050	0.161	7.777	0.144
0.100	0.645	8.398	0.455
0.150	1.452	9.343	0.769
0.200	2.581	10.612	1.020
0.250	4.033	12.204	1.206
0.300	5.808	14.121	1.340
0.350	7.906	16.361	1.438
0.400	10.326	18.926	1.511
0.450	13.068	21.814	1.567
0.500	16.134	25.026	1.610
0.550	19.522	28.562	1.644
0.600	23.233	32.422	1.671
0.650	27.266	36.606	1.693
0.700	31.622	41.114	1.711
0.750	36.301	45.946	1.726
0.800	41.303	51.101	1.739
0.850	46.627	56.581	1.750
0.900	52.274	62.384	1.759
0.950	58.243	68.511	1.768

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