STRUCTURAL EQUATION MODELS

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More than a decade ago, methods for modeling the structure of relationships among variables with systems of equations began to diffuse among sociologists. Expositions and applications have typically referred to *causal models* or *path analysis,* and we use those terms and *structural equation models* interchangeably. We prefer the latter term, since we do not attempt to impose a specific definition of cause. Rather, we take the heuristic view that the meaning of cause resides in the mechanisms thought to be embodied in an equation system. On this matter we are in substantial agreement with the French econometrician Malinvaud (1966): "A model is the formal representation of the notions that we have about a phenomenon." We think that efforts to impose a narrow definition of cause or effect on the potential application of structural equation models are unproductive (Lindsey 1973; Guttman, unpublished manuscript 1976). Sociologists speak of "causal models" because the term provides a convenient description of what a structural equation system does.

In early sociological discussion of these models, Simon (1975) and Blalock (1961, 1962, 1964) employed systems of equations to derive predictions about zero-order and partial correlations. Boudon (1965) noted that coefficients of the system of equations could be estimated and interpreted; in general, such coefficients are not partial correlations. A year earlier, Duncan & Hodge (1964) had estimated coefficients of a two-equation model of educational and occupational attainment. Indeed, it appears to have been the fruitful application of structural equation models to research in social stratification [as exemplified in the work of Blau & Duncan (1967)] rather than an increased statistical sophisticiation among sociologists, that accounts for the rapid diffusion of the use of causal models throughout the discipline. By the late 1960s, a number of expository papers (Duncan 1966, Heise 1968, Land 1968)

and elementary applications (B. Duncan 1967; Sewell & Shah 1967, 1968; Spaeth 1968; Werts 1968; Sewell, Haller & Portes 1969) of path analysis had appeared.

The substance and style of this work originated in the exposition of path analysis by the geneticist Wright (1934, 1954, 1960a), primarily through Duncan's (1966) expository paper. Wright's "principle of path analysis" provides an algorithm for expressing moments of the joint distribution of observable variables in terms of structural parameters. The isomorphism between path diagrams and systems of structural equations was exploited by Wright in both deductive and inductive applications of path analysis. The path diagram has continued to be a significant aid in teaching, exposition, and interpretation of structural equation models. Most of Wright's work expresses variables in standard form—as departures from the mean in standard deviation units—and path analysis is sometimes identified with this convention. However, as Wright (1960a) himself has pointed out, the method of path analysis does not require the use of standardized variables.

Wright's applications of path analysis covered a wide range of issues in genetics, psychology, and economics (1923, 1925, 1934). Wright (1934;204–13) was well aware of problems of statistical estimation and inference, and he anticipated later developments with respect to overidentification (1934), simultaneity (1960b), and unobserved variables (1925, 1960a). See Goldberger (1972) for a detailed appreciation of Wright's methodological contributions. Wright's perspective on path analysis is reflected in Li's recent (1975) introductory text. Through the early 1970s, the influence of Wright's approach on sociologists was evident both in expository treatments of more complex models (Blalock 1969a, 1970, 1971; Costner 1969, 1971; Duncan 1968, 1969, 1972; Heise 1969; Land 1970; Althauser & Heberlein 1970; Althauser, Heberlein & Scott 1971) and in more sophisticated empirical applications (Duncan, Haller & Portes 1968; Hodge & Treiman 1968; Siegel & Hodge 1968; Hauser 1969a, b, 1971; Siegel 1970; Duncan, Featherman & Duncan 1972). Some of this work incorporated specifications of simultaneous causation and latent variables that were later given better statistical treatment.

The accessibility of a variety of models and techniques for estimation and inference has increased since the early 1970s. The distinction between population and sample has been observed more carefully, and more reliance has been placed upon general analytic approaches to statistical estimation and inference. The period has been one of self-teaching among individual sociologists and for the discipline as a whole. It has been characterized by rediscovery, review, and exposition of ideas developed in other fields, with perhaps a few innovations.

The development of structural equation modeling within the sociological literature is no less important to the field because it derives from econometrics, psychometrics, and mathematical statistics. Indeed, sociology has provided the arena for a synthesis of the diverse approaches to structural models in econometrics and psychometrics. The substantive application of structural models to educational and socioeconomic inequality and the associated issues of measurement have contributed to a useful recombination of classical econometric treatments of structure with factor-analytic approaches. The empirical paper by Duncan, Haller & Portes (1968) was an important stimulus to this work because its interpretation of the influence of peers on

ambition combined the economic notion of simultaneity with the use of unobserved variables. Hauser (1971) used econometric psychometric notions of structure in models of academic achievement and aspiration; these ideas were further developed by Hauser & Goldberger (1971; also see Wold 1975). Subsequent work in social stratification (Bielby, Hauser & Featherman 1976, Chamberlain 1976) has applied models synthesizing econometric and psychometric approaches to structure such as those developed by Jöreskog (1970a, 1973, 1976b). The econometric/psychometric synthesis is discussed in detail by Goldberger (1971, 1972). Beyond stimulating the synthesis of statistical models developed elsewhere, the development of structural equation modeling by sociologists has also led to its diffusion in psychological and educational research (Werts & Linn 1970, Anderson & Evans 1974).

The recent sociological literature on structural equation models has been of uneven quality. Powerful statistical models have often been applied inappropriately or unpersuasively to empirical data. Some have argued that this reflects a faddish tendency of journal editors and reviewers to encourage quantitative empirical analyses regardless of substantive merit (Coser 1975). On the other hand, one might count it a virtue that poor theories and sloppy empirical work become more obvious when exposed by an explicit model. Some methodological developments, presented as innovative and useful, have in our opinion been unoriginal, misdirected, or simply incorrect. An unfortunate paper has sometimes stimulated refinements or extensions of equally questionable value. Many issues in structural equation modeling involve advanced mathematical statistics. Our methodological literature would be less error-prone if we let those problems be addressed by persons with advanced training. The fact is that sociologists are mining a well-developed territory, and naive readers should be skeptical of both the appearance of sophistication and claims to innovation.

There has been a substantial lag between the exposition of new or more powerful methods and sound empirical applications of them. For example, there have been many more expositions than substantive applications of models containing unobservable variables (Costner & Schoenberg 1973, Hauser 1973, Alwin & Tessler 1974, Otto & Featherman 1975, Mason et al 1976). Yet there are some encouraging signs. The specification of structural equation models and the values of their parameters have increasingly become the focus of important substantive debates. This is especially true of social stratification, where the use of causal models diffused early and rapidly. Jencks et al (1972) and the economist Bowles and his colleagues (Bowles 1972, Bowles & Nelson 1974, Bowles & Gintis 1976) have employed structural equation models to reassess earlier work of Duncan and others (Blau & Duncan 1967, Duncan, Featherman & Duncan 1972) on the determinants of economic success. Structural models have in turn been used to criticize these reassessments (Sewell 1973; Hauser & Dickinson 1974; Taylor 1973; Jencks 1973, 1974; Olneck 1977; Bielby, Hauser & Featherman 1976). Also, there has been a lengthy and sometimes heated controversy about the identifiability and the magnitude of structural parameters in models of hereditary and environmental effects on cognitive ability (Wright 1934; Jencks et al 1972; Jensen 1972, 1975; Jinks & Eaves 1974; Hogarth 1974; Rao, Morton & Yee 1974; Goldberger 1977).

The recent appearance of a number of texts on structural equation models written by or for sociologists (Van de Geer 1971, Namboodiri, Carter & Blalock 1975, Duncan 1975a, Heise 1975) is another encouraging development. This should result in increased exposure of sociologists to causal models and more competent use of the models within the discipline. Van de Geer uses path diagrams as an aid to mathematical exposition of techniques of multivariate analysis. Namboodiri, Carter & Blalock include introductory reviews of recursive and nonrecursive models and measurement error in their exhaustive treatment of linear models and experimental design. Heise exposits elementary statistical ideas and uses flowgraphs to present a condensed and comprehensive catalog of issues in structural equation modeling and systems analysis (also see Davis 1975). Duncan presumes the statistical competence of the reader and gives an integrated overview of the specification, identification, and interpretation of structural equation models. Many topics in structural equation modeling—especially methods of statistical inference and estimation—are developed in more detail and, indeed, may be more accessible in texts written for other fields. Thorough treatments of the classical econometric simultaneous equation model may be found in texts by Rao & Miller (1971), Wonnacott & Wonnacott (1970), Johnston (1972), Theil (1971), Pindyck & Rubinfeld (1976), and Goldberger (1964). Statistical issues in factor analysis are discussed in detail by Lawley & Maxwell (1971).

SCOPE OF THE REVIEW

The theme of structural models could be interpreted to cover a variety of topics that this review ignores. We limit our discussion to systems of equations describing causally interpreted structures. We ignore the descriptive use of multivariate analysis for data reduction and decomposition; for example, see the texts by Tatsuoka (1971), Hope (1969), Cooley & Lohnes (1971), and Morrison (1967). While we do not discuss issues of classical test theory and psychometric measurement (Lord & Novick 1968, Bohrnstedt 1970), the same measurement issues arise in the context of structural equation models containing unobserved variables (Jöreskog 1971a, Heise & Bohrnstedt 1970, Alwin 1977).

Furthermore, we do not cover issues of structure arising in single-equation models; specifically, we have not reviewed the voluminous literature on the general linear model that has appeared over the past decade. Excellent overview articles on the general linear model include those by Cohen (1968), Fennessey (1968), and Bohrnstedt & Carter (1971). The text by Searle (1971) contains a thorough conceptual treatment, and applications are emphasized by Kerlinger & Pedhazur (1973), while both issues are dealt with adequately in the recent book by Cohen & Cohen (1975). However, many issues like functional form, autocorrelation (Hibbs 1974), and aggregation (Hannan 1971, Hannan & Burstein 1974) can be interpreted as problems of structural specification and estimation.

The literature on the general linear model is especially useful in elaborating assumptions used for estimation and inference in structural equation models and procedures for overcoming violations of those assumptions. The researcher must

appreciate the assumptions required in structural modeling. Analysts who are unable or unwilling to make the necessary assumptions are referred to recent developments in discrete multivariate analysis. (For a comprehensive introduction, see Bishop, Fienberg & Holland 1975; Goodman 1972a, b.) Recursive, simultaneous, and latent causation have also been treated in the discrete case (Goodman 1973a, b, 1974).

SPECIFICATION OF STRUCTURAL EQUATION MODELS

A structural equation model specifies the process underlying the joint distribution of a set of observable variables. In this section we discuss the specification of these models; identification, estimation, and hypothesis testing are reviewed in later sections. The idea that structural parameters are fundamental or invariant appears throughout the literature. According to Goldberger:

> By structural equation models I refer to stochastic models in which each equation represents a causal link, rather than a mere empirical association . . . Generally speaking the structural parameters do not coincide with coefficients of regressions among observable variables, but the model does impose constraints on those regression coefficients. As a consequence, we face subtle issues of identification and draw upon elaborate methods of statistical inference. (1972:979). . . . the search for structural parameters is a search for invariant features of the mechanisms that generate observable variables. Invariant features are those which remain stable—or vary individually—over the set of populations in which we are interested. When regression parameters have this invariance, they are the proper objects of research, and regression is an appropriate tool. But when, as appears to be the case in many social science areas, regression parameters lack this invariance, the proper objects of research are more fundamental parameters; and statistical tools which go beyond conventional regression are required. (1973a:6).

A similar conceptualization of structural equation models is expressed by Duncan:

> The structural form of the model is that parameterization—among the various possible ones—in which the coefficients are (relatively) unmixed, invariant, and autonomous . . . if the coefficients in the model are indeed relatively invariant across populations, somewhat autonomous, and not inseparable mixtures of the coefficients that "really" govern how the world works—then your model is actually in the "structural" form. (1975a:151).

Duncan's statement introduces an important qualification of the view expressed by Goldberger: "correct" specification is not an all-or-nothing proposition. It is possible to specify a reasonable structural model characterized by parameters that are orderly or well-behaved combinations of parameters of a more elaborate model that better represents underlying causal mechanisms. Indeed, successful applications often involve the initial specification of a basic model, followed by conceptual and empirical developments that encourage the elaboration of that model; for example, see Duncan, Featherman & Duncan (1972).

Explicit references to fundamental mechanisms or to invariant and autonomous parameters almost never appear in sociological expositions and applications of structural equation models, although these notions usually motivate the use of the

models. Often, causal models are first applied to a given research problem because earlier models of analysis are believed to misrepresent underlying processes. Subsequently, more complex models are introduced to represent underlying structures more accurately. For example, Costner (1971) and Blalock (1971) have argued that structural models with unobservable variables more accurately represent the processes underlying experiments than do the traditional statistical designs for experimentation like those based on analysis of variance or analysis of covariance. Their suggestions were later elaborated and applied by Alwin & Tessler (1974).

The issue of invariance has been raised explicitly in discussion of the appropriateness of standardized parameters in structural equation models (Blalock 1967, Schoenberg 1972, Duncan 1975a, Kim & Mueller 1976). Suppose parameters are to be compared across populations. If there exists a meaningful metric (unstandardized) specification, then comparisons of standardized parameters involve inseparable combinations of more fundamental metric parameters. Duncan (1975a:64–65) similarly argues the inappropriateness of interpreting multiple correlation coefficients as structural parameters. However, Hargens (1976) notes that standardized parameters may be invariant when metric parameters are not.

Structural equation models have been used to represent a variety of causal systems. We offer a brief review of such models that illustrates some of their features.

Recursive Models in Observable Variables

Models in observable variables can be expressed by the following system of equations:

$$\Gamma y_i = B x_i + u_i$$
$$(L \times L)(L \times 1) \quad (L \times K)(K \times 1) \quad (L \times 1)' \qquad \qquad 1.$$

where x_i represents the ith observation of K exogenous variables, y_i the ith observation of L endogenous variables, u_i the structural disturbances for the ith observation in each of L structural equations, and Γ and B the structural coefficients. In a recursive system, Γ is lower-triangular, each structural disturbance is stochastically independent of the exogenous and predetermined endogenous variables in each equation, and the L structural disturbances do not covary with one another. The path diagram in Figure 1A shows a simple recursive model with two exogenous variables (x_1 and x_2) and two endogenous variables (y_1 and y_2). The unidirectional arrows correspond to structural coefficients, and the curved two-headed arrow indicates the possibility of unanalyzed correlation between the exogenous variables. The negative sign of γ_{21} compensates for the placement of all endogenous variables on the left-hand side of Equation 1; disturbances are assigned unit slopes. (Readers who are unfamiliar with matrix notation may find it helpful to write out the structural equations corresponding to the path diagrams in Figure 1; for example, see Hauser & Goldberger 1971, Jöreskog 1976b, and Long 1976).

Recursive models can routinely be estimated by ordinary least-squares and are relatively easy to interpret (Finney 1972, Lewis-Beck 1974, Alwin & Hauser 1975). Several writers have succumbed to the temptation of producing "novel" decompositions of variance in recursive models. This subject has been exploited thoroughly,
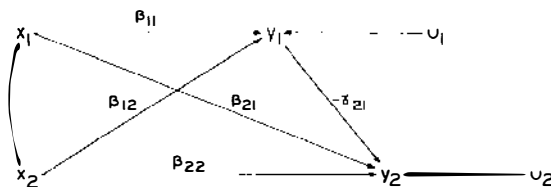
Figure 1A: A recursive model in observable variables.

and is of limited substantive utility (Duncan 1970). One reason for special caution in the interpretation of recursive models is their tendency to yield plausible estimates of parameters even when they are grossly misspecified.

Nonrecursive Models in Observable Variables

A nonrecursive model allows simultaneous or reciprocal causation between endogenous variables. The model can still be expressed by Equation 1, but Γ is no longer constrained to be lower-triangular nor need the structural disturbances be mutually uncorrelated. Figure 1B shows a simple two-equation nonrecursive model in observable variables.

The econometric literature dealt extensively with identification, estimation, and inference in nonrecursive models (Goldberger 1964, Theil 1971, Johnston 1972). While the possibility of mutual causation suggests appealing structural representations of sociological ideas (Stinchcombe 1968), seldom do the underlying theories and research designs permit the specification of identifiable nonrecursive models (Duncan 1975a:88–90). However, there have been some sound sociological applications of nonrecursive models in observable variables (Mason & Halter 1968, Henry & Hummon 1971, Land 1971, Hauser 1971, Anderson 1973, Kohn & Schooler 1973, Beck 1974, Pugh 1976); see Erlanger & Winsborough (1976) for a simple didactic treatment of such models.

Models with Unobservable Variables

It is useful to specify unobservable or latent variables in two contexts: (*a*) concrete variables, e.g. age and annual earnings, are subject to measurement error induced by factors like faulty recall and other response biases or inaccurate coding and record-keeping; (*b*) accurately measured variables are thought to reflect variation
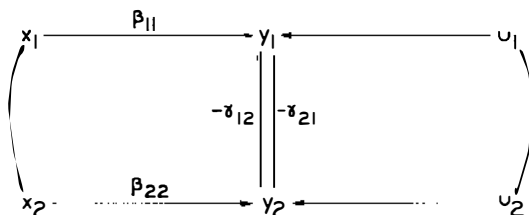

Figure 1B: A nonrecursive model in observable variables.

in an underlying theoretic construct that is inherently unobservable. For example, this occurs in domain sampling, where measured variables, e.g. mental abilities or political attitudes, are regarded as instances of a theoretical variable. It is difficult in practice to distinguish between these two interpretations, both of which may occur in the context of a single structural model. From a statistical point of view, the two are treated in the same way. (For a contrasting view, see Burt 1973, 1976).

Structural relationships among unobservable variables can be expressed in a way that parallels our treatment of observables:

$$\Gamma\eta_i \quad = \quad B\xi_i \quad + \quad \zeta_i$$
$$(L \times L)(L \times 1) \quad (L \times K)(K \times 1) \quad (L \times 1)' \qquad\qquad 2.$$

where vector η_i represents the ith case of L unobservable endogenous variables, ξ_i the ith case of K unobservable exogenous variables, and ζ_i the ith case of L structural disturbances. As in models containing only observable variables, the structural relationships among latent variables may be recursive or nonrecursive, depending upon the configuration of Γ and covariation among structural disturbances. A measurement structure relates latent variables to their observable indicators:

$$y_i \quad = \quad \mu_y \quad + \quad \Lambda_y\eta_i \quad + \quad \delta_i$$
$$(P \times 1) \quad (P \times 1) \quad (P \times L)(L \times 1) \quad (P \times 1)' \qquad\qquad 3.$$

$$x_i \quad = \quad \mu_x \quad + \quad \Lambda_x\xi \quad + \quad \epsilon_i$$
$$(Q \times 1) \quad (Q \times 1) \quad (Q \times K)(K \times 1) \quad (Q \times 1)\cdot \qquad\qquad 4.$$

Equations 3 and 4 specify that the P measurements in y_i, are linear functions of L latent endogenous variables plus a vector of means, μ_y, and a vector of disturbances, δ_i. A similar structure relates the Q indicators of exogenous variables to K latent exogenous variables and a vector of disturbances. Under this specification, the disturbances in the measurement of Equations 3 and 4 are independent of the disturbances in the structural Equations 2. The structural and statistical properties of this model and computational methods for estimation and inference have been developed by Karl Jöreskog (1973, 1976b); also see the expository review by Long (1976). In a typical application, multiple indicators are specified for each latent variable, so P exceeds L and Q is greater than K. Each row of Λ_y and Λ_x contains only one nonzero parameter, that is, no observable variable is an indicator of more than one latent variable. Also, the structural disturbances of the measurement equations are mutually independent. These specifications are highly restrictive, and under some conditions they may be relaxed. For example, observables may indicate more than one latent variable, and selected correlations may exist among the disturbances in the measurement equations. Moreover, correlations between errors and latent variables may be represented by nonunit coefficients in Λ_y and Λ_x (Bielby, Hauser & Featherman 1977).

The path diagram in Figure 1C represents a confirmatory factor model (Jöreskog 1970a) in which there are two indicators for each of three latent variables. In this
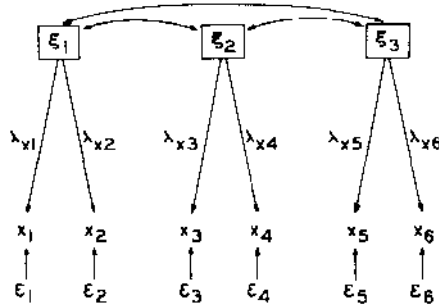
Figure 1C: A confirmatory factor model.

special case, the model is specified completely by Equation 4. We have already referred to the extensive sociological discussion of this and similar models that was initiated by Costner (1969) and Blalock (1969a, 1970). Considering the effort devoted to the exposition of such models, we were surprised to find only a few numerical illustrations in methodological papers (Hauser & Goldberger 1971, Althauser, Heberlein & Scott 1971, Costner & Schoenberg 1973, Burt 1973) and no extended substantive applications. However, as we note below, there have been more interesting applications of less restrictive models. The model has proven useful in representing and correcting Campbell & Fiske's (1959) intuitive exposition of validation by the "multitrait-multimethod matrix" (Alwin 1974, Kalleberg & Kluegel 1975, Morgan 1975).

Equations 2, 3, and 4 are required to describe the model specified in Figure 1D. In the model, there is a fully recursive structure among the four latent variables. Thus the structural portion of the model is just-identified—the moments among the latent variables provide just enough information to determine the structural parame-
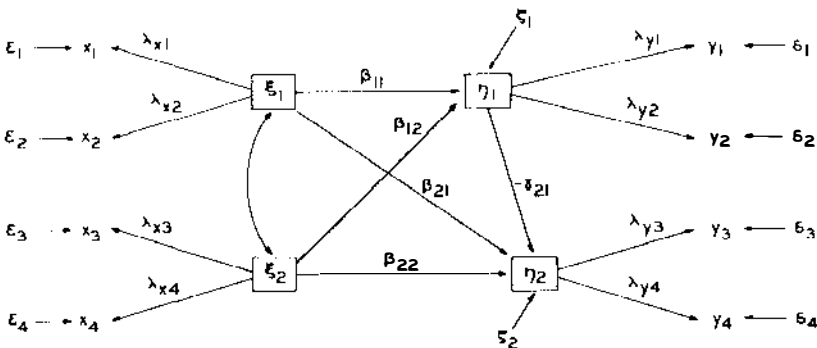

Figure 1D: A recursive model in unobservable variables.

ters uniquely. Two indicators for each latent variable comprise a measurement structure that is overidentified: it constrains the population moments among observable variables. The specification and estimation of models with just-identified and overidentified structural relationships among latent variables have been discussed by Jöreskog (1970a) and by Werts, Jöreskog & Linn (1973). In research on social stratification, such models have been applied in the assessment of survey response error (Siegel & Hodge 1968, Jencks et al 1972, Bowles 1972, Bowles & Nelson 1974, Mason et al 1976, Bielby, Hauser & Featherman 1976), in the measurement of global family effects on achievement (Jencks et al 1972, Duncan, Featherman & Duncan 1972, Bowles 1973, Hauser & Featherman 1976, Olneck 1977), in locating the social and psychological sources of alienation (Otto & Featherman 1975), and in testing the validity of scales of occupational status (Featherman, Jones & Hauser 1975). Similar models have also been applied to attitude-behavior consistency (Alwin 1973), to the measurement of job satisfaction (Kalleberg 1974), and to the validation of experimental manipulations (Alwin & Tessler 1974).

The path diagram in Figure 1E is similar to that in Figure 1D, except the structural portion of the model is a just-identified nonrecursive model in which each of two latent endogenous variables is affected by one of the two latent exogenous variables. Duncan, Haller & Portes (1968) first applied a nonrecursive model with latent variables, long before the general model and techniques for estimation had been developed. Similar models of the development of adolescent aspiration have been estimated in several student populations by Hout & Morgan (1975). Duncan & Featherman (1973) estimated a complex nonrecursive model of latent psychological factors in occupational achievement in a Detroit sample. Williams has developed nonrecursive models of the influence of parent-child interaction on intellectual development (1976) and of teacher-expectation effects on high school students (1975). Kohn & Schooler (1976) have complemented their earlier analysis (1973) of the reciprocal effects of job complexity and the intellectual flexibility of male workers with a model based on panel data in which the same reciprocally interacting variables appear as latent constructs.

While the measurement Equations 3 and 4 and our examples suggest that observables appear as indicators (reflections or effects) of latent variables, the specification
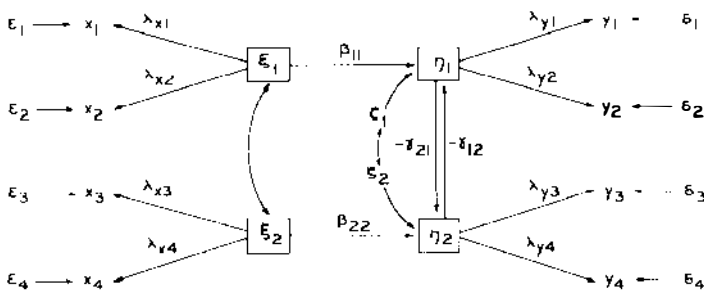

Figure 1E: A nonrecursive model in unobservable variables.

can be adapted to models where observables appear as causes of latent variables. For example, Hauser (1973) specified a model of student aspirations in which social influence was a consequence of the diverse expectations of parents, teachers, and peers (also see Hauser 1971). Blalock (1969b:42–43) discussed "multiple indicator-multiple cause" models (also see Wold 1975); their statistical properties have been elaborated by Hauser & Goldberger (1971) and by Jöreskog & Goldberger (1975); also see Kenny (1974).

Models for Panel Data

Methodological prescriptions for the analysis of panel data—repeated observations of one or more variables on several units of analysis—provide an interesting case study of issues of specification in structural equation models. Several psychologists had suggested that "cross-lagged correlations" be used to detect causation in two-wave, two-variable panel observations (Campbell & Stanley 1963, Pelz & Andrews 1964). The notion underlying this proposal was that over an appropriate interval of causation, the earlier measure of the cause would be more highly correlated with the later measure of the effect than the earlier measure of the effect would be correlated with the later measure of the cause. This suggestion was appealingly consonant with the tendency of sociological researchers to believe that the specification of cause and effect is easily resolvable in longitudinal data.

Heise (1970) argued that assumptions about the causal structures underlying the interpretation of panel data be made explicit in a structural equation model (also see Goldberger 1971). Heise proposed a simple recursive model for two-wave, two-variable panel analysis in which each variable at the second time was regressed on both prior measures. (A recursive model can incorporate a form of two-way causation by incorporating lagged effects; we restrict the term "nonrecursive" to instantaneous reciprocal causation.) He demonstrated that the logic of cross-lagged panel correlation was only valid under highly restrictive conditions. Without benefit of an explicit model, Rozelle & Campbell (1969) had also suggested limitations of the cross-lagged correlation technique. Duncan (1969) elaborated this observation by showing that no fewer than nine recursive or nonrecursive models in observable variables could be specified with two-wave, two-variable panel data. Also, he noted that the model might be specified to include simple measurement error or latent variables. Kenny (1973) formally elaborated some conditions under which a latent factor could be postulated as an alternative to causation among observable variables. In two later papers, Duncan (1972, 1975b) gave an exhaustive algebraic treatment (with some numerical examples) of recursive two-wave, two-variable models with and without latent factors and measurement error. The implication to be drawn from the large number of models that may be specified in even the two-wave, two-variable case is that taking repeated observations is absolutely no guarantee of valid causal interpretation.

As in the two-wave, two-variable case, several authors have exposited models for repeated measures of a single variable. In this case attention has focused on the implications of measurement error for the interpretation of change. In the case of three measurements of the same variable, Heise (1969) showed that a zero-order

causal chain in the latent variable could be specified by assuming a temporally constant correlation between the indicator and the latent variable. [Similar restrictions on standardized coefficients were used by Duncan (1969, 1972, 1975b).] He used this model to argue that test-retest correlations combine separable elements of reliability and stability.

Wiley & Wiley (1970) argued that it was more realistic to assume invariant components of error variance in the indicators than to assume constant reliability as Heise had done, because reliability varies both with true variance and error variance. Werts, Jöreskog, & Linn (1971) showed that observations at four or more time points permitted a test of the restrictions imposed by Heise and by Wiley & Wiley. Jöreskog (1970b) provides a general treatment of zero-order causal chain models with measurement error.

Blalock (1970) specified two- and three-wave, zero-order chain models with multiple indicators of a single unobservable variable at each time. While noting the Wileys' argument about constant error variances, Blalock did not exploit this in his algebraic treatment, which followed that of Heise. Hannan, Rubinson, & Warren (1974) extended Blalock's discussion to recursive models with two and three latent variables and multiple indicators of those variables in two- and three-wave panel models. They noted that when both substantive and measurement structures are elaborated, no easy generalizations follow from the simpler models treated earlier. For example, it may be very difficult to evaluate the identifiability of structural parameters in complex models. Consequently, aside from statistical efficiency, ad hoc estimation procedures for overidentified models are unsatisfactory. Hannon, Rubinson & Warren discuss some models in which measurement quality changes systematically over time, but these changes are specified to occur in (standardized) reliabilities rather than in error variances.

Under Jöreskog's (1973, 1976b) model of linear structural relations (LISREL), a large class of the panel models can be described and estimated efficiently. Jöreskog & Sörbom (1976b) exposit a number of examples in detail. Hargens, Reskin & Allison (1976) use LISREL to reexamine the appropriateness of a zero-order chain model in a latent variable to represent change in scientific productivity. While such a model fits well, it provides implausibly high stability in the latent variable and implausibly low error variance in the observed variable. Better results were obtained when they respecified the model to include an autoregressive process in the disturbances. Finally, Wheaton et al (1977) also emphasized the interplay between specification of substantive and measurement structures. In this way they elaborate the important observation made in several of the earlier papers: there are no stock or universal models for analyzing panel data. In addition, they present examples of the efficient estimation and testing of overidentified models. Like Hargens, Reskin & Allison (1976) they used the LISREL scheme to specify parameters of the measurement model in terms of structural coefficients and error variances rather than reliabilities. Thus, the development of models for the analysis of panel data clearly shows an evolution from naive intuitive prescription, through algebraic exposition of simple and highly restrictive models, to flexible application and sound statistical inference based on a very general analytic model.

IDENTIFICATION

The parameters of a structural equation model are the structural coefficients and the moments of exogenous variables and disturbances. Parameters are identified when they are uniquely determined by population moments of observable variables. When a structural parameter (or combination of parameters) is identified by more than one function of observable population moments, the structural model imposes constraints (overidentifying restrictions) on those moments. In this case the parameter (or combination of parameters) is overidentified, and the overidentifying restrictions must hold in the population when the specified model is correct. When a parameter is not uniquely determined by population moments, that is, when more than one value of the parameter is consistent with a given set of population moments, the parameter is underidentified. It is useful to think of identification of parameters and functions of parameters, not identification of models, for in a given model some parameters may be overidentified and others, underidentified (Jöreskog 1970b, Duncan 1975a:84–86).

While the concept of identification is straightforward, it is difficult to assess the identifiability of structural parameters in complex models. Of course, there must be at least as many moments (variances and covariances) among observable variables as there are structural parameters in a model, or some of the parameters cannot be identified. However, this "order" condition is only necessary, not sufficient, for identification. Again, it is possible to specify a structural model where the number of moments among observable variables greatly exceeds the number of structural parameters, but some parameters are not identified.

By definition, the identifiability of a structural parameter can be established conclusively by expressing it as function of the moments among observable variables. In addition, the exercise of deriving such functions can also make explicit the overidentifying restrictions imposed by the structural model. Such functions were typically obtained in the early literature of path analysis, and ad hoc estimates of parameters were computed from sample analogues to those functions. This was accompanied by confusion over the estimation of overidentified parameters. In complex models, the observable moments are sometimes complicated nonlinear functions of structural parameters, and, as Hannan, Rubinson, & Warren (1974:309) have noted, it is likely to be tedious, if not impossible, to solve for the parameters directly.

As computer programs for maximum-likelihood estimation of structural equation models have become available, the presentation of structural parameters as functions of population moments has become less frequent. The expository paper by Jöreskog & Sörbom (1976b) is an exception to this trend. However, it is dangerous to rely on the computer to resolve problems of identification, for the iterative numerical methods used in these programs will sometimes yield plausible estimates of underidentified parameters (for example, see Burt 1973, Fig. 5 and 8).

An obverse algebraic procedure for establishing identification may be less cumbersome: Express the moments in terms of structural parameters. If it can be demonstrated that two distinct values of a parameter reproduce the same moments, then

by definition the parameter is underidentified. See Wheaton et al (1977) for several applications of this idea. However, in a complex model, this procedure too can be troublesome, and it is less likely to expose overidentifying restrictions than is the procedure of expressing parameters as functions of population moments.

Other criteria for the assessment of identification have been developed for specific classes of structural equation models. Parameters of recursive models in observable variables (with uncorrelated structural disturbances) are always identified; a formal proof is presented by Land (1973). The order and rank conditions discussed in econometrics texts (Theil 1971; Goldberger 1964, Johnston 1972, Wonnacott & Wonnacott 1970) apply to identification in nonrecursive models in observable variables where no constraints are imposed upon structural disturbances. Jöreskog (1969) discusses sufficient conditions for identification in confirmatory factor analysis (but see the comment by Dunn 1973). Wiley (1973) presents sufficient conditions for identification in a structural model with multiple indicators of unobserved variables and random measurement error. Nonrecursive models with unobservable variables require a blending of psychometric and econometric approaches to identification that are discussed by Goldberger (1971). Geraci (1976) shows how overidentifying restrictions in the conventional nonrecursive econometric model may identify measurement error in a single indicator of an exogenous variable. Duncan (1975a) shows how multiple indicators of exogenous and endogenous variables may or may not identify structural parameters in a nonrecursive model.

Identification of structural parameters is not an all-or-nothing proposition. Duncan (1975b:89) and others have noted that an instrumental variable (a variable that does not enter a given structural equation, but whose moments identify parameters of that equation) is useful in estimation only if it has a nontrivial indirect effect on the endogenous variable. Instrumental variables that are weakly associated with endogenous variables lead to "weakly identified" structural parameters, and in this respect problems of identification and estimation are merged. For example, see Heyns's (1977) review of Hauser (1971:77–80) or Nolle (1973).

Also, bounded values of underidentified structural parameters may sometimes be obtained by varying constraints on a subset of structural parameters. The remaining parameters may be determined subject to each set of constraints. Such a sensitivity analysis may prove useful if an underidentified parameter of substantive interest is narrowly bounded. This procedure has been treated formally by Marschak & Andrews (1944), Nerlove (1965), Zellner (1972), Genberg (1972), and Rothenberg (1973) in the econometric literature, and more recently by sociologists Land & Felson (1976). For example, Siegel & Hodge (1968) used this approach to bound estimates of measurement error in socioeconomic variables. It was used by Hauser (1969a) to obtain bounded estimates of the effects of teachers' discrimination on academic achievement. Duncan, Featherman, & Duncan (1972) estimated a model of the influence of motivation in the stratification process under a series of assumptions about the validity of retrospective measurements of ambition and work orientation. Jencks et al (1972: Append. A) and Goldberger (1977) used this method to assess effects of heredity and environment on cognitive ability. Jencks et al (1972: Append. B) also generated bounded estimates of the influence of ability, family background, and schooling on economic success.

ESTIMATION AND TESTING

Identified and overidentified structural parameters may be estimated from sample moments, and overidentifying restrictions may be tested by assessing the degree to which those restrictions are violated by sample moments. Principles of statistical inference are involved in both estimation and testing, and the most important contributions in this area have been made by persons with advanced training in mathematical statistics. Indeed, issues of statistical inference in structural equation models were virtually ignored by sociologists until they were raised in the early 1970s by econometricians (e. g. Goldberger 1970) and psychometricians (e. g. Jöreskog 1970a, 1973).

An early and persisting flaw in sociological treatment of overidentifying restrictions was the tendency to interpret tests of restrictions as tests of a model as a whole (Duncan 1975a:46–50). The most extreme form of this tendency is the belief that tests of overidentifying restrictions can resolve the causal ordering among variables [see Rehberg, Schafer & Sinclair (1970) and the corrective comment by Alwin & Mueller (1971)].

The statistical treatment of overidentifying restrictions in structural equation models was first introduced to sociologists in the case of recursive models in observable variables. Blalock (1964) suggested that partial correlations be used to test restrictions on structural coefficients in three- and four-variable models. Duncan (1966) recommended that when structural coefficients took on negligible values, they could be set equal to zero and the equations reestimated by ordinary least-squares (also see Heise 1968). Boudon (1968) proposed that squared errors of reproduced correlations be minimized to estimate overidentified recursive models. However, Goldberger (1970) demonstrated that Boudon's estimators were less efficient statistically (i. e. had greater sampling variability) than those obtained by ordinary least-squares.

Most econometric texts (Goldberger 1964:354–55, Johnston 1972:377–80) discuss the consistency and efficiency of ordinary least-squares estimators for just-identified and overidentified recursive models with uncorrelated structural disturbances. Maximum-likelihood estimation leads to tests of overidentifying restrictions that are straightforward extensions of testing procedures under the general linear model. Statistical issues of estimation and testing in recursive models in observables were summarized by Land (1973). Recent attempts by sociologists to develop new procedures for hypothesis testing in simple recursive models have yielded results equivalent or nearly identical to the well-known maximum-likelihood procedures (McPherson & Huang 1974, Specht 1975, Specht & Warren 1975).

In estimating models containing unobservable variables, sociologists initially followed Wright in obtaining ad hoc estimators from sample analogs of equations relating parameters to population moments. Typically, alternative estimates for overidentified parameters were either arbitrarily ignored (Hodge & Treiman 1968, Hauser 1969b, Blalock 1970, Land 1970) or arbitrarily averaged (Duncan, Haller & Portes 1968, Hauser 1969a, Duncan, Featherman & Duncan 1972). Similarly, overidentifying restrictions were evaluated by qualitative assessments of the degree to which "consistency criteria" were violated (Costner 1969, Blalock 1970, Al-

thauser, Heberlein & Scott 1971, Van Valey 1971, Sullivan 1974). Hauser & Gold-berger (1971) drew the attention of sociologists to the efficient estimation techniques that had been developed for overidentified confirmatory factor models and multiple-indicator, multiple-cause models. Their suggestion that efficient estimators of the parameters of overidentified models could be interpreted as approximately weighted averages of conflicting estimators was elaborated by Goldberger (1973b).

Jöreskog has developed statistical methods for confirmatory factor models (1970a), including models with overidentifying restrictions on the factor moments, and for cross-population comparisons of these models (1971b). A computer pro-gram (COFAMM) for maximum-likelihood estimation of these models is available (Jöreskog & Sörbom 1976a). Jöreskog's (1973) general model for a linear structural equation system subsumes recursive and nonrecursive models in observable and unobservable variables (see Equations 2, 3, and 4 above). Maximum-likelihood estimates may be obtained under this specification, and a computer program (LIS-REL) is available (Jöreskog and van Thillo 1973). The LISREL model includes all of the features of the confirmatory factor models, but the program is limited to estimation and testing in a single population. Under these models, the maximum-likelihood procedures yield parameter estimates, standard errors, and a likelihood ratio test statistic. The latter statistic has degrees of freedom equal to the number of overidentifying restrictions in the model and permits a global test of those restrictions; that is, it contrasts the constraints imposed by the model (the null hypothesis) with an unrestricted moments matrix. In a series of hierarchical models, that is, a set of models in which restrictions are successively added or eliminated, the likelihood-ratio statistics may be compared to test the significance of the restric-tions imposed at each level of the hierarchy. Jöreskog has described (1970a:241) and applied (1969, 1971a) this feature of maximum-likelihood test statistics (also see Werts, Jöreskog & Linn 1973), and there have been several applications of it in the sociological literature (Werts, Jöreskog & Linn 1971, Mason et al 1976, Bielby, Hauser & Featherman 1976). Mayer & Younger (1974) exposited the same idea, apparently without recognizing its earlier development and application.

The LISREL model subsumes the classical nonrecursive econometric model in observables (Jöreskog 1973:93–99); for such models it yields "full-information maxi-mum-likelihood" (FIML) estimates. Before programs for maximum-likelihood esti-mation were widely available, econometricians had developed other estimation methods for overidentified models, e.g. two- and three-stage least-squares. While these methods are numerically simpler (allowing direct rather than iterative com-putation), maximum-likelihood estimation is statistically at least as efficient (and generally more efficient). The other methods differ in the types of overidentifying restrictions they incorporate, the way they reconcile alternative estimates and (for those reasons) the efficiency of the estimators. Aside from statistical efficiency, procedures for hypothesis testing are not as well developed for the traditional econometric methods as in maximum-likelihood estimation. The several economet-ric estimation techniques are developed in detail in econometric textbooks (Gold-berger 1964, Theil 1971, Johnston 1972); also see Duncan (1975a) for an exposition of the principle of instrumental variable and two-stage least-squares estimation in nonrecursive models with and without unobservable variables.

The programs developed by Jöreskog produce maximum-likelihood estimators only if the distribution of observable variables is multivariate normal. Little is known about the robustness of the statistical properties of the estimators with respect to violation of the multivariate normality assumption (Jöreskog 1976a:16), although the computing algorithm remains a flexible and reasonable fitting criterion. Most of the large-sample statistical properties of FIML estimators can be shown not to depend upon the distributional assumption for the classical econometric model in observable variables (Goldberger 1964:352), but to our knowledge no similar results have been presented for complex models with unobservable variables. In addition to the robustness of existing computational procedures, a related area deserving further research is maximum-likelihood estimation procedures under distributional assumptions other than multivariate normality. Muthen (1976) presents some of the first research in this area, exploring maximum-likelihood estimation and testing procedures in models with dichotomous indicators of unobservable dependent variables.

As in other areas of applied statistics, use of inferential statistics in structural equation modeling has focused on nominal probabilities of Type I error—rejecting a null hypothesis when it is true (Walster & Cleary 1970). However, the interpretation of a structural model seldom rests on a single test of a structural coefficient or of a global feature of the model. Almost always, there are several hypotheses about single coefficients or linear combinations of coefficients. In other cases, the test of one hypothesis is conditional upon the outcome of another, as when a hierarchy of likelihood-ratio tests is used to refine the specification of a model (Jöreskog 1971a, Bielby, Hauser & Featherman 1976). Where multiple or conditional tests are carried out, true Type I error rates will be larger than nominal rates. McPherson (1976) has raised some of these issues in a critique of "theory trimming" in causal models. Standard procedures of simultaneous statistical inference such as those developed by Scheffé (1959) are reviewed by Miller (1966) and by Bielby & Kluegel (1977). Sequential estimation and inference has also been addressed by econometricians (Wallace & Ashar 1972, Bock, Yancey & Judge 1973).

The probability of Type II error—failing to reject a null hypothesis when it is false—is virtually ignored in applications of structural equation models. Although rarely applied, power functions of some test statistics in the general linear model have been tabulated (Cohen 1969) and can be applied to recursive models in observables (Bielby & Kluegel 1977; also see Cleary, Linn & Walster 1970, Tretter & Walster 1975). However, little, if any, analysis exists of the power of tests for more complex simultaneous-equation models.

CONCLUSION

Since 1970, the sociological literature on structural equation modeling has shown two parallel trends. The treatment of specification, identification, and statistical inference has progressed. We now have general and flexible analytic models that have desirable and well-known statistical properties, and for which computer programs are available. While the use of structural equation models by sociologists has

increased greatly, most applications do not reflect recent methodological developments. We have referred to several exemplary uses of structural equation models. Yet most applications show little statistical skill beyond an introductory acquaintance with multiple regression analysis, and some are inexcusably thoughtless in concept, sloppy in execution, or primitive in technique.

This rapid diffusion of causal modeling in sociology has been strongly criticized. Some critics have shown a good deal less methodological acumen than the subjects of their criticism. For example, Boudon (1974:xv, 137–38) confuses the ability of a model to account for sample moments with its ability to account for interunit variance. Miller & Stokes (1975) purport to evaluate published applications of path analysis on the strength of a frequency distribution of standardized residual coefficients. As we have noted earlier, proportions of variance explained have little, if any, relevance to the validity of structural equation models. Two recent Presidential addresses to the American Sociological Association have offered pessimistic evaluations of structural equation modeling (Coser 1975) and of quantitative sociology generally (Lee 1976). For further discussion, see the responses to Coser by Featherman (1976) and Treiman (1976), and Coser's (1976) reply to them.

We do not share the pessimism of these critics. We see nothing unusual or reprehensible in the lag between the exegesis of structural equation methods and their application, even if the latter exhibits some of the undesirable characteristics of a fad. We think the quality of sociological applications is improving, and quantitative sociology certainly has no monopoly on thoughtless or shoddy work.

In his study of the development of structural equation modeling from 1962 to 1971, Mullins (1973) characterizes it as "the new causal theory." Coser's (1975) critique of modeling is permeated by a similar notion, that modeling is associated with specific theoretical ideas. On the contrary, we believe that the methods are merely tools, that they may be useful in diverse areas of sociological inquiry. We do not feel compelled to defend structural equation models as being useful or legitimate; despite some criticisms, we feel these matters are no longer controversial. But, by the same token, we are not interested in discrediting any other method or style of research. When theories and methods of measurement are sufficiently advanced to make formalization of interpretations profitable, the methods discussed here may be quite attractive.

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