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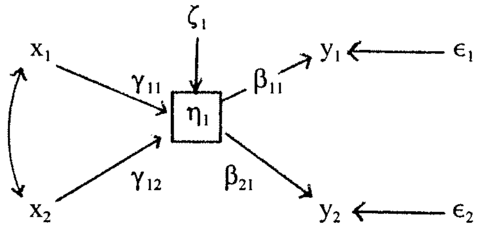
LECTURE 9: MULTIPLE-INDICATOR MULTIPLE-CAUSE (MIMIC) MODELS

I. A SIMPLE ONE-FACTOR TWO-CAUSE TWO-INDICATOR MODEL.
II. MOMENTS IN TERMS OF PARAMETERS.
III. OVERIDENTIFYING RESTRICTIONS.
IV. ILLUSTRATIVE EXAMPLE.

In this lecture, we will examine an important model in covariance structure analysis, the multiple-indicator multiple cause (MIMIC) model (see Hauser and Goldberger 1971; Joreskog and Goldberger 1975). This class of models is characterized by at least one latent factor that has both multiple indicators and multiple causes.

I. A SIMPLE ONE-FACTOR TWO-CAUSE TWO-INDICATOR MODEL.



This model has ½(p)(p+1) = ½ (4)(5) = 10 moments and 10 parameters -- ϕ_{11}, ϕ_{22}, ϕ_{21}, γ_{11}, γ_{12}, $\sigma_{\zeta 1}^2$, β_{11}, β_{21}, $\sigma_{\epsilon 1}^2$, $\sigma_{\epsilon 2}^2$ -- and therefore, one overidentifying restriction.

$y_1 = \beta_{11}\eta_1 + \epsilon_1$ $\qquad$ $\eta_1 = \gamma_{11}x_1 + \gamma_{12}x_2 + \zeta_1$
$y_2 = \beta_{21}\eta_1 + \epsilon_2$

We need to normalize the latent variable. We can either set $\beta_{11} = 1$ or $\beta_{21} = 1$ or we can set $\psi_{11} = 1$ (or some other constant; e.g., if η_1 were I.Q., we could set $\psi_{11} = 225$, since we know that the standard deviation of I.Q. is 15). We can now compute moments in terms of parameters in the usual way.

II. MOMENTS IN TERMS OF PARAMETERS.

To simplify things, I'm going to use LISREL notation but with no ξs.

$E(x_1^2) = \sigma_{x1}^2 = \phi_{11}$, since x is exogenous.

1. $\sigma_{x1}^2 = \phi_{11}$

2. $\sigma_{x2}^2 = \phi_{22}$

3. $\sigma_{x2x1} = \phi_{21}$

$E(y_1\ x_1) = \beta_{11}\, E(\eta_1 x_1)$ $\qquad$ But $E(\eta_1 x_1) = (\gamma_{11}\phi_{11} + \gamma_{12}\phi_{21})$, therefore,

4. $\sigma_{y1x1} = \beta_{11}\,(\gamma_{11}\phi_{11} + \gamma_{12}\phi_{21})$

5. $\sigma_{y1x2} = \beta_{11}\,(\gamma_{11}\phi_{21} + \gamma_{12}\phi_{22})$

6. $\sigma_{y2x1} = \beta_{21}\,(\gamma_{11}\phi_{11} + \gamma_{12}\phi_{21})$

7. $\sigma_{y2x2} = \beta_{21}\,(\gamma_{11}\phi_{21} + \gamma_{12}\phi_{22})$

$E(x_1^2) = \sigma_{y1}^2 = \beta_{11}\, E(\eta_1^2) + \theta_{11}^{\epsilon}$ $\qquad$ But since $E(\eta_1^2) = (\gamma_{11}^2\phi_{11} + \gamma_{12}^2\phi_{22} + 2\,\gamma_{11}\gamma_{12}\phi_{21} + \psi_{11})$, then

8. $\sigma_{y1}^2 = \beta_{11} (\gamma_{11}^2 \phi_{11} + \gamma_{12}^2 \phi_{22} + 2 \gamma_{11} \gamma_{12} \phi_{21} + \psi_{11}) + \theta_{11}^{\epsilon}$

9. $\sigma_{y2}^2 = \beta_{21} (\gamma_{11}^2 \phi_{11} + \gamma_{12}^2 \phi_{22} + 2 \gamma_{11} \gamma_{12} \phi_{21} + \psi_{11}) + \theta_{22}^{\epsilon}$

10. $\sigma_{y2y1}^2 = \beta_{11} \beta_{21} (\gamma_{11}^2 \phi_{11} + \gamma_{12}^2 \phi_{22} + 2 \gamma_{11} \gamma_{12} \phi_{21} + \psi_{11}) + \theta_{21}^{\epsilon}$ But here $\theta_{21}^{\epsilon} = 0$.

From these equations we can identify the overidentifying restriction.

III. OVERIDENTIFYING RESTRICTIONS.

Divide equation 5 by equation 7:

$$\sigma_{y1x2} / \sigma_{y2x2} = \beta_{11} (\gamma_{11} \phi_{21} + \gamma_{12} \phi_{22}) / \beta_{21} (\gamma_{11} \phi_{21} + \gamma_{12} \phi_{22})$$

$$\sigma_{y1x2} / \sigma_{y2x2} = \beta_{11} / \beta_{21}$$

Divide equation 4 by equation 6:

$$\sigma_{y1x1} / \sigma_{y2x1} = \beta_{11} (\gamma_{11} \phi_{11} + \gamma_{12} \phi_{21}) / \beta_{21} (\gamma_{11} \phi_{11} + \gamma_{12} \phi_{21})$$

Then equate the two:

$\sigma_{y1x2} / \sigma_{y2x2} = \sigma_{y1x1} / \sigma_{y2x1}$ or $\sigma_{y1x2}\, \sigma_{y2x1} = \sigma_{y2x2} / \sigma_{y1x1}$

Note that this is the *identical* overidentifying restriction on our ten population moments that we found for our walking dog model (and that we find for a confirmatory factor model in two factors with two indicators each)! But these models are very very different substantively. Our walking dog model specified that two latent variables, each with two fallible indicators generated by latent variables, were causally related to each other. That model corrected for attenuation due to unreliability in the regression of latent variables. This MIMIC model treats the xs as direct *causes* of the latent variable, which has two fallible indicators. In other words, the xs here are *causes* rather than *reflections* of a latent variable. Thus, the MIMIC model is not correcting for attenuation due to possible unreliability in the xs; here the xs are assumed to be perfectly-measured. There is a very important general lesson to be learned from this example:

> Very different parametric structures sometimes imply the *identical* overidentifying constraints on observed population moments. Therefore, a test on the moments themselves cannot distinguish between the two models. If you think about it, one can always specify multiple parametric structures that give rise to a given set of constraints on observed moments. Therefore, it is not the constraints that are so critical, but rather the parameteric structure of the model, which is based on a substantive theory of the process under study. That is the key to covariance structure models.

Our MIMIC model's overidentifying constraint can be expressed in a different, more substantive way (unlike our walking dog model). That is, we can specify our overidentifying constraint in terms of constraints on the reduced-fom coefficients, rather than on observable moments. It turns out that historically econometricians specified the identification problem in terms of overidentifying restrictions on reduced form parameters (mainly because they were interested in simultaneous equation models in which there were no measurement equations). We can express our MIMIC model in reduced form by replacing endogenous variables with their equations expressed in terms of exogenous variables:

$y_1 = \beta_{11} \eta_1 + \epsilon_1$ $y_2 = \beta_{21} \eta_1 + \epsilon_2$ $\eta_1 = \gamma_{11} x_1 + \gamma_{12} x_2 + \zeta_1$

$y_1 = \beta_{11} (\gamma_{11} x_1 + \gamma_{12} x_2 + \zeta_1) + \epsilon_1$ $y_2 = \beta_{21} (\gamma_{11} x_1 + \gamma_{12} x_2 + \zeta_1) + \epsilon_2$

$y_1 = \beta_{11}\gamma_{11}x_1 + \beta_{11}\gamma_{12}x_2 + \beta_{11}\zeta_1 + \epsilon_1$ $\quad\quad$ $y_2 = \beta_{21}\gamma_{11}x_1 + \beta_{21}\gamma_{12}x_2 + \beta_{21}\zeta_1 + \epsilon_2$

$y_1 = \pi_{11}x_1 + \pi_{12}x_2 + \pi_{\epsilon 1}$ $\quad\quad$ $y_2 = \pi_{21}x_1 + \pi_{22}x_2 + \pi_{\epsilon 2}$ where

$\pi_{11} = \beta_{11}\gamma_{11}$ $\quad$ $\pi_{12} = \beta_{11}\gamma_{12}$ $\quad$ $\pi_{\epsilon 1} = \beta_{11}\zeta_1 + \epsilon_1$ $\quad\quad$ $\pi_{21} = \beta_{21}\gamma_{11}$ $\quad$ $\pi_{22} = \beta_{21}\gamma_{12}$ $\quad$ $\pi_{\epsilon 2} = \beta_{11}\zeta_1 + \epsilon_2$

In matrix form, the reduced-form equations look like:

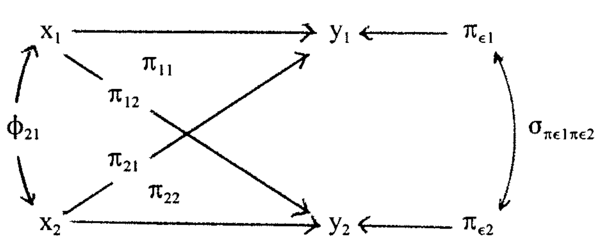
$$\underset{(2\times 1)}{\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}} = \underset{(2\times 2)}{\begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix}} \underset{(2\times 1)}{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} + \underset{(2\times 1)}{\begin{bmatrix} \pi_{\epsilon 1} \\ \pi_{\epsilon 2} \end{bmatrix}}$$

In the general case, the matrix form looks like:

$$\underset{(p\times 1)}{\begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_2 \end{bmatrix}} = \underset{(p\times q)}{\begin{bmatrix} \pi_{11} & \pi_{12} & \cdots & \pi_{1q} \\ \pi_{21} & \pi_{22} & \cdots & \pi_{2q} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \pi_{p1} & \pi_{p2} & \cdots & \pi_{pq} \end{bmatrix}} \underset{(q\times 1)}{\begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_q \end{bmatrix}} + \underset{(p\times 1)}{\begin{bmatrix} \pi_{\epsilon 1} \\ \pi_{\epsilon 2} \\ \cdot \\ \cdot \\ \cdot \\ \pi_{\epsilon p} \end{bmatrix}}$$

$$Y = \Pi \quad X + \Pi_\epsilon$$

The reduced-form model has the following path diagram:



Notice that the reduced-form disturbances are necessarily correlated, since they both include the structural disturbance, ζ_1. From the reduced-form, we can obtain expressions for our overidentifying restriction.

$\pi_{11} = \beta_{11}\gamma_{11}$ $\quad$ $\pi_{12} = \beta_{11}\gamma_{12}$ $\quad\quad$ $\pi_{21} = \beta_{21}\gamma_{11}$ $\quad$ $\pi_{22} = \beta_{21}\gamma_{12}$

From these expressions, we can obtain an equivalence:

$\pi_{11}/\pi_{21} = \beta_{11}\gamma_{11}/\beta_{21}\gamma_{11}$ $\quad\quad$ $\pi_{12}/\pi_{22} = \beta_{11}\gamma_{12}/\beta_{21}\gamma_{12}$

It follows that:

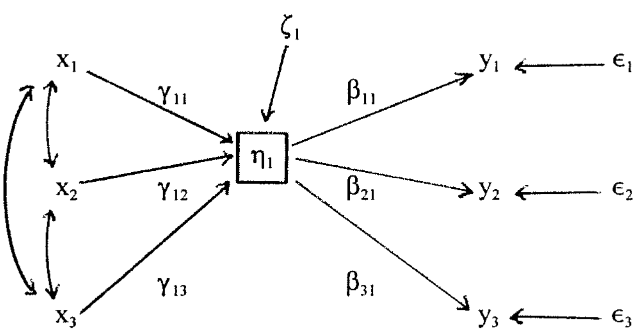
$\pi_{11}/\pi_{21} = \pi_{12}/\pi_{22}$ $\quad\quad$ $\pi_{11}\pi_{22} = \pi_{21}\pi_{12}$

This means that one can run the reduced form model using equation-by-equation OLS and get unbiased, efficient estimates of the πs. Then one can test the constraint on the πs -- does $\pi_{11}/\pi_{21} = \pi_{12}/\pi_{22}$ hold within the bounds of sampling variability? Or, better yet, one can run the LISREL model on the *structural* form and test whether the model fits the data, which would amount to a one degree-of-freedom χ^2 test.

In the general case, with many xs and ys, the MIMIC model contains two kinds of overidentifying constraints: tetrad difference constraints of the form, $\sigma_{y1x2}\,\sigma_{y2x1} = \sigma_{y2x2}\,/\sigma_{y1x1}$, and reduced-form constraints of the form, $\pi_{11}\,\pi_{22} = \pi_{21}\,\pi_{12}$. But if we allowed all measurement errors, ϵs, to be correlated -- that is, allowed the off-diagonals of the Θ_ϵ matrix to be nonzero -- then two things happen. First, we no longer have tetrad-difference restrictions on observable moments; we're left only with reduced-form constraints. Second, the disturbance term, ζ_1, disappears, becoming absorbed into the covariances among the measurement errors, and consequently, ψ_{11} is no longer a parameter. In our model above, if we allowed θ_{21}^{ϵ} to be a free parameter, ζ_1 would disappear, ψ_{11} would no longer be a parameter (and we'd have to normalize in a different way).

IV. ILLUSTRATIVE EXAMPLE.

Here is an example of a simple MIMIC model from Hodge and Treiman (1968) and reanalyzed by Hauser and Goldberger (1971):



x_1 = Family Income
x_2 = Main Earner's Occupation
x_3 = Respondent's Education
y_1 = Frequency of Church Attendance
y_2 = Membership in Voluntary Organizations
y_3 = Number of Friends Seen

I first estimated a model that normalizes η by fixing its variance to be 1, and constrains the measurement errors in y to be uncorrelated. Here's the output from that run:

```
The following lines were read from file C:\219\MIMIC1A.LR8:
SOCIAL STATUS AND PARTICIPATION
DA NI=6 NO=530 MA=KM
KM
*
1.00
.304 1.00
.305 .344 1.00
.100 .156 .158 1.00
.284 .192 .324 .360 1.00
.176 .136 .226 .210 .265 1.00
SD
```



```
*
1 1 1 1 1 1
LA
INCOME OCCUPATION EDUCATION CHURCHAT MEMBERSH FRIENDS
SE
4 5 6 1 2 3
MO NY=3 NX=3 NE=1 LY=FU,FR FI PS=DI,FI GA=FU,FR
LE
SOCIALPAR
VA 1.0 PS 1 1
OU ME=ML SE TV  EF MR SC NDEC=3

SOCIAL STATUS AND PARTICIPATION
              NUMBER OF INPUT VARIABLES  6
              NUMBER OF Y - VARIABLES    3
              NUMBER OF X - VARIABLES    3
              NUMBER OF ETA - VARIABLES  1
              NUMBER OF KSI - VARIABLES  3
              NUMBER OF OBSERVATIONS   530
    CORRELATION MATRIX TO BE ANALYZED
           CHURCHAT  MEMBERSH  FRIENDS   INCOME  OCCUPATI  EDUCATIO
           --------  --------  --------  --------  --------  --------
 CHURCHAT    1.000
 MEMBERSH    0.360     1.000
  FRIENDS    0.210     0.265     1.000
   INCOME    0.100     0.284     0.176     1.000
 OCCUPATI    0.156     0.192     0.136     0.304     1.000
 EDUCATIO    0.158     0.324     0.226     0.305     0.344     1.000
PARAMETER SPECIFICATIONS
    LAMBDA-Y
           SOCIALPA
           --------
 CHURCHAT       1
 MEMBERSH       2
  FRIENDS       3
    GAMMA
             INCOME  OCCUPATI  EDUCATIO
           --------  --------  --------
 SOCIALPA       4         5         6
    THETA-EPS
           CHURCHAT  MEMBERSH   FRIENDS
           --------  --------  --------
                7         8         9
Number of Iterations =  9
LISREL ESTIMATES (MAXIMUM LIKELIHOOD)
    LAMBDA-Y
           SOCIALPA
           --------
 CHURCHAT    0.401
            (0.046)
             8.697
```

```
 MEMBERSH    0.633
           (0.060)
            10.558

 FRIENDS     0.346
           (0.046)
             7.553
      GAMMA
           INCOME  OCCUPATI  EDUCATIO
          --------  --------  --------
SOCIALPA    0.269    0.113    0.387
          (0.066)  (0.065)  (0.070)
            4.075    1.749    5.532
      COVARIANCE MATRIX OF ETA AND KSI
         SOCIALPA   INCOME  OCCUPATI  EDUCATIO
         --------  --------  --------  --------
SOCIALPA    1.348
  INCOME    0.422    1.000
OCCUPATI    0.328    0.304    1.000
EDUCATIO    0.508    0.305    0.344    1.000
      PHI
           INCOME  OCCUPATI  EDUCATIO
          --------  --------  --------
  INCOME    1.000
OCCUPATI    0.304    1.000
EDUCATIO    0.305    0.344    1.000
      PSI
        SOCIALPA
        --------
          1.000
      SQUARED MULTIPLE CORRELATIONS FOR STRUCTURAL EQUATIONS
        SOCIALPA
        --------
          0.258
      THETA-EPS
        CHURCHAT  MEMBERSH   FRIENDS
        --------  --------  --------
          0.783     0.459     0.839
        (0.058)   (0.075)   (0.058)
         13.608     6.096    14.512
      SQUARED MULTIPLE CORRELATIONS FOR Y - VARIABLES
        CHURCHAT  MEMBERSH   FRIENDS
        --------  --------  --------
          0.217     0.541     0.161
                 GOODNESS OF FIT STATISTICS

        CHI-SQUARE WITH 6 DEGREES OF FREEDOM = 12.498 (P = 0.0517)
           ESTIMATED NON-CENTRALITY PARAMETER (NCP) = 6.498
         90 PERCENT CONFIDENCE INTERVAL FOR NCP = (0.0 ; 20.716)
                MINIMUM FIT FUNCTION VALUE = 0.0236
          POPULATION DISCREPANCY FUNCTION VALUE (F0) = 0.0123
         90 PERCENT CONFIDENCE INTERVAL FOR F0 = (0.0 ; 0.0392)
```

ROOT MEAN SQUARE ERROR OF APPROXIMATION (RMSEA) = 0.0452
90 PERCENT CONFIDENCE INTERVAL FOR RMSEA = (0.0 ; 0.0808)
P-VALUE FOR TEST OF CLOSE FIT (RMSEA < 0.05) = 0.533

EXPECTED CROSS-VALIDATION INDEX (ECVI) = 0.0803
90 PERCENT CONFIDENCE INTERVAL FOR ECVI = (0.0681 ; 0.107)
ECVI FOR SATURATED MODEL = 0.0794
ECVI FOR INDEPENDENCE MODEL = 0.721

CHI-SQUARE FOR INDEPENDENCE MODEL WITH 15 DEGREES OF FREEDOM = 369.314
INDEPENDENCE AIC = 381.314
MODEL AIC = 42.498
SATURATED AIC = 42.000
INDEPENDENCE CAIC = 412.951
MODEL CAIC = 121.591
SATURATED CAIC = 152.730

ROOT MEAN SQUARE RESIDUAL (RMR) = 0.0256
STANDARDIZED RMR = 0.0256
GOODNESS OF FIT INDEX (GFI) = 0.992
ADJUSTED GOODNESS OF FIT INDEX (AGFI) = 0.974
PARSIMONY GOODNESS OF FIT INDEX (PGFI) = 0.284

NORMED FIT INDEX (NFI) = 0.966
NON-NORMED FIT INDEX (NNFI) = 0.954
PARSIMONY NORMED FIT INDEX (PNFI) = 0.386
COMPARATIVE FIT INDEX (CFI) = 0.982
INCREMENTAL FIT INDEX (IFI) = 0.982
RELATIVE FIT INDEX (RFI) = 0.915
CRITICAL N (CN) = 712.594

MODIFICATION INDICES FOR THETA-EPS

	CHURCHAT	MEMBERSH	FRIENDS
CHURCHAT	- -		
MEMBERSH	3.951	- -	
FRIENDS	0.664	7.021	- -

EXPECTED CHANGE FOR THETA-EPS

	CHURCHAT	MEMBERSH	FRIENDS
CHURCHAT	- -		
MEMBERSH	0.153	- -	
FRIENDS	0.036	-0.169	- -

COMPLETELY STANDARDIZED EXPECTED CHANGE FOR THETA-EPS

	CHURCHAT	MEMBERSH	FRIENDS
CHURCHAT	- -		
MEMBERSH	0.153	- -	
FRIENDS	0.036	-0.169	- -

MODIFICATION INDICES FOR THETA-DELTA-EPS

CHURCHAT MEMBERSH FRIENDS
-------- -------- --------

```
   INCOME    5.200    2.247    0.359
 OCCUPATI    2.362    1.908    0.020
 EDUCATIO    2.550    0.092    1.993
        EXPECTED CHANGE FOR THETA-DELTA-EPS
           CHURCHAT  MEMBERSH   FRIENDS
           --------  --------  --------
   INCOME    -0.092     0.081     0.024
 OCCUPATI     0.060    -0.070     0.006
 EDUCATIO    -0.065     0.017     0.056
        COMPLETELY STANDARDIZED EXPECTED CHANGE FOR THETA-DELTA-EPS
           CHURCHAT  MEMBERSH   FRIENDS
           --------  --------  --------
   INCOME    -0.092     0.081     0.024
 OCCUPATI     0.060    -0.070     0.006
 EDUCATIO    -0.065     0.017     0.056
 MAXIMUM MODIFICATION INDEX IS    7.02 FOR ELEMENT ( 3, 2) OF THETA-EPS
 COMPLETELY STANDARDIZED SOLUTION
        LAMBDA-Y
           SOCIALPA
           --------
 CHURCHAT     0.466
 MEMBERSH     0.735
  FRIENDS     0.402
        GAMMA
             INCOME  OCCUPATI  EDUCATIO
           --------  --------  --------
 SOCIALPA     0.232     0.097     0.334
        CORRELATION MATRIX OF ETA AND KSI
           SOCIALPA    INCOME  OCCUPATI  EDUCATIO
           --------  --------  --------  --------
 SOCIALPA     1.000
   INCOME     0.363     1.000
 OCCUPATI     0.283     0.304     1.000
 EDUCATIO     0.438     0.305     0.344     1.000
        PSI
           SOCIALPA
           --------
              0.742
        THETA-EPS
           CHURCHAT  MEMBERSH   FRIENDS
           --------  --------  --------
              0.783     0.459     0.839
        REGRESSION MATRIX ETA ON X (STANDARDIZED)
             INCOME  OCCUPATI  EDUCATIO
           --------  --------  --------
 SOCIALPA     0.232     0.097     0.334
 TOTAL AND INDIRECT EFFECTS
        TOTAL EFFECTS OF X ON Y
             INCOME  OCCUPATI  EDUCATIO
           --------  --------  --------
 CHURCHAT     0.108     0.045     0.155
            (0.028)   (0.026)   (0.031)
```

```
        3.818    1.726    4.934

 MEMBERSH    0.171    0.072    0.245
       (0.041)  (0.041)  (0.042)
        4.214    1.759    5.896

 FRIENDS    0.093    0.039    0.134
          (0.025)  (0.023)  (0.028)
           3.707    1.716    4.703

 STANDARDIZED TOTAL AND INDIRECT EFFECTS
      THE PROBLEM USED    5800 BYTES (=  0.1% OF AVAILABLE WORKSPACE)
                  TIME USED:    0.4 SECONDS
```

I then estimated this model but allowed all of the covariances among ϵs to correlate. Out of curiosity, I simply allowed the errors to covary, without changing anything, even though I knew that the structural disturbance should disappear, and become absorbed into the error covariances. I obtained the following error message:

```
 Number of Iterations = 14

 LISREL ESTIMATES (MAXIMUM LIKELIHOOD)

 W_A_R_N_I_N_G: TE 3,3 may not be identified.
                Standard Errors, T-Values, Modification Indices,
                and Standardized Residuals cannot be computed.
```

The program wouldn't give me standard errors or t-values. So I fixed the variance of the latent variable to be zero and reran the model. Again, I received an error message:

```
 W_A_R_N_I_N_G: PSI is not positive definite

 SOCIAL STATUS AND PARTICIPATION:  CORRELATED ERRORS

 W_A_R_N_I_N_G: The solution was found non-admissible after  20 iterations.
                The following solution is preliminary and is provided only
                for the purpose of tracing the source of the problem.
                Setting AD> 20 or AD=OFF may solve the problem

 LISREL ESTIMATES(Intermediate Solution)
```

So I then realized that I needed to turn off the admissibility test, which won't allow a variance to be zero. But I know that in this model, a zero variance is not a problem. So I added the parameter AD=OFF to the output card:

```
 OU ME=ML SE TV  EF MR SC  NDEC=3 AD=OFF
```

and reran the model expecting the program to succeed. However, I again received an error message:

```
 W_A_R_N_I_N_G: The solution has not converged after  33 iterations.
                The following solution is preliminary and is provided only
                for the purpose of tracing the source of the problem.
                Setting IT> 33 may solve the problem.
```

LISREL ESTIMATES(Intermediate Solution)

Therefore, I again modified the output card by adding IT=100 to allow the program to iterate longer to converge:

```
OU ME=ML SE TV  EF MR SC IT=100 NDEC=3 AD=OFF
```

This time, the program worked. Note, however, that because the variance of ζ_1 is zero and no longer a parameter, we need a different way of normalizing η. The program fixed the first lambda to normalize the latent variable. The value it is fixed to is its starting value obtained through LISREL's algorithm for obtaining consistent start values. Here is some selected output from that run:

```
 PARAMETER SPECIFICATIONS
      LAMBDA-Y
        SOCIALPA
        --------
 CHURCHAT      0
MEMBERSH       1
    FRIENDS    2
      GAMMA
          INCOME   OCCUPATI   EDUCATIO
        --------  --------  --------
SOCIALPA       3        4        5
      THETA-EPS
        CHURCHAT  MEMBERSH   FRIENDS
        --------  --------  --------
 CHURCHAT      6
MEMBERSH       7        8
    FRIENDS    9       10       11

SOCIAL STATUS AND PARTICIPATION:  CORRELATED ERRORS  PS = ZERO
Number of Iterations = 37

LISREL ESTIMATES (MAXIMUM LIKELIHOOD)
     LAMBDA-Y
        SOCIALPA
        --------
CHURCHAT    0.176

MEMBERSH    0.380
        (0.088)
          4.312

 FRIENDS    0.255
        (0.069)
          3.715
      GAMMA
         INCOME   OCCUPATI   EDUCATIO
        --------  --------  --------
SOCIALPA     0.482     0.146     0.664
           (0.149)   (0.110)   (0.182)
             3.229     1.332     3.656
      COVARIANCE MATRIX OF ETA AND KSI
```

	SOCIALPA	INCOME	OCCUPATI	EDUCATIO
SOCIALPA	1.000			
INCOME	0.729	1.000		
OCCUPATI	0.521	0.304	1.000	
EDUCATIO	0.862	0.305	0.344	1.000

PHI

	INCOME	OCCUPATI	EDUCATIO
INCOME	1.000		
OCCUPATI	0.304	1.000	
EDUCATIO	0.305	0.344	1.000

SQUARED MULTIPLE CORRELATIONS FOR STRUCTURAL EQUATIONS

SOCIALPA
1.000

THETA-EPS

	CHURCHAT	MEMBERSH	FRIENDS
CHURCHAT	0.969 (0.060) 16.263		
MEMBERSH	0.293 (0.042) 7.048	0.855 (0.053) 16.263	
FRIENDS	0.165 (0.042) 3.932	0.168 (0.040) 4.249	0.935 (0.057) 16.263

SQUARED MULTIPLE CORRELATIONS FOR Y - VARIABLES

CHURCHAT	MEMBERSH	FRIENDS
0.031	0.145	0.065

GOODNESS OF FIT STATISTICS

CHI-SQUARE WITH 4 DEGREES OF FREEDOM = 4.621 (P = 0.328)

Note that this model fits much better than the first: the difference in chi-squares of 7.88 with 2 degrees of freedom is significant at the .001 level. It is only a two-degree of freedom test because of the need to normalize η by fixing a lambda after losing the variance of ζ_1. The test implies that the tetrad-difference overidentifying restrictions fail to hold.