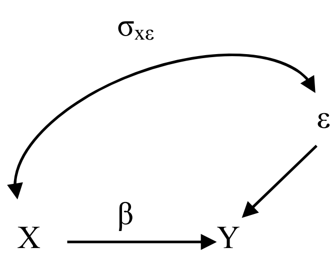
LECTURE 8: INSTRUMENTAL VARIABLES AND NONRECURSIVE MODELS

I. ENDOGENEITY BIAS (SPECIFICATION ERROR)
II. FORMS OF ENDOGENEITY BIAS
III. INSTRUMENTAL VARIABLES (IV) ESTIMATOR
IV. WHAT ARE GOOD INSTRUMENTS?

I. ENDOGENEITY BIAS (SPECIFICATION ERROR)

The most important assumption for a general linear model is that the disturbance (ε) is uncorrelated with the predictor variable(s) (X). If this assumption is violated OLS estimates of the model will be biased and inconsistent.

Here is the equation for the correct model (Model I):



Model I

$$Y = \beta X + \varepsilon \quad and\ \sigma_{x\varepsilon} \neq 0$$

$$Z_y = P_{yx} Z_x + P_{x\varepsilon} Z_\varepsilon, \qquad where\ P_{yx} = \sigma_{xy}/\sigma_x\sigma_y\ ,\ P_{y\varepsilon} = \sigma_{y\varepsilon}/\sigma_x\sigma_\varepsilon\ ,$$
$$Z_y = y/\sigma_y,\ \ Z_x = x/\sigma_x\ , and\ Z_\varepsilon = y/\sigma_\varepsilon$$

If we incorrectly assumed that $\sigma_{x\varepsilon} = 0$, then we have the wrong model (Model (II):

$$Y = \beta^* X + \varepsilon \quad when\ \ \sigma_{x\varepsilon} = 0$$ Here, $\beta^* = \sigma_{xy}/\sigma_x^2$

Let's begin with the correct model:

$$Y = \beta X + \varepsilon$$

$$E(XY) = \beta E(X^2) + E(X\varepsilon)$$

$$\sigma_{xy} = \beta \sigma_x^2 + \sigma_{x\varepsilon}$$

Therefore it follows that the correct β in the population is

$$\sigma_{xy} = \beta \sigma_x^2 + \sigma_{x\varepsilon}$$

$$\sigma_{xy} - \sigma_{x\varepsilon} = \beta \sigma_x^2$$

$$\sigma_{xy}/\sigma_x^2 - \sigma_{x\varepsilon}/\sigma_x^2 = \beta \qquad Note\ that\ \sigma_{xy}/\sigma_x^2 = \beta^*\ from\ above$$

$$\boxed{\beta = \beta^* - \sigma_{x\varepsilon}/\sigma_x^2}$$

$$\boxed{\beta^* = \beta + \sigma_{x\varepsilon}/\sigma_x^2}$$

Aside: we can also express the incorrect parameter in terms of the correct parameter and the correlation between X and ε. Note that $\rho_{x\varepsilon} = \sigma_{x\varepsilon}/\sigma_x\sigma_\varepsilon$:
$\beta^* = \beta + \rho_{x\varepsilon}\, \sigma_\varepsilon/\sigma_x$

Therefore, the degree to which the bivariate regression coefficient is wrong depends on the correlation between X and ε, and the size of the error variance σ_ε^2.

It follows that the OLS estimator $\hat{\beta}^*$ of β^* under Model II is unbiased and efficient, as we have learned. But for Model I, the OLS estimator $\hat{\beta}^*$ of β is biased:

$$E(\beta^*) = \beta + \sigma_{x\varepsilon}/\sigma_x^2$$

and inconsistent (biased as the sample size goes to infinity):

$$plim\ \beta^* = \beta + \sigma_{x\varepsilon}/\sigma_x^2$$

II. FORMS OF ENDOGENEITY BIAS

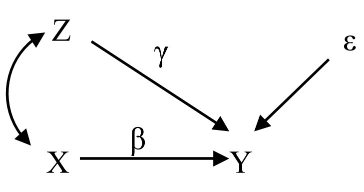
Economists refer to the problem that $\sigma_{x\varepsilon} \neq 0$ as endogeneity bias. The reason is that when X is exogenous—that is, truly predetermined with respect to the system of equations being estimated—then $\sigma_{x\varepsilon} = 0$ must hold. Note that controlled experiments manipulate the values of X, either randomly assigning values of X or fixing the values of X. In either case, $\sigma_{x\varepsilon} = 0$. When we are unable to assume the values of X are either fixed or determined by a process exogenous to our system of equations, then it becomes possible that the disturbance is correlated with the values of X—i.e., $\sigma_{x\varepsilon} \neq 0$. There are two categories of endogeneity bias: (1) unobserved heterogeneity (or omitted variable bias), and (2) reciprocal causation.

Unobserved Heterogeneity

When the problem of omitted-variable bias arises because of the omission of time-invariant (stable) covariate(s), the problem is referred to as unobserved heterogeneity.

Aside: I think the term arose in the context of time-series or event history data, in which one wants to distinguish state dependence (the effect of lagged dependent variables) from unobserved heterogeneity (the effect of omitted and unobserved individual characteristics). For example, in models of individual unemployment over time, over time dependence (stability) in unemployment could be due to state dependence in which previous spells of unemployment (undermines health and demoralizes the individual) or unobserved heterogeneity, in which some unmeasured characteristics of the individual (such as genetic endowment) puts individuals at risk of unemployment at all times.

We know from our regression course that an omitted variable will bias the estimate of a regression coefficient when each of two conditions hold: (1) the omitted variable Z has a non-zero effect on Y, when controlling for other regressors (Xs), and (2) the omitted variable Z is correlated with included regressors (X). If (2) holds but (1) does not, OLS is still unbiased and inefficient. If (1) holds but (2) does not, OLS is still unbiased but is no longer efficient (since Z is now pooled with ε). This is easy to show by starting with our bivariate regression equation and adding a confounding variable Z:



$$Y = \beta X + \gamma Z + \varepsilon$$

$$E(XY) = \beta E(X^2) + \gamma E(XZ) + E(X\varepsilon)$$

$$\sigma_{xy} = \beta\, \sigma_x^2 + \gamma \sigma_{xz}$$

$$\sigma_{xy} - \gamma\sigma_{xz} = \beta\, \sigma_x^2$$

$$\beta = (\sigma_{xy}/\sigma_x^2) - (\gamma\sigma_{xz}/\sigma_x^2)$$

The first term on the right-hand side is the bivariate regression coefficient $\beta_{yx} = (\sigma_{xy}/\sigma_x^2)$; that is, it is the wrong population parameter contaminated with omitted variable bias. The second term is the ratio of the covariance between the confounder Z and the regressor X divided by the variance in X. Therefore,

$$\beta = \beta_{yx} - (\gamma\sigma_{xz}/\sigma_x^2) \quad where\ \beta_{yx}\ is\ the\ bivariate\ regression\ coefficien\ from\ the\ wrong\ model$$

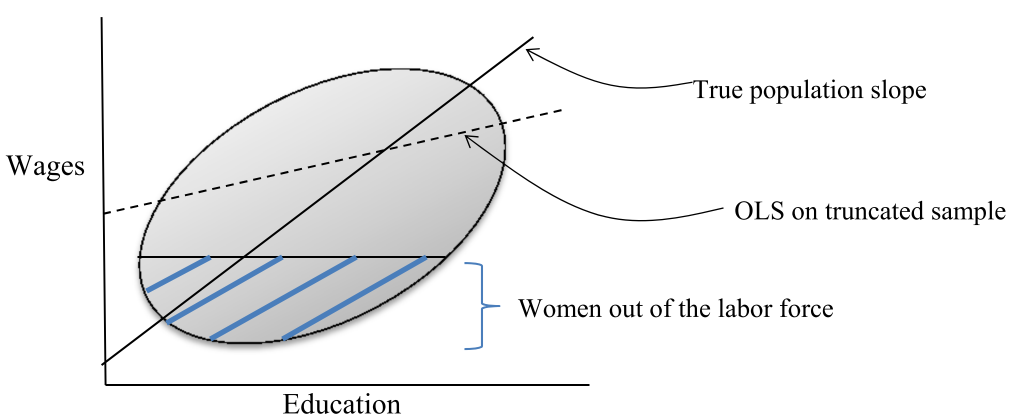
Here we can see clearly that if either $\gamma = 0$ (meaning the counfounder Z has no effect on Y) or $\sigma_{xz} = 0$ (meaning the confounder is uncorrelated with X, then our bivariate regression coefficient $\beta_{yx} = \beta$ is correct in the population. It follows that in either of these situations $\gamma = 0$ or $\sigma_{xz} = 0$, our OLS estimator $\hat{\beta}_{yx}$ will be an unbiased and consistent estimator of β. If neither holds, OLS will give a biased estimate of β.

Longitudinal data (e.g., panel data) gives the researcher more leverage over the problem of unobserved heterogeneity. If there is sufficient variation over time (compared to across individuals), one can estimate fixed-effects models to help eliminate unobserved heterogeneity. The idea here is that each individual is her own control and one examines within-individual, across-time variation in the dependent variable. With cross-sectional data, solutions are more difficult. Obviously, the best solution would be to measure the relevant variables and include them in the regression equation. Or design a randomized experiment in which the researcher randomly assigns individuals to values of X. Another potential solution would be to use instrumental variables methods, but this assumes that an instrumental variable exists.

One can often characterize different kinds of modeling problems as an issue of omitted variable bias or unobserved heterogeneity.

Sample Selection Bias. James Heckman won the Nobel Prize in Economics largely on his work on sample selection bias. Heckman showed that sample selection bias can be conceptualized as an omitted variable problem—the omitted variable is "propensity to be in the sample"—and therefore, the problem can be addressed using instrumental variables.

Aside: Heckman's classic example was estimating gender differences in returns to education in wage equations. He noted that because many women are housewives and out of the labor force, the sample of women for which returns to education is estimated is a biased sample of all women. This isn't a problem if we are only interested in returns to education for working women. Typically, however, in estimating the role of gender, we want to equalize the samples and ask the hypothetical: what would returns to education be if all women were in the labor force (like all men are, more or less). Heckman's solution is to estimate a selection equation for the propensity to be in the labor force, and then include that variable in the wage equation to eliminate the correlation of this propensity and wages.



Data Missing and not at Random. It follows that when one has missing data that cannot be assumed to be missing at random, one can conceptualize the problem as an omitted-variable bias, in which the omitted-variable is the propensity of the value to be non-missing. In principle, an instrumental variables approach could be followed.

Random Measurement Error. Similarly, random measurement error in variables can be conceptualized as a missing value problem in which the omitted variable is propensity to respond perfectly to the measuring instrument.

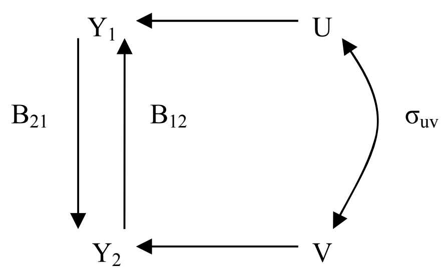
Non-Compliance in a Randomized Experiment. Finally, non-compliance in a randomized experiment can be conceived of as an omitted-variables problem, in which the omitted variable is propensity to remain in the assigned group (treatment or control).

Non-Recursive Effects (Simultaneity Bias)

When we have a non-recursive relationship—that is the arrow goes in both directions—applying equation-by-equation OLS yields biased and inconsistent parameter estimates. An example is diagrammed below:

$Y_1 = B_{12} Y_2 + U$

$Y_2 = B_{21} Y_1 + V$

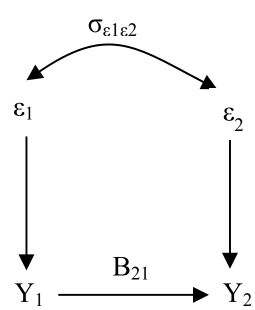


You can see that in the Y_1 equation, the endogenous regressor Y_2 is correlated with V, which is correlated with U, and therefore, $E(Y_2 U) \neq 0$, which violates the OLS assumption, rendering OLS estimates of B_{12} biased and inconsistent. This particular model has another problem: it is underidentified. There are three observed moments (the variances and covariances of Ys), but five parameters to be estimated.

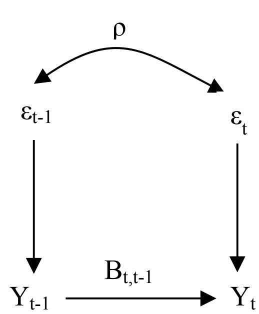
A related special case of this model would be one in which the arrow is going in one only one direction but the errors are correlated:

$Y_1 = \varepsilon_1$

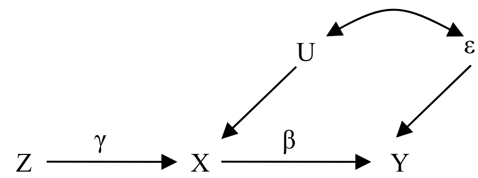
$Y_2 = B_{21} Y_1 + \varepsilon_2$



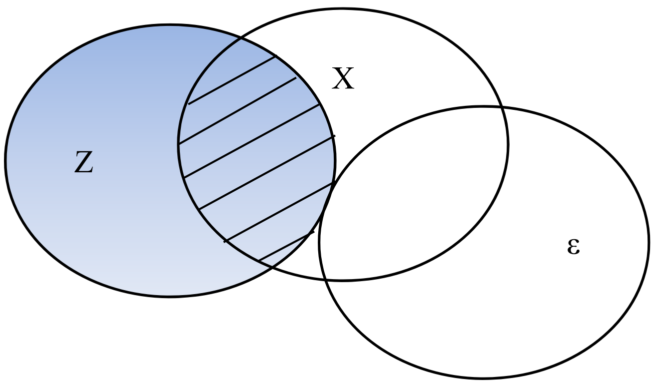
Here again, $E(Y_1 \varepsilon_2) \neq 0$, and therefore, using OLS to estimate B_{21} will yield biased and inconsistent estimates. Note that if these are time series data, and B is he lagged effect of Y on itself, this describes a situation of a lagged endogenous predictor in the presence of serial correlation, which yields an OLS estimate of $B_{t,t-1}$ that is biased and inconsistent.



III. INSTRUMENTAL VARIABLES (IV) ESTIMATOR



Instrumental variables were discovered by the economist Phillip Wright in 1928 (with the possible help of his son, the geneticist Sewall Wright), rediscovered by Rejersøl in 1945, who named them using a term coined by Frisch. The logic of instrumental variables (IV) is based on the violation of the OLS assumption that $\sigma_{x\varepsilon} = 0$, due to endogeneity. The solution is to find another exogenous variable—termed an "instrumental variable"—that is correlated with X but uncorrelated with ε. This IV then can stand in for X. More precisely, because the IV is uncorrelated with ε, the variance shared between the IV and X will be uncorrelated with ε. Then if we use this "shared variance" as a variable "standing in" for X, we can run the regression on the new variable and obtain consistent estimates of the effect of X. In the path diagram, Z is an instrument for X, which allows us to overcome the bias from the correlation between U and ε, which violates the assumption that E(X ε) = 0.



This Venn diagram shows that we want to use the overlapping variance between Z and X as a "stand-in" for X. Because Z is uncorrelated with ε, the overlapping variance between Z and X will be uncorrelated with ε.

Aside: A two-step approach, such as 2SLS literally does this: (1) Estimate a first stale model $X = \gamma Z + U$, and then compute the predicted values of X, $\hat{X} = \gamma Z$. Note that $\hat{X}$ is orthogonal to U and corresponds to the hatched region on the Venn diagram. (2) Estimate a second stage model, replacing X with $\hat{X}$, $Y = \beta\hat{X} + \varepsilon$.

Let's look more closely at this model using the tools we have used thus far:

$$Y = \beta X + \varepsilon \qquad\qquad X = \gamma Z + U$$

In this model, we have three observed variables and therefore (3 x 4)/2 = 6 observed moments and 6 parameters $(\gamma, \beta, \sigma_x^2, \sigma_\varepsilon^2, \sigma_u^2, \sigma_{\varepsilon u})$. Because Z has a nonzero effect on X and a zero direct effect on Y, the model is just-identified. Again, Z, the instrumental variable, is standing in for X and purging X of its correlation with ε. If we made the incorrect assumption that $\sigma_{\varepsilon u} = 0$, we would incorrectly assume $\beta = \sigma_{xy}/\sigma_x^2$. We can compute the correct equation for β in terms of population moments by multiplying the Y equation by Z and taking expectations:

$$E(YZ) = \beta E(XZ) + E(Z\varepsilon)$$

$$\sigma_{yz} = \beta\sigma_{xz} + \sigma_{z\varepsilon}$$

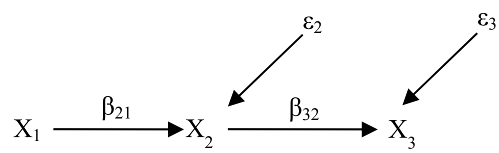
If Z has a direct effect on Y, it is not an instrumental variable for X and then $\sigma_{z\varepsilon} \neq 0$. In that case, the equation for β would be problematic:

$$\beta = \frac{\sigma_{yz}}{\sigma_{xz}} - \frac{\sigma_{z\varepsilon}}{\sigma_{xz}}$$

But if Z is a proper instrument for X, then $\sigma_{z\varepsilon} = 0$ and $\sigma_{xz} \neq 0$, and we can obtain an expression for β in terms of moments:

$$\beta = \sigma_{yz}/\sigma_{xz}$$

Aside: note that when we discussed a simple three-variable, two-equation model with no direct effect (Model II from Lecture 3), we found that the overidentifying restriction meant there were two ways of expressing one of the parameters in terms of moments:
$\beta_{32} = {}^{\sigma_{31}}/_{\sigma_{21}} = {}^{\sigma_{32}}/_{\sigma_{22}}$
which implies two ways of estimating β_{32} from sample moments:
${}^{s_{31}}/_{s_{21}}$ *or* ${}^{s_{32}}/_{s_{22}}$
We argued that the second estimator, the OLS estimator, is preferred because it is efficient. Note that the first estimator is the IV estimator considered here!



When we draw a sample and attempt to estimate β from sample data, we cannot assume that the sample estimate of $\sigma_{z\varepsilon}$, $s_{z\varepsilon} = 0$, and we cannot assume that the sample estimate of σ_{xz}, $s_{xz} \neq 0$. We can, however, assume that each holds asymptotically, as the sample size approaches infinity. That is, $plim\ s_{z\varepsilon} = 0$ and $plim\ s_{xz} \neq 0$. Therefore, it follows that the instrumental variables estimator $\hat{\beta}_{IV}$ is biased in finite samples:

$$\mathrm{E}(\hat{\beta}_{IV}) = \beta \frac{s_{z\varepsilon}}{s_{xz}}$$

However, $\hat{\beta}_{IV}$ is consistent:

$$plim\ \hat{\beta}_{IV} = \beta$$

Finally, the asymptotic variance of $\hat{\beta}_{IV}$ can be shown to be:

$$asymptotic\ var(\hat{\beta}_{IV}) = \frac{\sigma_\varepsilon^2}{n\sigma_x^2} \cdot \frac{1}{\rho_{zx}^2}$$

$$where\ \frac{\sigma_\varepsilon^2}{n\sigma_x^2}\ is\ the\ variance\ of\ \hat{\beta}_{OLS}\ and\ \rho_{zx}^2 = \frac{\sigma_{xz}}{\sigma_x\sigma_z}$$

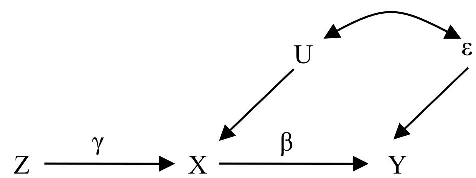
The square root of the asymptotic variance of $\hat{\beta}_{IV}$ is the asymptotic standard error. Therefore, we can see that more the variance of X is explained by Z, the smaller the asymptotic standard error of the IV estimator. A weak instrument is one with a low correlation with X and results in an estimate $\hat{\beta}_{IV}$ with a large sampling variability. A strong instrument will result in a more precise estimate. Note that, for a somewhat weak instrument, this imprecision can be offset if n is larger, the variance of X is larger, or the error variance is smaller. In general, we don't have access to the population error variance σ_ε^2 and observed moments $\sigma_{xz}, \sigma_x, \sigma_z$ and have to estimate them from sample data, and therefore, we must estimate the asymptotic standard error.

Aside: Startz and Nelson discovered a very interesting property for IV estimators. When the correlation between X and Z tends toward zero, the estimate is not only inefficient, but also inconsistent.

The instrumental variable estimator can be generalized to the multivariate case. In the above model, if we had multiple instruments for X, we would obtain a more precise estimator $\hat{\beta}_{IV}$ because the explained variance of X would be greater and asymptotic standard errors smaller. This would also yield overidentifying restrictions, which can be subjected to empirical test. If we have an endogeneity problem for multiple Xs, we would need at least one instrument for every problematic X for the model to be identified. Having fewer that one instrument for each problematic X means that the model is underidentified; having exactly one instrument for every problematic X means he model is just-identified, and having more than one instrument for an X means the model is overidentified. In the multivariate case, strong instruments are highly-correlated with X after controlling for other predictors of X that are required in a properly-specified model. Most SEM software provide ML estimates of identified models using instrumental variables. ML estimates are consistent and asymptotically efficient.

IV. WHAT ARE GOOD INSTRUMENTS?

Remember that a good instrument is one that is (1) strongly exogenous (2) strongly correlated with X—even after controlling for other exogenous determinants of X, and (3) uncorrelated with the disturbance ε, meaning it has no direct effect on Y once other relevant variables are controlled. The third requirement—sometimes referred to as an "exclusion restriction" because we have excluded it from the Y-equation—is the most difficult. One needs strong a priori theoretical or substantive knowledge to make this exclusion restriction. A strong case can be made for an instrument whose values have been assigned by the researcher, for example, via randomization. In the treatment effects literature, when there is noncompliance to treatment conditions in a randomized experiment, the random assignment to treatment can be used as an instrument for the actual treatment experienced. By virtue of random assignment, the treatment assignment will not be correlated with an outcome, controlling for actual treatment experienced. This has been the model used to estimate the effects of the Moving to Opportunity experimental effects, in which families were randomly-assigned to treatment of being given vouchers to move out of bad neighborhoods.



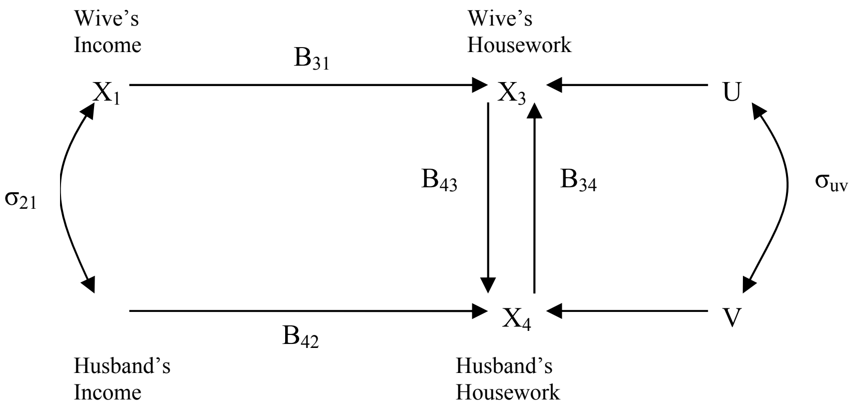
For example, suppose we randomly-assigned individuals to a treatment condition of having a job versus a control group that does not receive a job. But some subjects quit their jobs, a form of noncompliance that will bias the experimental results. If one were only interested in the treatment of giving subjects the opportunity to have a job, then OLS estimates will be unbiased. But if we are really interested in the experience of having a job, then OLS will be biased as in the diagram above, in which Z is the random assignment to jobs, X is the actual experience of having a job (including quitting). Z should have no effect on Y once we control for X. Z should also have a strong effect on X, assuming that most subjects complied. Therefore Z is likely to be a very strong instrument for X.

Other examples from economics attempt to identify instruments that appear to be relatively random. For example, during the latter months of the Vietnam era, military service was determined in part by a lottery, providing a strong instrumental variable for the effect of military experience on various outcomes, such as health, wages, etc. Because part of military service is determined by a random process orthogonal to ε, and part is determined voluntarily or by another process such as draft dodging, going to college, or having a health problem (which could be correlated with ε, we can use the lottery assignment as an instrumental variable. In a study of the effects of teen child-bearing on future earnings, critics argued that teen mothers are likely very different from non-teen mothers in ways that are unmeasured—such as having a stronger propensity to have unprotected sex. To address this problem, Hotz used miscarriages during the teenage years as an instrumental variable for teen motherhood. His argument was that compared to unprotected sex as a determinant of teen childbearing, miscarriages are relatively random and less likely to affect future earnings directly when controlling for teen childbearing. Another example from economics is to use sales taxes as an instrument for cigarette prices in models of cigarette consumption.

V. A JUST-IDENTIFIED NON-RECURSIVE (SIMULTANEOUS EQUATIONS) MODEL

1. $X_3 = B_{31} X_1 + B_{34} X_4 + U$

2. $X_4 = B_{41} X_1 + B_{43} X_3 + V$



10 PARAMETERS: B_{31}, B_{34}, B_{42}, B_{43}, σ_{11}, σ_{22}, σ_{12}, σ_{UU}, σ_{VV}, σ_{UV}

10 MOMENTS K (K-1)/2 = 5 (6)/2 = 10

$$\Sigma = \begin{bmatrix} \sigma_{11} & & \\ \sigma_{21} & \sigma_{22} & \\ \vdots & & \ddots \\ \sigma_{51} & \cdots & \sigma_{55} \end{bmatrix}$$

5 x 5

COMPUTE MOMENTS ALMOST IN TERMS OF PARAMETERS

For example, multiply the X_3 equation by X_1:

$E(X_1X_3) = B_{31}\,E(X_1^2) + B_{34}\,E(X_1X_4) + E(X_1U)$

1. $\sigma_{13} = B_{31}\sigma_{11} + B_{34}\sigma_{14}$

2. $\sigma_{23} = B_{31}\sigma_{12} + B_{34}\sigma_{24}$

3. $\sigma_{14} = B_{42}\sigma_{12} + B_{43}\sigma_{13}$

4. $\sigma_{24} = B_{42}\sigma_{22} + B_{43}\sigma_{23}$

For purposes of computing regression coefficients, these are the important ones. Others are:

$$\sigma_{33} = B_{31}\sigma_{13} + B_{34}\sigma_{34} + \sigma_{3U}$$
$$\sigma_{34} = B_{31}\sigma_{14} + B_{34}\sigma_{44} + \sigma_{4U}$$
$$\sigma_{3U} = B_{34}\sigma_{4U} + \sigma_{UU}$$
$$\sigma_{3V} = B_{34}\sigma_{4V} + \sigma_{UV}$$
$$\sigma_{34} = B_{42}\sigma_{23} + B_{43}\sigma_{33} + \sigma_{3V}$$
$$\sigma_{44} = B_{42}\sigma_{24} + B_{43}\sigma_{34} + \sigma_{4V}$$
$$\sigma_{4U} = B_{43}\sigma_{3U} + \sigma_{UV}$$
$$\sigma_{4V} = B_{43}\sigma_{3V} + \sigma_{VV}$$

From the above, we can see why using equation-by-equation OLS will yield biased and inconsistent estimates of parameters. Our two original equations are:

$X_3 = B_{31}\,X_1 + B_{34}\,X_4 + U$
$X_4 = B_{41}\,X_1 + B_{43}\,X_3 + V$

The usual OLS assumptions require that $E(X_4U) = \sigma_{4U} = E(X_3V) = \sigma_{3V} = 0$. But from above, we have

$$\sigma_{4U} = B_{43}\sigma_{3U} + \sigma_{UV}$$
$$\sigma_{3V} = B_{34}\sigma_{4V} + \sigma_{UV}$$

This suggests that the key OLS assumption required for unbiasedness could be violated if either the first or second terms in the two equations above are non-zero. We probably can't expect $\sigma_{UV} = 0$ because the omitted variables for an endogenous variable that is reciprocally related to another endogenous variable are very likely to be similar (e.g., omitted characteristics in supply and demand or husband's household labor and wife's household labor). Clearly, we don't expect either B_{34} or B_{43} to be zero, since that is what we're most interested in substantively. But what about the remaining term, covariances between the disturbance term and the dependent variable in the other equation? We have expressions for each: $\sigma_{3U} = B_{34}\sigma_{4U} + \sigma_{UU}$ and $\sigma_{4V} = B_{43}\sigma_{3V} + \sigma_{VV}$ **.** Let's substitute these two equations for σ_{3V} and σ_{4U}, respectively into the equations above them:

$$\sigma_{4U} = B_{43}(B_{34}\sigma_{4U} + \sigma_{UU}) + \sigma_{UV}$$

$$\sigma_{3V} = B_{34}(B_{43}\sigma_{3V} + \sigma_{VV}) + \sigma_{UV}$$

Now we can see when the OLS assumptions of our two original equations for X_3 and X_4 are violated. Let's assume that we cannot assume that B_{34} or B_{43} is zero, since we are substantively interested in their magnitudes. First, take the X_3 equation: when will the OLS assumption that $\sigma_{4U} = 0$ be violated? If we cannot assume that B_{34} or B_{43} is zero, then σ_{4U} (σ_{3V}) will be zero only when σ_{4U} (σ_{3V}) is zero (uh, okay) *and* σ_{UV} is zero *and* σ_{UU} (σ_{VV}) is zero. When will all three of these variances be zero? Perhaps in bizarro-world. It is unlikely that σ_{UV} is zero: When things simultaneously affect each other, you would think that the omitted variables in each equation would have some similarity (e.g., supply and demand, husbands' and wives' household work). But more importantly, it makes absolutely no sense to assume that σ_{UU} (σ_{VV})is zero – that's part of our structural model, and the part that allows for uncertainty. So, it's very, very unlikely that the OLS assumptions hold. Therefore OLS methods will not work and alternatives (2SLS, 3SLS, ML) are required to get optimal estimates.

COMPUTE SOME PARAMETERS IN TERMS OF MOMENTS:

Start with the two normal equations for X_3, equations (1) and (2) above. There are two equations and two unknowns (B_{31} and B_{34}):

$$\sigma_{13} = B_{31}\sigma_{11} + B_{34}\sigma_{14}$$

$$\sigma_{23} = B_{31}\sigma_{12} + B_{34}\sigma_{24}$$

Isolate B_{31} in each of the two equations:

$$(\sigma_{13} - B_{34}\sigma_{14})/\sigma_{11} = B_{31}$$

$$(\sigma_{23} - B_{34}\sigma_{24})/\sigma_{12} = B_{31}$$

Set the two equations equal, and solve for B_{34}:

$$(\sigma_{13} - B_{34}\sigma_{14})/\sigma_{11} = (\sigma_{23} - B_{34}\sigma_{24})/\sigma_{12}$$ Cross-multiply:

$$\sigma_{12}\sigma_{13} - B_{34}\sigma_{12}\sigma_{14} = \sigma_{11}\sigma_{23} - B_{34}\sigma_{11}\sigma_{24}$$ By subtraction, put B_{34} on the left side

$$B_{34}\sigma_{11}\sigma_{24} - B_{34}\sigma_{12}\sigma_{14} = \sigma_{11}\sigma_{23} - \sigma_{12}\sigma_{13}$$ Factor out B_{34}

$$B_{34}(\sigma_{11}\sigma_{24} - \sigma_{12}\sigma_{14}) = \sigma_{11}\sigma_{23} - \sigma_{12}\sigma_{13}$$ Isolate B_{34} by division

1. $$B_{34} = \frac{\sigma_{11}\sigma_{23} - \sigma_{12}\sigma_{13}}{\sigma_{11}\sigma_{24} - \sigma_{12}\sigma_{14}}$$

Use the same two equations, but now obtain an equation for B_{31}:

2. $$B_{31} = \frac{\sigma_{13}\sigma_{24} - \sigma_{14}\sigma_{23}}{\sigma_{11}\sigma_{24} - \sigma_{12}\sigma_{14}}$$

Now take the normal equations for X_4, equations (3) and (4). Again two equations and two unknowns (B_{42} and B_{43}):

$$\sigma_{14} = B_{42}\sigma_{12} + B_{43}\sigma_{13}$$

$$\sigma_{24} = B_{42}\sigma_{22} + B_{43}\sigma_{23}$$

Follow the logic above to obtain the following:

3. $$B_{43} = \frac{\sigma_{12}\sigma_{24} - \sigma_{14}\sigma_{22}}{\sigma_{12}\sigma_{23} - \sigma_{13}\sigma_{22}}$$

2. $$B_{42} = \frac{\sigma_{14}\sigma_{23} - \sigma_{13}\sigma_{24}}{\sigma_{12}\sigma_{23} - \sigma_{13}\sigma_{22}}$$

Aside: If we did not have access to population moments, we could substitute sample moments s_{ij} and obtain moment estimators for each of the four parameters. These estimators would be equivalent to an *instrumental variable (IV) estimator*.

$$\hat{B}_{34} = \frac{s_{11}s_{23} - s_{12}s_{13}}{s_{11}s_{24} - s_{12}s_{14}} \quad \hat{B}_{31} = \frac{s_{13}s_{24} - s_{14}s_{23}}{s_{11}s_{24} - s_{12}s_{14}} \quad \hat{B}_{43} = \frac{s_{12}s_{24} - s_{14}s_{22}}{s_{12}s_{23} - s_{13}s_{22}} \quad \hat{B}_{42} = \frac{s_{14}s_{23} - s_{13}s_{24}}{s_{12}s_{23} - s_{13}s_{22}}$$

THE REDUCED FORM:

We can compute the reduced form for our two equations in the usual way, by substituting for the endogenous variables until we can express our dependent variable wholly in terms of exogenous variables:

$X_3 = B_{31} X_1 + B_{34} X_4 + U$
$X_4 = B_{42} X_2 + B_{43} X_3 + V$

$X_3 = B_{31} X_1 + B_{34} (X_4) + U$ Substitute for X_4
$X_3 = B_{31} X_1 + B_{34} (B_{42} X_2 + B_{43} X_3 + V) + U$ Multiply

$X_3 = B_{31} X_1 + B_{34} B_{42} X_2 + B_{34}B_{43} X_3 + B_{34}V + U$ Note that X_3 on the right side is endogenous. Let's put it on the left side by subtraction
$X_3 - B_{34}B_{43} X_3 = B_{31} X_1 + B_{34} B_{42} X_2 + B_{34}V + U$
$(1- B_{34}B_{43})X_3 = B_{31} X_1 + B_{34} B_{42} X_2 + B_{34}V + U$ We can isolate X_3 by division:

$$1.\ X_3 = \frac{B_{31}}{1 - B_{34}B_{43}} X_1 + \underbrace{\frac{B_{34}B_{42}}{1 - B_{34}B_{43}}}_{\pi_{32}} X_2 + \underbrace{\frac{B_{34}V + U}{1 - B_{34}B_{43}}}_{\pi_{3U}}$$

$X_3 = \pi_{31} X_1 + \pi_{32} X_2 + \pi_{3U}$

We can do the same for X_4:

$$2.\ X_4 = \underbrace{\frac{B_{43}B_{31}}{1 - B_{34}B_{43}}}_{\pi_{41}} X_1 + \underbrace{\frac{B_{42}}{1 - B_{34}B_{43}}}_{\pi_{42}} X_2 + \underbrace{\frac{B_{43}U + V}{1 - B_{34}B_{43}}}_{\pi_{4V}}$$

$X_4 = \pi_{41} X_1 + \pi_{42} X_2 + \pi_{4V}$

There are four reduced form coefficients (π_{31}, π_{32}, π_{41}, π_{42}) and four structural-form regression coefficients (parameters) (B_{43}, B_{31} , B_{34} , B_{41}), which suggests one way of computing structural parameters in terms of reduced-form parameters.

If we take the first term of equation (2) and divide it by the first term of equation (1), we can express B_{43} in terms of reduced-form coefficients:

$$\pi_{41} / \pi_{31} = \frac{B_{43}B_{31}}{1 - B_{34}B_{43}} \Big/ \frac{B_{31}}{1 - B_{34}B_{43}} = B_{43}$$

Likewise, divide the second term of equation (1) by the second term of equation (2) and we get an expression for B_{34} in terms of reduced-form coefficients:

$$\pi_{32} / \pi_{42} = \frac{B_{34}B_{42}}{1 - B_{34}B_{43}} \Big/ \frac{B_{42}}{1 - B_{34}B_{43}} = B_{34}$$

Given that we have expressions of B_{43} and B_{34} in terms of reduced-form parameters, we can express the multiplier effect, $1 - B_{34} B_{43}$, in terms of reduced form parameters:

$$1 - B_{34}B_{43} = 1 - \frac{\pi_{32}\pi_{41}}{\pi_{42}\pi_{31}}$$

We can use the above equation to substitute for 1 - $B_{34} B_{43}$ to get expressions for B_{31} and B_{42} in terms of reduced-form parameters:

From the first term of reduced-form equation (1), we substitute the denominator:

$$\pi_{31} = \frac{B_{31}}{1 - B_{34}B_{43}} = B_{31} \Big/ (1 - \frac{\pi_{32}\pi_{41}}{\pi_{42}\pi_{31}})$$

$$B_{31} = \pi_{31}(1 - \frac{\pi_{32}\pi_{41}}{\pi_{42}\pi_{31}}) = \pi_{31} - \frac{\pi_{32}\pi_{41}}{\pi_{42}} = \pi_{31} - B_{34}\pi_{41}$$ (because $B_{34} = \pi_{32} / \pi_{42}$)

And, using the second term of reduced-form equation (2), we again substitute the denominator to get an expression of B_{42} in terms of reduced-form parameters:

$$B_{42} = \pi_{42}(1 - \frac{\pi_{32}\pi_{41}}{\pi_{42}\pi_{31}}) = \pi_{42} - \frac{\pi_{32}\pi_{41}}{\pi_{31}} = \pi_{42} - B_{43}\pi_{32}$$

ESTIMATION FROM SAMPLE DATA:

For our *just-identified model*, there are two equivalent methods of obtaining consistent estimates of our parameters from sample data. The first follows from the method of moments, and is the instrumental variable estimator noted above:

$$\hat{B}_{34} = \frac{s_{11}s_{23} - s_{12}s_{13}}{s_{11}s_{24} - s_{12}s_{14}} \quad \hat{B}_{31} = \frac{s_{13}s_{24} - s_{14}s_{23}}{s_{11}s_{24} - s_{12}s_{14}} \quad \hat{B}_{43} = \frac{s_{12}s_{24} - s_{14}s_{22}}{s_{12}s_{23} - s_{13}s_{22}} \quad \hat{B}_{42} = \frac{s_{14}s_{23} - s_{13}s_{24}}{s_{12}s_{23} - s_{13}s_{22}}$$

The second method uses a two-step method involving the reduced form coefficients:

1. Estimate the reduced-form equations of endogenous variables on exogenous variables using OLS:

$$X_3 = \hat{\pi}_{31}X_1 + \hat{\pi}_{32}X_2 + \hat{\pi}_{3U}$$
$$X_4 = \hat{\pi}_{41}X_1 + \hat{\pi}_{42}X_2 + \hat{\pi}_{4V}$$

Compute predicted values for endogenous variables X_3 and X_4:

$$\hat{X}_3 = \hat{\pi}_{31}X_1 + \hat{\pi}_{32}X_2$$
$$\hat{X}_4 = \hat{\pi}_{41}X_1 + \hat{\pi}_{42}X_2$$

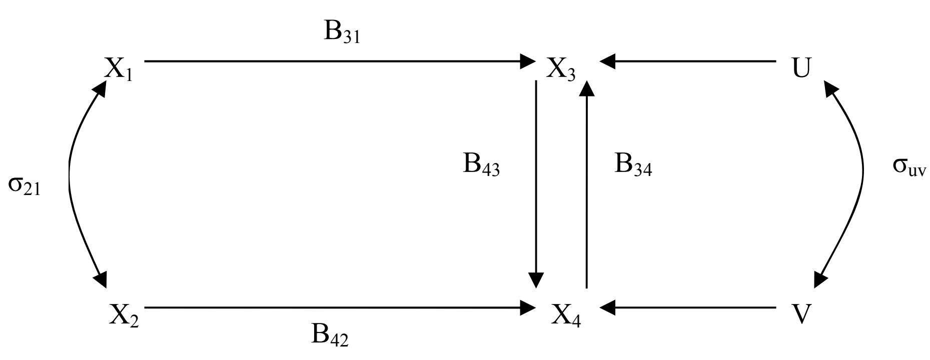
2. Plug in the predicted values, $\hat{X}_3, \hat{X}_4$, into the right hand side of the structural equations:

$$X_3 = B_{31}X_1 + B_{34}\hat{X}_4 + U$$
$$X_4 = B_{42}X_2 + B_{43}\hat{X}_3 + V$$

Now estimate these equations using OLS – this is the "second stage" of two-stage least-squares estimation:

$$X_3 = \hat{B}_{31}X_1 + \hat{B}_{34}\hat{X}_4 + U$$
$$X_4 = \hat{B}_{42}X_2 + \hat{B}_{43}\hat{X}_3 + V$$

Then adjust the standard errors to take into consideration that you estimated the first stage equations to get predicted values for X_3 and X_4.



It is instructive to examine the mechanics of this estimation method.

Looking at the path diagram, we can see that the problem of estimation involves a violation of OLS assumptions.

$$X_3 = B_{31}X_1 + B_{34}X_4 + U$$
$$X_4 = B_{42}X_2 + B_{43}X_3 + V$$

Recall that the problem of estimating the model using OLS had to do with the assumption that the disturbance of an equation is uncorrelated with the exogenous variables. This is not a problem for exogenous (instrumental) variables $X_1$and X_2 but is problematic for the non-recursive relationship among endogenous predictors ($\sigma_{4U} = \sigma_{3V} = 0$). Violations of this assumption cause OLS estimates to be biased and inconsistent. The fact that the OLS assumption for the instrumental variables holds provides the key to the solution of the problem of estimation.

Consider the X_3 equation. The problem is that X_4 is correlated with U in part because of σ_{UU} being non-zero and in part because of σ_{UV} being non-zero (in addition to B_{43} and B_{34} being non-zero).

$$\sigma_{4U} = B_{43}(B_{34}\sigma_{4U} + \sigma_{UU}) + \sigma_{UV}$$

So, part of X_4, which affects X_3 is correlated with U and part is uncorrelated with U. If we could eliminate the portion of X_4 that is correlated with U (i.e., σ_{4U}), then OLS would work fine because now the "new" X_4 would be orthogonal to U ($\sigma_{4U} = 0$). How do we construct the new X_4? Well, we know X_1 is uncorrelated with U and we know it has a non-zero effect on X_3—that is, we assume B_{31} is non-zero (that's why X_1 is in the equation!). It follows that if X_1 is also correlated with X_4, then perhaps we could capitalize on their "overlapping variance" to purge X_1 of its correlation with U. We can do this by first regressing X_4 on X_1 (as well as X_2) using OLS:

1. $X_4 = \pi_{41}X_1 + \pi_{42}X_2 + \pi_{4v}$

where $\pi_{4V} = (B_{43}U + V)/(1 - B_{34}B_{43})$ is the reduced-form disturbance as above. Call this the "first stage regression." Note also that this first stage regression is the reduced-form regression for X_4. Compute the predicted scores from this regression,

$\hat{X}_4$: $\hat{X}_4 = \pi_{41}X_1 + \pi_{42}X_2$

Note that the predicted score for X_4 is "purged" of the reduced-form disturbance term, and consequently purged of the problem terms, B_{34}, U, and V. Now use OLS to estimate the structural form equation for X_3, but using $\hat{X}_4$ in place of X_4. Call this the "second stage regression:"

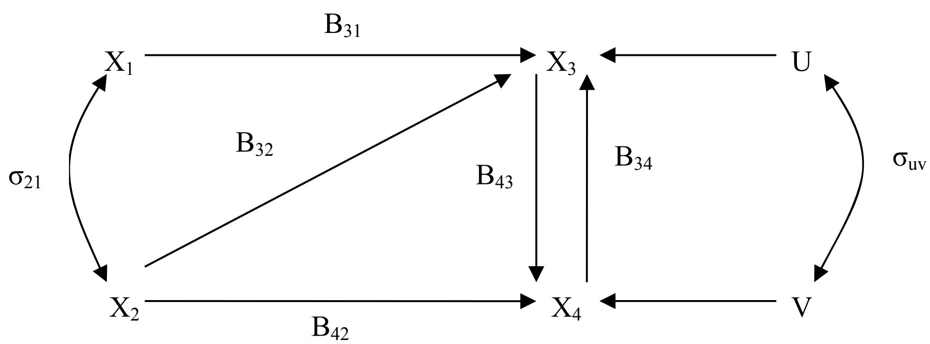
2. $X_3 = B_{31}X_1 + B_{34}\hat{X}_4 + U$

Now we need to adjust the standard errors to take into consideration that we estimated a first stage regression to get predicted values for X_4. Computer packages that use 2SLS will do this adjustment. Of course, we can also do the same for the X_3 equation to get 2SLS estimators. (Do this yourself.)

The 2SLS estimator is consistent but inefficient. A more efficient estimator can be obtained by applying Zellner's (1962) seemingly unrelated regression GLS in a third stage to adjust the estimator for the covariance between U and V. This is termed 3SLS, which gives consistent and asymptotically efficient parameter estimates. Note that 3SLS is asymptotically equivalent to ML using LISREL 8 for non-recursive models in observables. They are each "system" estimators in that they estimate all equations as a single system. Like all "system" estimators, they have the advantage of providing asymptotically efficient estimates, but the drawback that misspecification in one portion of the model can "spill over" and bias estimates in another portion.

VI. **A PARTIALLY IDENTIFIED MODEL:**

Often in empirical applications we are interested in an effect of X_3 on X_4 but want to control for the biasing effect of possible reciprocal causation—that is, that X_4 may affect X_3, even though we are not interested in estimating the latter coefficient. All we would need is an instrument for the variable we're interested in—in this case, X_3. As an example, suppose I'm interested in estimating the effect of reciprocated exchange in 2002 on collective efficacy in 2002 (B_{43} in the diagram) but I'm worried that it will be biased because of reciprocal causation: Collective efficacy in 2002 may simultaneously affect exchange in 2002. I happen to have measured exchange in 1990 and, under the assumptions that (1) exchange in 1990 affects exchange in 2002; exchange in 2002 affects collective efficacy in 2002; and exchange in 1990 does not affect collective efficacy in 2002 net of exchange in 2002, I can estimate B_{43}.



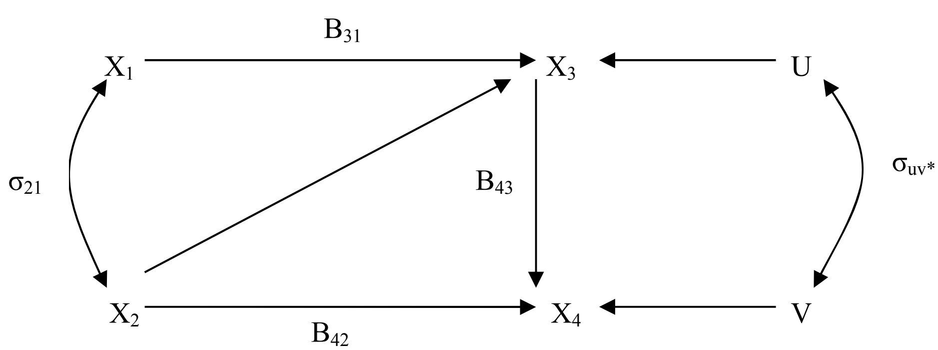
$$X_3 = B_{31}X_1 + B_{32}X_2 + B_{34}X_4 + U$$
$$X_4 = B_{42}X_2 + B_{43}X_3 + V$$

If we look at the X_3 equation, it is obvious that neither X_1 nor X_2 qualify as instruments for X_4 necessary to estimate B_{34}: X_1 is not causally-related to X_4 and while X_2 is related to X_4, it is also related to X_3. Thus, we cannot estimate B_{34}. Looking at the X_4 equation, however, we have an instrument for X_3, namely X_1, which is causally related to X_3 and excluded from the X_4 equation. Consequently, we can estimate B_{43}, the coefficient of interest, even in the presence of reciprocal causation between X_3 and X_4. That estimator will appear as above:

$\hat{B}_{43} = \hat{\pi}_{41}/\hat{\pi}_{31}$, or equivalently,

$$\hat{B}_{43} = \frac{s_{12}s_{24} - s_{14}s_{22}}{s_{12}s_{23} - s_{13}s_{22}}$$

Note that using the counting rule this model is under-identified: there are 10 moments and 11 parameters. We can estimate the model if we don't try to disentangle B_{34} from σ_{uv}, but pool them into one term (call it σ_{uv}*).



AN EXAMPLE OF NEIGHBORHOOD EXCHANGE AND COLLECTIVE EFFICACY

Model I:

Here is a LISREL run estimating the a model like the first one above, in which I have reasonably identified B_{12} using lagged Exchange 1990 as an instrumental variable for Exchange 2003, but have *arbitrarily identified* B_{21} using Percent Hispanic as an instrumental variable. Note this model is *empirically underidentified* and very unstable.

```
The following lines were read from file C:\529 examples\soccap1a.spl:

 Non-Recursive Model of Obligations and Child-Centered Control:soccap1a
 DA NI=9 NO=99 MA=CM
 CM
 *
 0.064
 0.029  0.023
 0.022  0.013  0.013
 -0.129 -0.036 -0.033  0.625
 0.165  0.061  0.051 -0.528  0.888
 -0.005 -0.001 -0.002  0.016 -0.025  0.002
 -0.223 -0.079 -0.054  1.023 -1.248  0.033  3.956
 0.168  0.101  0.082 -0.265  0.327 -0.008 -0.119  0.895
 -0.081 -0.056 -0.044  0.093 -0.071 -0.001 -0.142 -0.409 0.416

 LA
 *
 COLL02 EXCHAN02 EXCHAN90 CONDIS CONAFF2 PHISP ASIMM RESSTAB DENSITY
 SE
 COLL02 EXCHAN02 EXCHAN90 CONDIS CONAFF2 PHISP ASIMM RESSTAB DENSITY
 MO NY=2 NX=7 BE=FU,FI GA=FU,FR PS=FU,FI FI
 Fi BE 2 1
 FR BE 1 2
 FI GA 1 1
 Fr GA 2 4
 FR PS 1 1 PS 2 2 PS 2 1
 OU ME=ML SE TV SC PC

 Non-Recursive Model of Obligations and Child-Centered Control:soccap1a

                          Number of Input Variables  9
                          Number of Y - Variables    2
                          Number of X - Variables    7
                          Number of ETA - Variables  2
                          Number of KSI - Variables  7
                          Number of Observations    99

 Non-Recursive Model of Obligations and Child-Centered Control:soccap1a

         Covariance Matrix

               COLL02   EXCHAN02   EXCHAN90     CONDIS    CONAFF2      PHISP
             --------   --------   --------   --------   --------   --------
   COLL02       0.06
 EXCHAN02       0.03       0.02
 EXCHAN90       0.02       0.01       0.01
   CONDIS      -0.13      -0.04      -0.03       0.62
  CONAFF2       0.17       0.06       0.05      -0.53       0.89
    PHISP      -0.01       0.00       0.00       0.02      -0.03       0.00
    ASIMM      -0.22      -0.08      -0.05       1.02      -1.25       0.03
  RESSTAB       0.17       0.10       0.08      -0.27       0.33      -0.01
  DENSITY      -0.08      -0.06      -0.04       0.09      -0.07       0.00

         Covariance Matrix

                ASIMM    RESSTAB    DENSITY
             --------   --------   --------
    ASIMM       3.96
  RESSTAB      -0.12       0.90
  DENSITY      -0.14      -0.41       0.42


 Non-Recursive Model of Obligations and Child-Centered Control:soccap1a

 Parameter Specifications
```

BETA

	COLL02	EXCHAN02
COLL02	0	1
EXCHAN02	2	0

GAMMA

	EXCHAN90	CONDIS	CONAFF2	PHISP	ASIMM	RESSTAB
COLL02	0	3	4	5	6	7
EXCHAN02	9	10	11	0	12	13

GAMMA

	DENSITY
COLL02	8
EXCHAN02	14

PSI

	COLL02	EXCHAN02
COLL02	15	
EXCHAN02	16	17

Non-Recursive Model of Obligations and Child-Centered Control:soccap1a

Number of Iterations =129

LISREL Estimates (Maximum Likelihood)

BETA

	COLL02	EXCHAN02
COLL02	- -	1.21
		(0.33)
		3.67
EXCHAN02	-15.16	- -
	(135.10)	
	-0.11	

GAMMA

	EXCHAN90	CONDIS	CONAFF2	PHISP	ASIMM	RESSTAB
COLL02	- -	-0.11	0.03	-0.82	0.01	0.00
		(0.03)	(0.03)	(0.36)	(0.01)	(0.03)
		-3.70	1.25	-2.30	1.11	0.09
EXCHAN02	11.67	-1.01	1.08	- -	-0.11	0.92
	(99.69)	(9.34)	(9.52)		(0.79)	(7.74)
	0.12	-0.11	0.11		-0.13	0.12

GAMMA

	DENSITY
COLL02	0.00
	(0.03)
	0.11
EXCHAN02	-0.58
	(4.74)
	-0.12

Covariance Matrix of Y and X

	COLL02	EXCHAN02	EXCHAN90	CONDIS	CONAFF2	PHISP
COLL02	0.06					
EXCHAN02	0.03	0.02				
EXCHAN90	0.02	0.01	0.01			
CONDIS	-0.13	-0.04	-0.03	0.62		
CONAFF2	0.16	0.06	0.05	-0.53	0.89	
PHISP	0.00	0.00	0.00	0.02	-0.03	0.00
ASIMM	-0.22	-0.08	-0.05	1.02	-1.25	0.03
RESSTAB	0.17	0.10	0.08	-0.27	0.33	-0.01
DENSITY	-0.08	-0.06	-0.04	0.09	-0.07	0.00

Covariance Matrix of Y and X

	ASIMM	RESSTAB	DENSITY
ASIMM	3.96		
RESSTAB	-0.12	0.90	
DENSITY	-0.14	-0.41	0.42

PHI

	EXCHAN90	CONDIS	CONAFF2	PHISP	ASIMM	RESSTAB
EXCHAN90	0.01					
CONDIS	-0.03	0.62				
CONAFF2	0.05	-0.53	0.89			
PHISP	0.00	0.02	-0.03	0.00		
ASIMM	-0.05	1.02	-1.25	0.03	3.96	
RESSTAB	0.08	-0.27	0.33	-0.01	-0.12	0.90
DENSITY	-0.04	0.09	-0.07	0.00	-0.14	-0.41

PHI

	DENSITY
DENSITY	0.42

PSI

	COLL02	EXCHAN02
COLL02	0.01 (0.00) 3.75	
EXCHAN02	0.12 (1.09) 0.11	3.25 (56.57) 0.06

Squared Multiple Correlations for Structural Equations

COLL02	EXCHAN02
0.99	1.04

NOTE: Rý for Structural Equatios are Hayduk's (2006) Blocked-Error Rý

Reduced Form

	EXCHAN90	CONDIS	CONAFF2	PHISP	ASIMM	RESSTAB
COLL02	0.73	-0.07	0.07	-0.04	-0.01	0.06

	(0.20)	(0.02)	(0.02)	(0.37)	(0.01)	(0.02)
	3.70	-2.75	3.01	-0.11	-0.61	2.50
EXCHAN02	0.60	0.03	0.03	0.64	-0.02	0.05
	(0.15)	(0.02)	(0.02)	(0.27)	(0.01)	(0.02)
	4.10	1.82	1.77	2.34	-2.22	2.63

Reduced Form

	DENSITY
COLL02	-0.04
	(0.03)
	-1.26
EXCHAN02	-0.03
	(0.02)
	-1.54

Squared Multiple Correlations for Reduced Form

COLL02	EXCHAN02
0.79	0.67

Goodness of Fit Statistics

Degrees of Freedom = 0
Minimum Fit Function Chi-Square = 0.0 (P = 1.00)
Normal Theory Weighted Least Squares Chi-Square = 0.00 (P = 1.00)

The Model is Saturated, the Fit is Perfect !

Covariance Matrix of Parameter Estimates

	BE 1_2	BE 2_1	GA 1_2	GA 1_3	GA 1_4	GA 1_5
BE 1_2	0.11					
BE 2_1	-8.82	18251.02				
GA 1_2	-0.01	0.64	0.00			
GA 1_3	0.00	-0.36	0.00	0.00		
GA 1_4	-0.03	-22.47	0.00	0.00	0.13	
GA 1_5	0.00	-0.28	0.00	0.00	0.00	0.00
GA 1_6	-0.01	0.68	0.00	0.00	0.00	0.00
GA 1_7	0.01	-0.87	0.00	0.00	0.00	0.00
GA 2_1	7.00	-13462.80	-0.49	0.24	16.26	0.21
GA 2_2	-0.62	1260.94	0.05	-0.02	-1.55	-0.02
GA 2_3	0.61	-1285.26	-0.04	0.03	1.59	0.02
GA 2_5	-0.05	105.00	0.00	0.00	-0.13	0.00
GA 2_6	0.47	-1044.04	-0.03	0.02	1.31	0.01
GA 2_7	-0.29	637.85	0.02	-0.01	-0.80	-0.01
PS 1_1	0.00	-0.08	0.00	0.00	0.00	0.00
PS 2_1	0.06	-147.29	0.00	0.00	0.18	0.00
PS 2_2	3.69	-7642.58	-0.27	0.15	9.41	0.12

Covariance Matrix of Parameter Estimates

	GA 1_6	GA 1_7	GA 2_1	GA 2_2	GA 2_3	GA 2_5
GA 1_6	0.00					
GA 1_7	0.00	0.00				
GA 2_1	-0.54	0.67	9938.99			
GA 2_2	0.05	-0.06	-930.31	87.26		
GA 2_3	-0.05	0.06	947.83	-88.75	90.62	
GA 2_5	0.00	0.00	-77.37	7.23	-7.38	0.63
GA 2_6	-0.03	0.05	769.59	-72.11	73.51	-6.02
GA 2_7	0.02	-0.02	-470.15	44.04	-44.94	3.68
PS 1_1	0.00	0.00	0.06	-0.01	0.01	0.00
PS 2_1	0.00	0.01	108.61	-10.18	10.37	-0.85
PS 2_2	-0.28	0.37	5637.52	-528.02	538.20	-43.97

Covariance Matrix of Parameter Estimates

	GA 2_6	GA 2_7	PS 1_1	PS 2_1	PS 2_2
GA 2_6	59.85				
GA 2_7	-36.44	22.47			
PS 1_1	0.00	0.00	0.00		
PS 2_1	8.43	-5.15	0.00	1.19	
PS 2_2	437.19	-267.10	0.03	61.68	3200.54

Non-Recursive Model of Obligations and Child-Centered Control:soccap1a

Correlation Matrix of Parameter Estimates

	BE 1_2	BE 2_1	GA 1_2	GA 1_3	GA 1_4	GA 1_5
BE 1_2	1.00					
BE 2_1	-0.20	1.00				
GA 1_2	-0.56	0.16	1.00			
GA 1_3	-0.47	-0.10	0.48	1.00		
GA 1_4	-0.23	-0.47	0.04	0.46	1.00	
GA 1_5	0.63	-0.17	-0.62	0.00	-0.06	1.00
GA 1_6	-0.82	0.15	0.52	0.26	0.21	-0.58
GA 1_7	0.62	-0.19	-0.43	-0.31	-0.02	0.49
GA 2_1	0.21	-1.00	-0.17	0.09	0.46	0.18
GA 2_2	-0.20	1.00	0.18	-0.09	-0.46	-0.18
GA 2_3	0.19	-1.00	-0.15	0.12	0.47	0.17
GA 2_5	-0.18	0.98	0.11	-0.08	-0.47	-0.07
GA 2_6	0.19	-1.00	-0.15	0.10	0.47	0.16
GA 2_7	-0.18	1.00	0.15	-0.11	-0.47	-0.15
PS 1_1	0.83	-0.16	-0.47	-0.39	-0.19	0.52
PS 2_1	0.18	-1.00	-0.15	0.11	0.47	0.15
PS 2_2	0.20	-1.00	-0.16	0.10	0.47	0.17

Correlation Matrix of Parameter Estimates

	GA 1_6	GA 1_7	GA 2_1	GA 2_2	GA 2_3	GA 2_5
GA 1_6	1.00					
GA 1_7	-0.23	1.00				
GA 2_1	-0.16	0.20	1.00			
GA 2_2	0.15	-0.19	-1.00	1.00		
GA 2_3	-0.15	0.18	1.00	-1.00	1.00	
GA 2_5	0.12	-0.16	-0.98	0.98	-0.98	1.00
GA 2_6	-0.13	0.19	1.00	-1.00	1.00	-0.98
GA 2_7	0.15	-0.14	-0.99	0.99	-1.00	0.98
PS 1_1	-0.68	0.52	0.18	-0.17	0.16	-0.15
PS 2_1	-0.13	0.18	1.00	-1.00	1.00	-0.98
PS 2_2	-0.15	0.19	1.00	-1.00	1.00	-0.98

Correlation Matrix of Parameter Estimates

	GA 2_6	GA 2_7	PS 1_1	PS 2_1	PS 2_2
GA 2_6	1.00				
GA 2_7	-0.99	1.00			
PS 1_1	0.15	-0.15	1.00		
PS 2_1	1.00	-1.00	0.16	1.00	
PS 2_2	1.00	-1.00	0.17	1.00	1.00

Non-Recursive Model of Obligations and Child-Centered Control:soccap1a

Standardized Solution

BETA

	COLL02	EXCHAN02
COLL02	- -	0.73
EXCHAN02	-25.29	- -

GAMMA

EXCHAN90	CONDIS	CONAFF2	PHISP	ASIMM	RESSTAB

```
          --------  --------  --------  --------  --------  --------
   COLL02      - -     -0.34      0.12     -0.14      0.11      0.01
 EXCHAN02     8.77     -5.24      6.74      - -      -1.39      5.73

         GAMMA

             DENSITY
            --------
   COLL02     0.01
 EXCHAN02    -2.45

         Correlation Matrix of Y and X

              COLL02  EXCHAN02  EXCHAN90    CONDIS   CONAFF2     PHISP
            --------  --------  --------  --------  --------  --------
   COLL02     1.00
 EXCHAN02     0.76      1.00
 EXCHAN90     0.76      0.75      1.00
   CONDIS    -0.64     -0.30     -0.37      1.00
  CONAFF2     0.69      0.43      0.47     -0.71      1.00
    PHISP    -0.44     -0.15     -0.39      0.45     -0.59      1.00
    ASIMM    -0.44     -0.26     -0.24      0.65     -0.67      0.37
  RESSTAB     0.70      0.70      0.76     -0.35      0.37     -0.19
  DENSITY    -0.50     -0.57     -0.60      0.18     -0.12     -0.03

         Correlation Matrix of Y and X

               ASIMM   RESSTAB   DENSITY
            --------  --------  --------
    ASIMM     1.00
  RESSTAB    -0.06      1.00
  DENSITY    -0.11     -0.67      1.00

         PSI

              COLL02  EXCHAN02
            --------  --------
   COLL02     0.21
 EXCHAN02     3.08    141.30

         Regression Matrix Y on X (Standardized)

            EXCHAN90    CONDIS   CONAFF2     PHISP     ASIMM   RESSTAB
            --------  --------  --------  --------  --------  --------
   COLL02     0.33     -0.21      0.26     -0.01     -0.05      0.22
 EXCHAN02     0.45      0.18      0.19      0.19     -0.21      0.28

         Regression Matrix Y on X (Standardized)

             DENSITY
            --------
   COLL02    -0.09
 EXCHAN02    -0.14

                          Time used:    0.047 Seconds
```

Model II:

In the model below, I have pooled B_{21} with σ_{uv}, and focused on estimating B_{12}, which is identified using lagged Exchange1990 as an instrument for Exchange 2003. I am assuming that B_{21} is not identified. Note that the estimates of B_{12}—the parameter of interest—is identical in the two models. Thus, even though the reciprocal effects in a true simultaneous equation model are not each identified, if you are interested in getting a consistent estimate of one effect—which is identified—you can get a consistent estimator of the effect even when the other reciprocal effect is not identified and not estimable from the data.

```
The following lines were read from file C:\529 examples\soccap1b.spl:

 Unbiased Model of Obligations and Child-Centered Control:soccap1b
 DA NI=9 NO=99 MA=CM
 CM
 *
 0.064
```

```
0.029  0.023
0.022  0.013  0.013
-0.129 -0.036 -0.033  0.625
0.165  0.061  0.051 -0.528  0.888
-0.005 -0.001 -0.002  0.016 -0.025  0.002
-0.223 -0.079 -0.054  1.023 -1.248  0.033  3.956
0.168  0.101  0.082 -0.265  0.327 -0.008 -0.119  0.895
-0.081 -0.056 -0.044  0.093 -0.071 -0.001 -0.142 -0.409 0.416

LA
*
COLL02 EXCHAN02 EXCHAN90 CONDIS CONAFF2 PHISP ASIMM RESSTAB DENSITY
SE
COLL02 EXCHAN02 EXCHAN90 CONDIS CONAFF2 PHISP ASIMM RESSTAB DENSITY
MO NY=2 NX=7 BE=FU,FI GA=FU,FR PS=FU,FI FI
FI BE 2 1
FR BE 1 2
FI GA 1 1
FR GA 2 4
FR PS 1 1 PS 2 2 PS 2 1
OU ME=ML SE TV SC PC

Unbiased Model of Obligations and Child-Centered Control:soccap1b

                          Number of Input Variables  9
                          Number of Y - Variables    2
                          Number of X - Variables    7
                          Number of ETA - Variables  2
                          Number of KSI - Variables  7
                          Number of Observations    99

Unbiased Model of Obligations and Child-Centered Control:soccap1b

        Covariance Matrix

             COLL02   EXCHAN02   EXCHAN90     CONDIS    CONAFF2      PHISP
           --------   --------   --------   --------   --------   --------
  COLL02       0.06
EXCHAN02       0.03       0.02
EXCHAN90       0.02       0.01       0.01
  CONDIS      -0.13      -0.04      -0.03       0.62
 CONAFF2       0.17       0.06       0.05      -0.53       0.89
   PHISP      -0.01       0.00       0.00       0.02      -0.03       0.00
   ASIMM      -0.22      -0.08      -0.05       1.02      -1.25       0.03
 RESSTAB       0.17       0.10       0.08      -0.27       0.33      -0.01
 DENSITY      -0.08      -0.06      -0.04       0.09      -0.07       0.00

        Covariance Matrix

              ASIMM    RESSTAB    DENSITY
           --------   --------   --------
   ASIMM       3.96
 RESSTAB      -0.12       0.90
 DENSITY      -0.14      -0.41       0.42


Unbiased Model of Obligations and Child-Centered Control:soccap1b

Parameter Specifications

        BETA

             COLL02   EXCHAN02
           --------   --------
  COLL02          0          1
EXCHAN02          0          0

        GAMMA

           EXCHAN90     CONDIS    CONAFF2      PHISP      ASIMM    RESSTAB
           --------   --------   --------   --------   --------   --------
  COLL02          0          2          3          4          5          6
EXCHAN02          8          9         10         11         12         13
```

GAMMA

	DENSITY
COLL02	7
EXCHAN02	14

PSI

	COLL02	EXCHAN02
COLL02	15	
EXCHAN02	16	17

Unbiased Model of Obligations and Child-Centered Control:soccap1b

Number of Iterations = 31

LISREL Estimates (Maximum Likelihood)

BETA

	COLL02	EXCHAN02
COLL02	- -	1.21
		(0.33)
		3.67
EXCHAN02	- -	- -

GAMMA

	EXCHAN90	CONDIS	CONAFF2	PHISP	ASIMM	RESSTAB
COLL02	- -	-0.11	0.03	-0.82	0.01	0.00
		(0.03)	(0.03)	(0.36)	(0.01)	(0.03)
		-3.70	1.25	-2.30	1.11	0.09
EXCHAN02	0.60	0.03	0.03	0.64	-0.02	0.05
	(0.15)	(0.02)	(0.02)	(0.27)	(0.01)	(0.02)
	4.10	1.82	1.77	2.34	-2.22	2.63

GAMMA

	DENSITY
COLL02	0.00
	(0.03)
	0.11
EXCHAN02	-0.03
	(0.02)
	-1.54

Covariance Matrix of Y and X

	COLL02	EXCHAN02	EXCHAN90	CONDIS	CONAFF2	PHISP
COLL02	0.06					
EXCHAN02	0.03	0.02				
EXCHAN90	0.02	0.01	0.01			
CONDIS	-0.13	-0.04	-0.03	0.62		
CONAFF2	0.17	0.06	0.05	-0.53	0.89	
PHISP	-0.01	0.00	0.00	0.02	-0.03	0.00
ASIMM	-0.22	-0.08	-0.05	1.02	-1.25	0.03
RESSTAB	0.17	0.10	0.08	-0.27	0.33	-0.01
DENSITY	-0.08	-0.06	-0.04	0.09	-0.07	0.00

Covariance Matrix of Y and X

ASIMM	RESSTAB	DENSITY

```
   ASIMM       3.96
 RESSTAB      -0.12        0.90
 DENSITY      -0.14       -0.41        0.42

        PHI

            EXCHAN90     CONDIS    CONAFF2      PHISP      ASIMM    RESSTAB
            --------   --------   --------   --------   --------   --------
EXCHAN90        0.01
  CONDIS       -0.03       0.62
 CONAFF2        0.05      -0.53       0.89
   PHISP        0.00       0.02      -0.03       0.00
   ASIMM       -0.05       1.02      -1.25       0.03       3.96
 RESSTAB        0.08      -0.27       0.33      -0.01      -0.12       0.90
 DENSITY       -0.04       0.09      -0.07       0.00      -0.14      -0.41

        PHI

             DENSITY
            --------
 DENSITY        0.42

        PSI

              COLL02   EXCHAN02
            --------   --------
  COLL02        0.01
              (0.00)
                3.75

EXCHAN02        0.00       0.01
              (0.00)     (0.00)
               -1.69       6.75

        Squared Multiple Correlations for Structural Equations

              COLL02   EXCHAN02
            --------   --------
                0.96       0.67

NOTE: Rý for Structural Equations are Hayduk's (2006) Blocked-Error Rý

        Reduced Form

            EXCHAN90     CONDIS    CONAFF2      PHISP      ASIMM    RESSTAB
            --------   --------   --------   --------   --------   --------
  COLL02        0.73      -0.07       0.07      -0.04      -0.01       0.06
              (0.20)     (0.02)     (0.02)     (0.37)     (0.01)     (0.02)
                3.70      -2.75       3.01      -0.11      -0.61       2.50

EXCHAN02        0.60       0.03       0.03       0.64      -0.02       0.05
              (0.15)     (0.02)     (0.02)     (0.27)     (0.01)     (0.02)
                4.10       1.82       1.77       2.34      -2.22       2.63


        Reduced Form

             DENSITY
            --------
  COLL02       -0.04
              (0.03)
               -1.26

EXCHAN02       -0.03
              (0.02)
               -1.54


        Squared Multiple Correlations for Reduced Form

              COLL02   EXCHAN02
            --------   --------
                0.79       0.67
```

Goodness of Fit Statistics

Degrees of Freedom = 0
Minimum Fit Function Chi-Square = 0.0 (P = 1.00)
Normal Theory Weighted Least Squares Chi-Square = 0.00 (P = 1.00)

The Model is Saturated, the Fit is Perfect !

Covariance Matrix of Parameter Estimates

	BE 1_2	GA 1_2	GA 1_3	GA 1_4	GA 1_5	GA 1_6
BE 1_2	0.11					
GA 1_2	-0.01	0.00				
GA 1_3	0.00	0.00	0.00			
GA 1_4	-0.03	0.00	0.00	0.13		
GA 1_5	0.00	0.00	0.00	0.00	0.00	
GA 1_6	-0.01	0.00	0.00	0.00	0.00	0.00
GA 1_7	0.01	0.00	0.00	0.00	0.00	0.00
GA 2_1	-0.02	0.00	0.00	0.01	0.00	0.00
GA 2_2	0.00	0.00	0.00	0.00	0.00	0.00
GA 2_3	0.00	0.00	0.00	0.00	0.00	0.00
GA 2_4	-0.01	0.00	0.00	-0.04	0.00	0.00
GA 2_5	0.00	0.00	0.00	0.00	0.00	0.00
GA 2_6	0.00	0.00	0.00	0.00	0.00	0.00
GA 2_7	0.00	0.00	0.00	0.00	0.00	0.00
PS 1_1	0.00	0.00	0.00	0.00	0.00	0.00
PS 2_1	0.00	0.00	0.00	0.00	0.00	0.00
PS 2_2	0.00	0.00	0.00	0.00	0.00	0.00

Covariance Matrix of Parameter Estimates

	GA 1_7	GA 2_1	GA 2_2	GA 2_3	GA 2_4	GA 2_5
GA 1_7	0.00					
GA 2_1	0.00	0.02				
GA 2_2	0.00	0.00	0.00			
GA 2_3	0.00	0.00	0.00	0.00		
GA 2_4	0.00	0.01	0.00	0.00	0.08	
GA 2_5	0.00	0.00	0.00	0.00	0.00	0.00
GA 2_6	0.00	0.00	0.00	0.00	0.00	0.00
GA 2_7	0.00	0.00	0.00	0.00	0.00	0.00
PS 1_1	0.00	0.00	0.00	0.00	0.00	0.00
PS 2_1	0.00	0.00	0.00	0.00	0.00	0.00
PS 2_2	0.00	0.00	0.00	0.00	0.00	0.00

Covariance Matrix of Parameter Estimates

	GA 2_6	GA 2_7	PS 1_1	PS 2_1	PS 2_2
GA 2_6	0.00				
GA 2_7	0.00	0.00			
PS 1_1	0.00	0.00	0.00		
PS 2_1	0.00	0.00	0.00	0.00	
PS 2_2	0.00	0.00	0.00	0.00	0.00

Unbiased Model of Obligations and Child-Centered Control:soccap1b

Correlation Matrix of Parameter Estimates

	BE 1_2	GA 1_2	GA 1_3	GA 1_4	GA 1_5	GA 1_6
BE 1_2	1.00					
GA 1_2	-0.56	1.00				
GA 1_3	-0.47	0.48	1.00			
GA 1_4	-0.23	0.04	0.46	1.00		
GA 1_5	0.63	-0.62	0.00	-0.06	1.00	
GA 1_6	-0.82	0.52	0.26	0.21	-0.58	1.00
GA 1_7	0.62	-0.43	-0.31	-0.02	0.49	-0.23
GA 2_1	-0.46	0.26	0.21	0.10	-0.29	0.37
GA 2_2	0.10	-0.42	-0.16	0.02	0.20	-0.11
GA 2_3	0.04	-0.14	-0.42	-0.19	-0.12	0.03

GA 2_4	-0.16	0.13	-0.08	-0.38	-0.14	0.12
GA 2_5	-0.11	0.21	-0.11	-0.02	-0.41	0.13
GA 2_6	0.25	-0.18	-0.03	-0.07	0.20	-0.42
GA 2_7	-0.16	0.14	0.09	-0.03	-0.16	-0.02
PS 1_1	0.83	-0.47	-0.39	-0.19	0.52	-0.68
PS 2_1	-0.90	0.51	0.42	0.20	-0.57	0.74
PS 2_2	0.00	0.00	0.00	0.00	0.00	0.00

Correlation Matrix of Parameter Estimates

	GA 1_7	GA 2_1	GA 2_2	GA 2_3	GA 2_4	GA 2_5
GA 1_7	1.00					
GA 2_1	-0.28	1.00				
GA 2_2	0.11	-0.22	1.00			
GA 2_3	0.03	-0.09	0.31	1.00		
GA 2_4	-0.15	0.35	-0.18	0.36	1.00	
GA 2_5	-0.12	0.24	-0.44	0.39	0.18	1.00
GA 2_6	-0.03	-0.55	0.22	-0.15	-0.16	-0.26
GA 2_7	-0.43	0.36	-0.20	-0.05	0.27	0.23
PS 1_1	0.52	-0.38	0.08	0.03	-0.13	-0.09
PS 2_1	-0.56	0.41	-0.09	-0.04	0.15	0.10
PS 2_2	0.00	0.00	0.00	0.00	0.00	0.00

Correlation Matrix of Parameter Estimates

	GA 2_6	GA 2_7	PS 1_1	PS 2_1	PS 2_2
GA 2_6	1.00				
GA 2_7	0.29	1.00			
PS 1_1	0.21	-0.14	1.00		
PS 2_1	-0.23	0.15	-0.89	1.00	
PS 2_2	0.00	0.00	0.12	-0.25	1.00

Unbiased Model of Obligations and Child-Centered Control:soccap1b

Standardized Solution

BETA

	COLL02	EXCHAN02
COLL02	- -	0.73
EXCHAN02	- -	- -

GAMMA

	EXCHAN90	CONDIS	CONAFF2	PHISP	ASIMM	RESSTAB
COLL02	- -	-0.34	0.12	-0.14	0.11	0.01
EXCHAN02	0.45	0.18	0.19	0.19	-0.21	0.28

GAMMA

	DENSITY
COLL02	0.01
EXCHAN02	-0.14

Correlation Matrix of Y and X

	COLL02	EXCHAN02	EXCHAN90	CONDIS	CONAFF2	PHISP
COLL02	1.00					
EXCHAN02	0.76	1.00				
EXCHAN90	0.76	0.75	1.00			
CONDIS	-0.64	-0.30	-0.37	1.00		
CONAFF2	0.69	0.43	0.47	-0.71	1.00	
PHISP	-0.44	-0.15	-0.39	0.45	-0.59	1.00
ASIMM	-0.44	-0.26	-0.24	0.65	-0.67	0.37
RESSTAB	0.70	0.70	0.76	-0.35	0.37	-0.19
DENSITY	-0.50	-0.57	-0.60	0.18	-0.12	-0.03

Correlation Matrix of Y and X

	ASIMM	RESSTAB	DENSITY
ASIMM	1.00		
RESSTAB	-0.06	1.00	
DENSITY	-0.11	-0.67	1.00

PSI

	COLL02	EXCHAN02
COLL02	0.21	
EXCHAN02	-0.12	0.33

Regression Matrix Y on X (Standardized)

	EXCHAN90	CONDIS	CONAFF2	PHISP	ASIMM	RESSTAB
COLL02	0.33	-0.21	0.26	-0.01	-0.05	0.22
EXCHAN02	0.45	0.18	0.19	0.19	-0.21	0.28

Regression Matrix Y on X (Standardized)

	DENSITY
COLL02	-0.09
EXCHAN02	-0.14

Time used: 0.016 Seconds