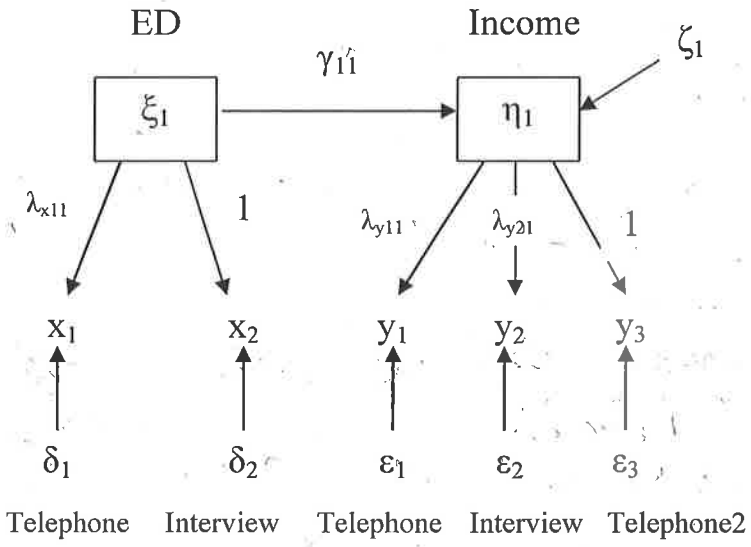
HOMEWORK 3

This assignment is due on Thursday May 29th at 5:30pm. You may turn it in to the Dropbox or at the beginning of class.

I. Consider the following walking dog model in six equations:



1. Write out the six equations in scalar algebra. (2 points)

$x_1 = \lambda_{x11}\xi_1 + \delta_1$ $\quad$ $y_2 = \lambda_{y21}\eta_1 + \varepsilon_2$

$x_2 = \xi_1 + \delta_2$ $\quad$ $y_3 = \eta_1 + \varepsilon_3$

$y_1 = \lambda_{y11}\eta_1 + \varepsilon_1$ $\quad$ $\eta_1 = \gamma_{11}\xi_1 + \zeta_1$

2. List out the seven parameter matrices (Φ, Γ, Ψ, Λ_x, Λ_y, Θ_δ, Θ_ε). (3 points)

$\Phi = [\phi_{11}]$ $\quad$ $\Theta_\delta = \begin{bmatrix} \theta^{\delta}_{11} & \\ & \theta^{\delta}_{22} \end{bmatrix}$

$\Gamma = [\gamma_{11}]$

$\Psi = [\psi_{11}]$

$\Lambda_x = \begin{bmatrix} \lambda_{x11} \\ 1.0 \end{bmatrix}$ $\quad$ $\Theta_\varepsilon = \begin{bmatrix} \theta^{\varepsilon}_{11} & & \\ & \theta^{\varepsilon}_{22} & \\ & & \theta^{\varepsilon}_{33} \end{bmatrix}$

$\Lambda_y = \begin{bmatrix} \lambda_{y11} \\ \lambda_{y21} \\ 1.0 \end{bmatrix}$

3. How many tetrad difference overidentifying restrictions on observed moments does this model imply? Give an example of **one** of these. (2 points)

15 moments
11 parameters
4 tetrad-difference restrictions

e.g., $\sigma_{x_1 y_2}\sigma_{x_2 y_1} = \sigma_{x_1 y_1}\sigma_{x_2 y_2}$

4. What would the overall goodness-of-fit χ^2 test? How many degrees of freedom does it have? (2 points)

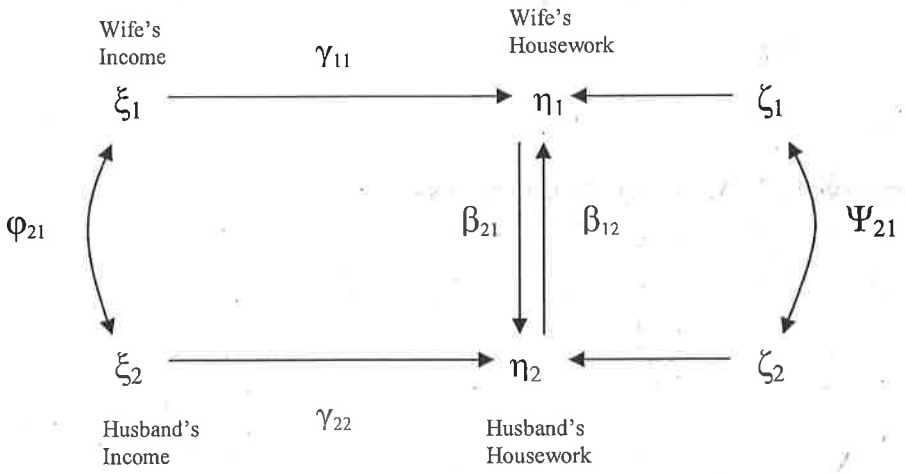
4 df. It tests the null hypothesis that all 4 restrictions hold in the population.

5. How is the metric of ξ_1 and η_1 set? Suppose that $\theta_{11}^{\varepsilon} = .20$, $\theta_{22}^{\varepsilon} = .10$, and $\theta_{33}^{\varepsilon} = .25$. Which measure of income would you say is most accurate? (3 points)

Reference indicators: Metric for ξ_1 = metric for x_2; metric for η_1 = metric for y_3

y_2 would be most accurate -- it has the smallest measurement error variance

III. Consider the following simultaneous equation model in two equations:



1. Write out the two structural form equations in scalar algebra. (2 points)

$\eta_1 = \gamma_{11}\xi_1 + \beta_{12}\eta_2 + \zeta_1$
$\eta_2 = \gamma_{22}\xi_2 + \beta_{21}\eta_1 + \zeta_2$

2. What are the properties of OLS for estimating the structural parameters (γs and βs) (i.e. is it consistent and unbiased)? Explain why. (2 points)

OLS is biased and inconsistent because for the η_1 equation, $E(\eta_2\zeta_1) \neq 0$ and for the η_2 equation $E(\eta_1\zeta_2) \neq 0$, violating the OLS assumption

3. Briefly discuss whether each equation is just-identified, underidentified, or overidentified. (3 points)

η_1 equation is just-identified. To overcome $E(\eta_2 \zeta_1) \neq 0$, we need an IV for η_2. ξ_2 qualifies as it affects η_2 but not η_1. η_2 equation is identified. To overcome $E(\eta_1 \zeta_2) \neq 0$, we have ξ_1 as an IV for η_1 (affects η_1 but not η_2). Also 10 moments, 10 parameters.

4. Does each of the endogenous predictor variables have at least one instrumental variable? If they do, identify them. Also, if they do, discuss very briefly whether you believe they are, substantively, good instruments. If there is no instrument(s), suggest a potential viable instrumental variable(s). (3 points)

Each has one IV: ξ_2 for η_2 and ξ_1 for η_1. The IVs seem reasonable — wife's income affects wife's housework and affects husband's housework only indirectly thru wife's housework. Husband's income affects husband's housework and affects wife's housework only indirectly thru husband's housework.

5. Suppose you estimated the model and found that $\beta_{21} > \beta_{12}$. What would you conclude substantively, given the labels on the variables above? (1 point)

Wife's housework has a stronger effect on husband's housework than vice-versa. Perhaps wives have more agency over household tasks, and have husbands do chores left over.

6. Suppose that γ_{11} is actually zero in the population. What does this imply substantively? What does this imply for estimation of the model's parameters? (2 points)

Wife's income has no effect on her housework. Wives who earn high incomes do as much housework as wives who earn low incomes. Perhaps because of gender roles, men are not picking up the slack when their wives bring home the bacon.

If $\gamma_{11} = 0$, then is no instrument for η_1 and β_{21} (and equation for η_2) is underidentified, and cannot be estimated from sample data.