## Thermodynamics and Chemical Equilibrium

- Consider a differential amount of the following chemical reaction in a closed system:

$$
a \mathrm{~A}+b \mathrm{~B} \leftrightarrow c \mathrm{C}+d \mathrm{D}
$$

Define one mole of reaction as $a$ moles of $A$ and $b$ moles of $B$ forming $c$ moles of $C$ and $d$ moles of $D$. Moles of reaction occurring in a given system is $\Delta \eta$.

$$
d n_{\mathrm{A}}=-a \Delta \eta ; \quad d n_{\mathrm{B}}=-b \Delta \eta ; \quad d n_{\mathrm{C}}=+c \Delta \eta ; \quad d n_{\mathrm{D}}=+d \Delta \eta
$$

$$
\begin{gathered}
\frac{d G_{\mathrm{sys}}}{d \eta}=-a \bar{G}_{\mathrm{A}}-b \bar{G}_{\mathrm{B}}+c \bar{G}_{\mathrm{C}}+d \bar{G}_{\mathrm{D}} \equiv \Delta \bar{G}_{r} \\
\Delta \bar{G}_{r}=-a\left(\bar{G}_{\mathrm{A}}^{\mathrm{o}}+R T \ln \{\mathrm{~A}\}\right)-b\left(\bar{G}_{\mathrm{B}}^{\mathrm{o}}+R T \ln \{\mathrm{~B}\}\right) \\
\\
+c\left(\bar{G}_{\mathrm{C}}^{\mathrm{o}}+R T \ln \{\mathrm{C}\}\right)+d\left(\bar{G}_{\mathrm{D}}^{\mathrm{o}}+R T \ln \{\mathrm{D}\}\right) \\
\Delta \bar{G}_{r}=-a \bar{G}_{\mathrm{A}}^{\mathrm{o}}-b \bar{G}_{\mathrm{B}}^{\mathrm{o}}+c \bar{G}_{\mathrm{C}}^{\mathrm{o}}+d \bar{G}_{\mathrm{D}}^{\mathrm{o}}+R T \ln \frac{\{\mathrm{C}\}^{c}\{\mathrm{D}\}^{d}}{\{\mathrm{~A}\}^{a}\{\mathrm{~B}\}^{b}} \\
\Delta \bar{G}_{r}=\Delta \bar{G}_{r}^{\mathrm{o}}+R T \ln Q=\Delta \bar{G}_{r}^{\mathrm{o}}+\left(5.71 \frac{\mathrm{~kJ}}{\mathrm{~mol}}\right) \log Q
\end{gathered}
$$

- Like in physical systems, the change (reaction) will occur if it decreases $P E_{\text {sys }}$ (in this case, $G_{\text {sys }}$ ).

$$
\begin{gathered}
G_{\mathrm{sys}, 1}=n_{\mathrm{A}, 1} \bar{G}_{\mathrm{A}}+n_{\mathrm{B}, 1} \bar{G}_{\mathrm{B}}+n_{\mathrm{C}, 1} \bar{G}_{\mathrm{C}}+n_{\mathrm{D}, 1} \bar{G}_{\mathrm{D}} \\
G_{\mathrm{sy}, 2}=\left(n_{\mathrm{A}, 1}+d n_{\mathrm{A}}\right) \bar{G}_{\mathrm{A}}+\left(n_{\mathrm{B}, 1}+d n_{\mathrm{B}}\right) \bar{G}_{\mathrm{B}} \\
\quad+\left(n_{\mathrm{C}, 1}+d n_{\mathrm{C}}\right) \bar{G}_{\mathrm{C}}+\left(n_{\mathrm{D}, 1}+d n_{\mathrm{D}}\right) \bar{G}_{\mathrm{D}} \\
d G_{\mathrm{sys}}=d n_{\mathrm{A}} \bar{G}_{\mathrm{A}}+d n_{\mathrm{B}} \bar{G}_{\mathrm{B}}+d n_{\mathrm{C}} \bar{G}_{\mathrm{C}}+d n_{\mathrm{D}} \bar{G}_{\mathrm{D}} \\
=- \\
=-d \eta\left(\bar{G}_{\mathrm{A}}\right)-b d \eta\left(\bar{G}_{\mathrm{B}}\right)+c d \eta\left(\bar{G}_{\mathrm{C}}\right)+d d \eta\left(\bar{G}_{\mathrm{D}}\right)
\end{gathered}
$$

- At equilibrium, $G_{\text {sys }}$ will be minimized, so $d G_{\text {sys }} / d \eta=0$ :

$$
\begin{aligned}
& 0=\left(\Delta \bar{G}_{r}^{\mathrm{o}}+R T \ln Q\right)_{e q}=\Delta \bar{G}_{r}^{\mathrm{o}}+R T \ln K_{e q} \\
& \Delta \bar{G}_{r}^{\mathrm{o}}=-R T \ln K_{e q} ; \quad K_{e q}=\exp \left(-\frac{\Delta \bar{G}_{r}^{\mathrm{o}}}{R T}\right)
\end{aligned}
$$



- Additive (Sequential) Reactions

$$
\begin{array}{cc}
\mathrm{H}_{2} \mathrm{CO}_{3} \leftrightarrow 2 \mathrm{H}^{+}+\mathrm{CO}_{3}^{2-} & \Delta \bar{G}_{r, 1}=\Delta \bar{G}_{r, 1}^{o}+R T \ln \frac{\left\{\mathrm{H}^{+}\right\}^{2}\left\{\mathrm{CO}_{3}^{2-}\right\}}{\left\{\mathrm{H}_{2} \mathrm{CO}_{3}\right\}} \\
\mathrm{Ca}^{2+}+\mathrm{CO}_{3}^{2-} \leftrightarrow \mathrm{CaCO}_{3}(s) & \Delta \bar{G}_{r, 2}=\Delta \bar{G}_{r, 2}^{o}+R T \ln \frac{\left\{\mathrm{CaCO}_{3}(s)\right\}}{\left\{\mathrm{Ca}^{2+}\right\}\left\{\mathrm{CO}_{3}^{2-}\right\}} \\
\hline \mathrm{Ca}^{2+}+\mathrm{H}_{2} \mathrm{CO}_{3} \leftrightarrow 2 \mathrm{H}^{+}+\mathrm{CaCO}_{3}(s) & \Delta \bar{G}_{r, 3}=\Delta \bar{G}_{r, 3}^{o}+R T \ln \frac{\left\{\mathrm{H}^{+}\right\}^{2}\left\{\mathrm{CaCO}_{3}(s)\right\}}{\left\{\mathrm{Ca}^{2+}\right\}\left\{\mathrm{H}_{2} \mathrm{CO}_{3}\right\}}
\end{array}
$$

- Additive (Sequential) Reactions

$$
\Delta \bar{G}_{r, 3}^{\mathrm{o}}=\Delta \bar{G}_{r, 1}^{\mathrm{o}}+\Delta \bar{G}_{r, 2}^{\mathrm{o}}
$$

$R T \ln Q_{1}+R T \ln Q_{2}=R T \ln Q_{1} Q_{2}=R T \ln Q_{3}$

$$
\Delta \bar{G}_{r, 3}=\Delta \bar{G}_{r, 1}+\Delta \bar{G}_{r, 2}
$$

- Molar Gibbs energy of Additive (Sequential) Reactions
- Equal to the sum of the molar Gibbs energy of constituent reactions


## Dependence of $K_{e q}$ on Temperature

Note: The relationship $\ln K_{e q}=-\Delta \bar{G}_{r}^{o} / R T$ is valid for any standard state $T$. How does $K_{e q}$ change when $T_{\text {std.state }}$ changes?

$$
d \ln K_{e q}=-d \frac{\Delta \bar{G}_{r}^{\circ}}{R T}=-d \frac{\Delta \bar{H}_{r}^{\circ}-T \Delta \bar{S}_{r}^{\circ}}{R T}=-d \frac{\Delta \bar{H}_{r}^{\circ}}{R T}+d \frac{\Delta \bar{S}_{r}^{\circ}}{R}
$$

If $\Delta \bar{H}_{r}{ }^{\circ}$ and $\Delta \overline{\mathrm{S}}_{r}{ }^{\circ}$ are insensitive to $T$ over the range of interest:

$$
\begin{gathered}
d \ln K_{e q}=-\frac{\Delta \bar{H}_{r}^{\circ}}{R} d\left(\frac{1}{T}\right) \\
\ln \frac{\left.K_{e q}\right|_{T_{2}}}{\left.K_{e q}\right|_{T_{1}}}=\frac{\Delta \bar{H}_{r}^{\circ}}{R}\left(\frac{1}{T_{1}}-\frac{1}{T_{2}}\right)
\end{gathered}
$$

## Acids and Bases

- An acid is a chemical that increases the $\mathrm{H}^{+}$activity in solution. A base is a chemical that decreases that activity.
- $\mathrm{An} \mathrm{H}^{+}$ion is a proton, which is very unstable in water; it becomes hydrated to form $\mathrm{H}_{3} \mathrm{O}^{+}$(hydronium ion) or other $\mathrm{H}_{2 n+1} \mathrm{O}_{\mathrm{n}}{ }^{+}$species.
- The simplest way that an acid can increase $\mathrm{H}^{+}$activity is to release an $\mathrm{H}^{+}$ion into solution (i.e., give it to an $\mathrm{H}_{2} \mathrm{O}$ to form $\mathrm{H}_{3} \mathrm{O}^{+}$. Similarly, the simplest way to decrease $\mathrm{H}^{+}$ activity is to acquire an $\mathrm{H}^{+}$from solution. Therefore, acids and bases are sometimes referred to as proton donors and proton acceptors, respectively.
- Everyday examples include citric acid, ammonia, phosphoric acid and other phosphate species, sodium benzoate, carbonic acid/ bicarbonate/ carbonate, amino acids, and many, many others.


## Acid/Base Properties of Water

- Water molecules react with one another, to a small extent, to donate and accept protons:

$\mathrm{H}_{2} \mathrm{O}(l)$

$\mathrm{H}_{2} \mathrm{O}(\mathrm{l})$
$\rightleftharpoons$


$$
K_{e q}=\frac{\left\{\mathrm{H}^{+}\right\}\left\{\mathrm{OH}^{-}\right\}}{\left\{\mathrm{H}_{2} \mathrm{O}\right\}^{2}} \equiv K_{\mathrm{w}}
$$

$$
K_{w}=10^{-14.0} ; \quad p K_{w}=14.0
$$

## Acid/Base Properties of Water

- Water dissociation reaction is fast, so assumption is made that it is always at equilibrium
- Activity of $\mathrm{H}_{2} \mathrm{O}$ is $\sim 1.0$, so:

$$
\begin{gathered}
K_{w}=\left\{\mathrm{H}^{+}\right\}\left\{\mathrm{OH}^{-}\right\}=10^{-14.0} \\
\log \left\{\mathrm{H}^{+}\right\}+\log \left\{\mathrm{OH}^{-}\right\}=-14.0 \\
p \mathrm{H}+p \mathrm{OH}=14.0
\end{gathered}
$$

where it is understood that $p \mathrm{H}$ and $p \mathrm{OH}$ are $-\log _{10}\left\{\mathrm{H}^{+}\right\}$and $-\log _{10}\left\{\mathrm{OH}^{-}\right\}$, respectively

## Acid/Base Properties of Water



