A $\log \mathrm{C}$ - pe diagram for a pH 5 system containing $1.1 \times 10^{-3} \mathrm{M}$ TOTCr and $3 \times 10^{-3} \mathrm{M}$ TOTFe is shown below. Based on the information in the diagram, answer the following questions.
a. What is $\mathrm{pe}^{0}$ for the $\mathrm{H}_{2} \mathrm{CrO}_{4} / \mathrm{Cr}^{3+}$ redox couple?
b. What is $\mathrm{p} K_{a 2}$ for chromic acid $\left(\mathrm{H}_{2} \mathrm{CrO}_{4}\right)$ ?
c. Write the TOTe equation for a system made by adding $3 \times 10^{-3} \mathrm{M} \mathrm{FeCl}_{2}$ to a solution initially containing $1.1 \times 10^{-3} \mathrm{M} \mathrm{Cr}(\mathrm{VI})$ and no $\mathrm{Cr}(\mathrm{III})$ at pH 5 . Assuming the system is well-buffered at this pH , what are the approximate values of pe and $\operatorname{TOTCr}(\mathrm{VI})$ at equilibrium? Consider only redox reactions of the $\mathrm{Fe}(\mathrm{III}) / \mathrm{Fe}(\mathrm{II})$ and $\mathrm{Cr}(\mathrm{VI}) / \mathrm{Cr}(\mathrm{III})$ couples.
d. Assuming that $\mathrm{Cr}(\mathrm{VI})$ species constitute the only weak acid/base system in the initial solution, how would the addition of the $\mathrm{FeCl}_{2}$ and the subsequent reactions affect the alkalinity and pH of the solution (increase, decrease, no change)? Explain briefly.


(a) The relevant half-reaction is as follows:

$$
\mathrm{H}_{2} \mathrm{CrO}_{4}+6 \mathrm{H}^{+}+3 \mathrm{e}^{-} \leftrightarrow \mathrm{Cr}^{3+}+4 \mathrm{H}_{2} \mathrm{O}
$$

The equilibrium constant for this reaction is:

$$
\begin{aligned}
& K=\frac{\left\{\mathrm{Cr}^{3+}\right\}}{\left\{\mathrm{H}_{2} \mathrm{CrO}_{4}\right\}\left\{\mathrm{H}^{+}\right\}^{6}\left\{\mathrm{e}^{-}\right\}^{3}} \\
& \log K=\log \frac{\left\{\mathrm{Cr}^{3+}\right\}}{\left\{\mathrm{H}_{2} \mathrm{CrO}_{4}\right\}}+6 \mathrm{pH}+3 \mathrm{pe}
\end{aligned}
$$

From the graph, we can obtain values for $\left\{\mathrm{H}_{2} \mathrm{CrO}_{4}\right\}$ and $\left\{\mathrm{Cr}^{3+}\right\}$ at any arbitrary pe and substitute those values (along with the known pH ) into the above equation to find $\log K$ for the reaction of interest. One convenient condition for the evaluation is at pe $=12.5$, where $\left\{\mathrm{H}_{2} \mathrm{CrO}_{4}\right\}=\left\{\mathrm{Cr}^{3+}\right\}$. Substituting these conditions into the equilibrium constant expression, we obtain:

$$
\log K=0+6(5.0)+3(12.5)=67.5
$$

pe ${ }^{0}$ equals $\log K$ for the reduction reaction for a one-electron transfer. The above reaction is for a three-electron transfer, so pe ${ }^{0}=67.5 / 3=22.5$.
(b) The graph indicates that, at the given pH of 5.0 , the value of $\left\{\mathrm{CrO}_{4}{ }^{2-}\right\}$ is always 1.5 log units less than that of $\left\{\mathrm{HCrO}_{4}{ }^{-}\right\}$, so the value of $K_{a 2}$ can be found as follows:

$$
\begin{aligned}
& K_{a 2}=\frac{\left\{\mathrm{H}^{+}\right\}\left\{\mathrm{CrO}_{4}^{2-}\right\}}{\left\{\mathrm{HCrO}_{4}^{-}\right\}}=\frac{\left\{\mathrm{H}^{+}\right\}\left(10^{-1.5}\left\{\mathrm{HCrO}_{4}^{-}\right\}\right)}{\left\{\mathrm{HCrO}_{4}^{-}\right\}}=10^{-1.5}\left\{\mathrm{H}^{+}\right\} \\
& \log K_{a 2}=-1.5+\log \left\{\mathrm{H}^{+}\right\}=-6.5
\end{aligned}
$$

(c) The species to be considered include all those shown on the graph. We need to choose a compound in each oxidation state for each element as a component. We can choose these components somewhat arbitrarily, although if we are going to use a $\log c$-pe diagram to determine the equilibrium pe, it is most convenient to choose chemicals in the dominant oxidation states. We can identify to dominant oxidation states by imagining the inputs to consist of oxidized species and electrons, and then allocating the electrons sequentially to the available oxidants from strongest to weakest. From the $\log c-$ pe diagram, we can see that the dominant oxidation state for Cr changes from +VI to +III at pe $\sim 11.4$, and that for Fe changes from + III to + II at pe $\sim 8.9$. Based on these values, the dominant oxidation states at equilibrium are expected to be $\mathrm{Cr}(+\mathrm{III})$ and $\mathrm{Fe}(+\mathrm{III})$, as shown in the following table.

| Oxidized/ <br> Reduced | $\mathbf{p} \mathbf{e}_{\mathbf{p H}}$ | TOTi | Fraction <br> Reduced | $\boldsymbol{e}^{-}$Remaining |
| :--- | :---: | :---: | :---: | :---: |
| $\mathrm{Cr}(\mathrm{VI}) / \mathrm{Cr}(\mathrm{III})$ | 11.4 | $1.1 \times 10^{-3}$ | $0 \%$ | $3.0 \times 10^{-3}$ |
| $\mathrm{Fe}(\mathrm{III}) / \mathrm{Fe}(\mathrm{II})$ | 8.9 | $1.0 \times 10^{-3}$ | $01 \%$ | 0 |

The tableau for deriving the TOTe equation using $\mathrm{Cr}(\mathrm{III})$ and $\mathrm{Fe}(\mathrm{III})$ as components is shown below. Because we only need to know the values in the $e^{-}$column to write the TOTe equation, the tableu can be condensed by including all the species containing Cr or Fe in a given oxidation state in a single row.

|  | $\mathrm{H}_{2} \mathrm{O}$ | $\mathrm{H}^{+}$ | $e^{-}$ | $\mathrm{Fe}(\mathrm{III})$ | $\mathrm{Cr}(\mathrm{III})$ | Conc'n |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{H}_{2} \mathrm{O}$ | 1 | 0 | 0 | 0 | 0 |  |
| $\mathrm{H}^{+}$ | 0 | 1 | 0 | 0 | 0 |  |
| $e^{-}$ | 0 | 0 | 1 | 0 | 0 |  |
| $\mathrm{Fe}(\mathrm{III})$ | 0 | var | 0 | 1 | 0 |  |
| $\mathrm{Fe}(\mathrm{II})$ | 0 | var | 1 | 1 | 0 |  |
| $\mathrm{Cr}(\mathrm{VI})$ | 0 | var | 0 | 0 | 0 |  |
| $\mathrm{Cr}(\mathrm{III})$ | 1 | 0 | 3 | 0 | 1 |  |
| Inputs |  |  |  |  |  |  |
| FeCl | 2 | 0 | var | 1 | 1 | 0 |
| $\mathrm{NaHCrO}_{4}$ | 0 | var | -3 | 0 | 1 | $1.1 \times 10^{-3}$ |

The TOTe is a mass-based equation, but assuming that the solution behaves ideally, so that the activities of dissolved species can be equated with their molar concentrations, it can be written as follows:

$$
\begin{aligned}
& \text { TOTe }_{\text {in }}=\text { TOTe }_{\text {eq }} \\
& \left(3 \times 10^{-3}\right)-3\left(1.1 \times 10^{-3}\right)=\sum(\mathrm{Fe}(\mathrm{II}))-3 \sum(\mathrm{Cr}(\mathrm{VI})) \\
& 3 \sum(\mathrm{Cr}(\mathrm{VI}))=\sum(\mathrm{Fe}(\mathrm{II}))+3 \times 10^{-4}
\end{aligned}
$$

The equation is satisfied at approximately pe = 11.1 (Figure 3). At this pe, $\operatorname{TOTCr}(\mathrm{VI})$ is approximately $1 \times 10^{-4} M$ and $T O T F e(I I)$ is approximately $10^{-5}$, i.e., about $99 \%$ of the $\mathrm{Fe}(\mathrm{II})$ has been oxidized, reducing most of the $\mathrm{Cr}(\mathrm{VI})$. Since the $\mathrm{Fe}(\mathrm{II})$ dose was less than the stoichiometric amount required to reduce all the Cr , some hexavalent Cr remains.


