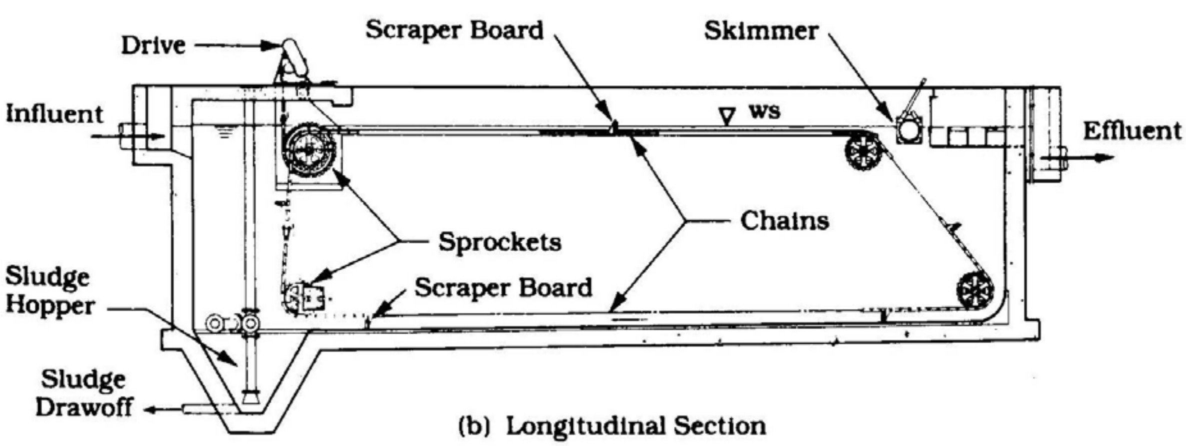
INTRODUCTION AND OVERVIEW

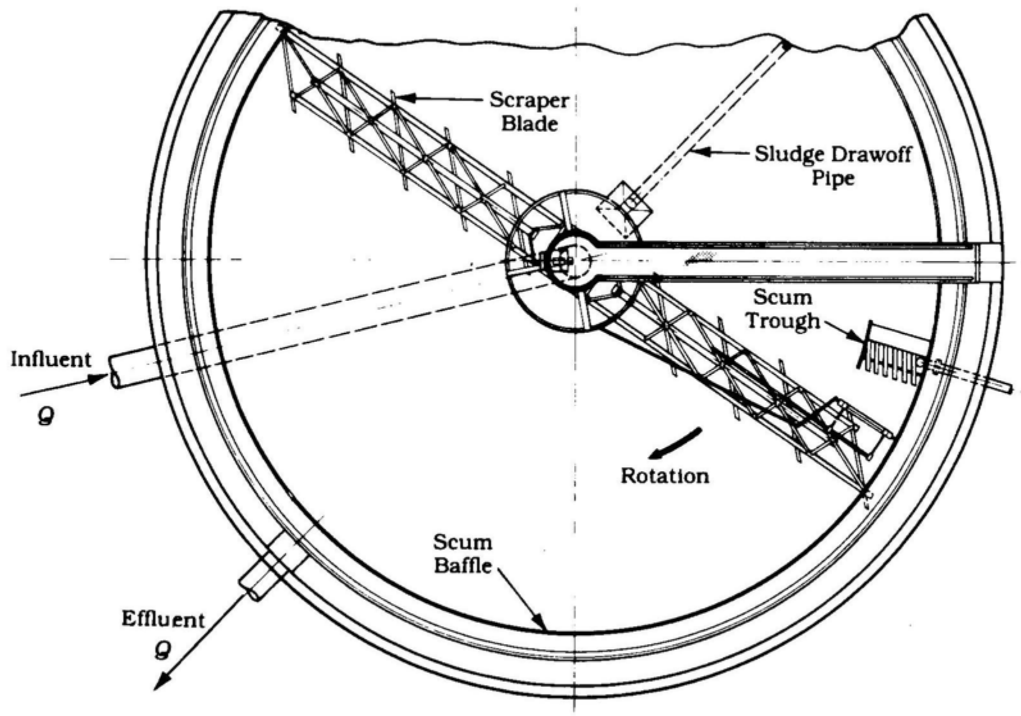
All natural waters and wastewaters contain particles with a wide range of physical and chemical characteristics. These particles can decrease the clarity of the water, and they may also have adverse health effects, for instance if they are pathogenic microorganisms or they contain heavy metals, pesticides, or other contaminants.

The simplest way to remove suspended particles is to allow them to settle out of the water, a process formally called *sedimentation*. Although sedimentation is a good technique for removing the larger and denser particles entering the sedimentation process, the time required for settling of particles with less favorable characteristics can far exceed the time available for settling in a water treatment plant (typically, one to a few hours). In those cases, it is sometimes possible to alter the physical and chemical conditions in the water to make the particles more likely to collide and stick to one another (*flocculate* or *coagulate*), so that they grow large enough to settle out. Even so, some particles cannot be made to "grow" to a settleable size, or the cost of causing them to do so would be prohibitive. In such a case, the most efficient removal approach is usually to filter them. In water treatment, passage of the water through a column packed with granular media such as sand is the most common filtration process, although filtration through membranes is rapidly gaining in popularity. Particles that are not removed by filtration enter the distribution system.

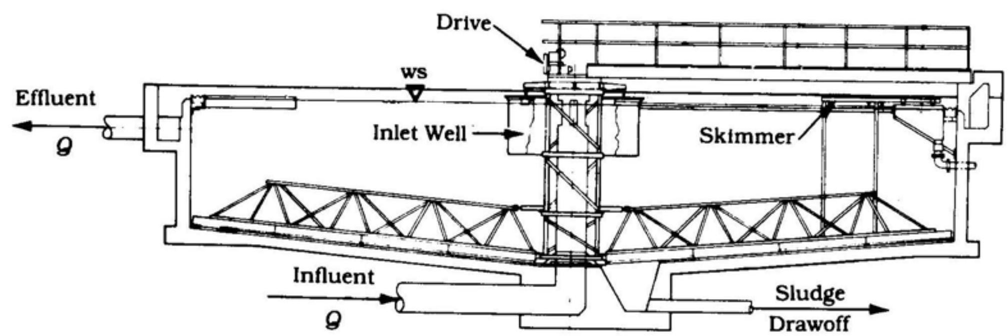
Diagrams showing the characteristics of some typical sedimentation tanks are shown in Figure 1. Figure 1a shows a rectangular tank with scrapers mounted on a continuous converyor. The scraper push the settled sludge to a hopper at the upstream end of the tank, from where it can be removed and processed further or disposed of. Figures 1b-d show the corresponding arrangements for circular tanks, which are somewhat more common in wastewater treatment than drinking water treatment.



(b) Longitudinal Section



(a) Plan



(b) Elevation

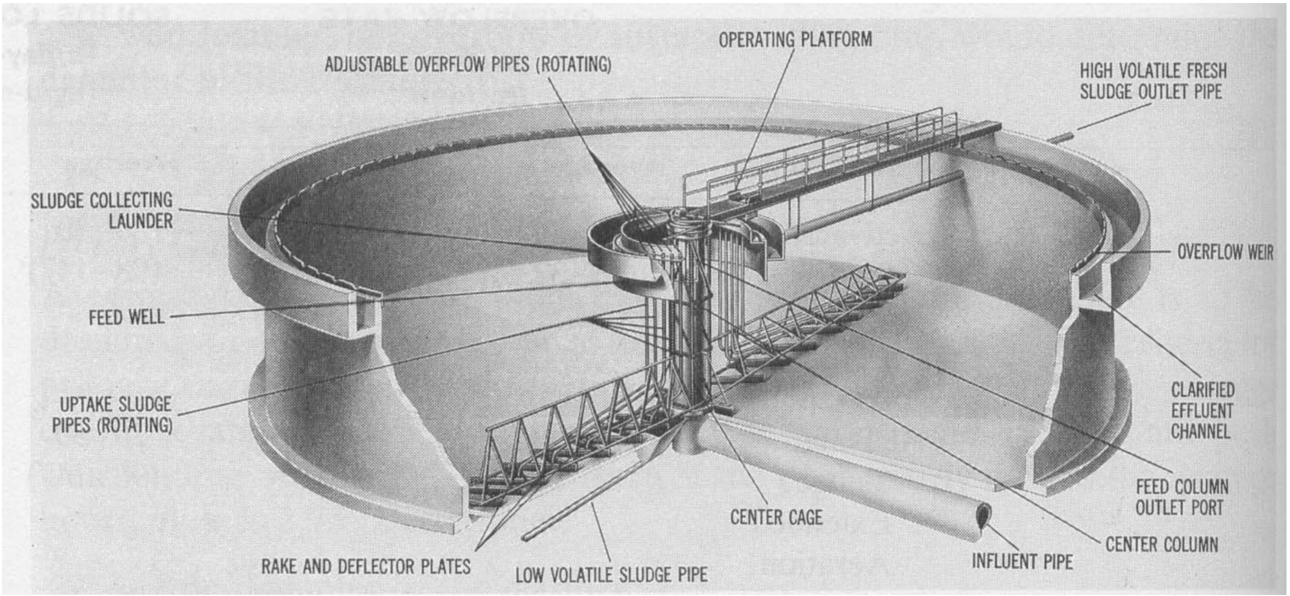


Figure 1. (a) Cross-section of a typical, rectangular sedimentation tank. (b) Plan view of a circular sedimentation tank (often called a clarifier). (c) Cross-section of a circular clarifier. (d) Cut-away view of a circular clarifier. (Parts *a*-*c* from Reynolds and Richards; not sure of original source for part *d*.)

For either type of tank, the optimal flow pattern is plug flow. In circular tanks, this pattern corresponds to radial flow from the center outward, or from the periphery inward, without any mixing between packets of water that entered at different times in the past. Such a flow pattern is desired, because it allows the greatest chance for particles to settle without being mixed vertically; since water near the upper surface gets clean before water that is deeper in the tank, any vertical mixing brings dirtier water upward from the bottom, while transporting cleaner water downward. Since the idea is to allow and encourage particles to move to the bottom of the tank, any process that brings more particles upward than downward is undesirable. Figure 2 shows that no tank behaves ideally, but rectangular tanks are more likely to approximate PFRs than circular tanks are, and circular tanks with peripheral feed are typically more PFR-like than circular tanks with feed entering the center.

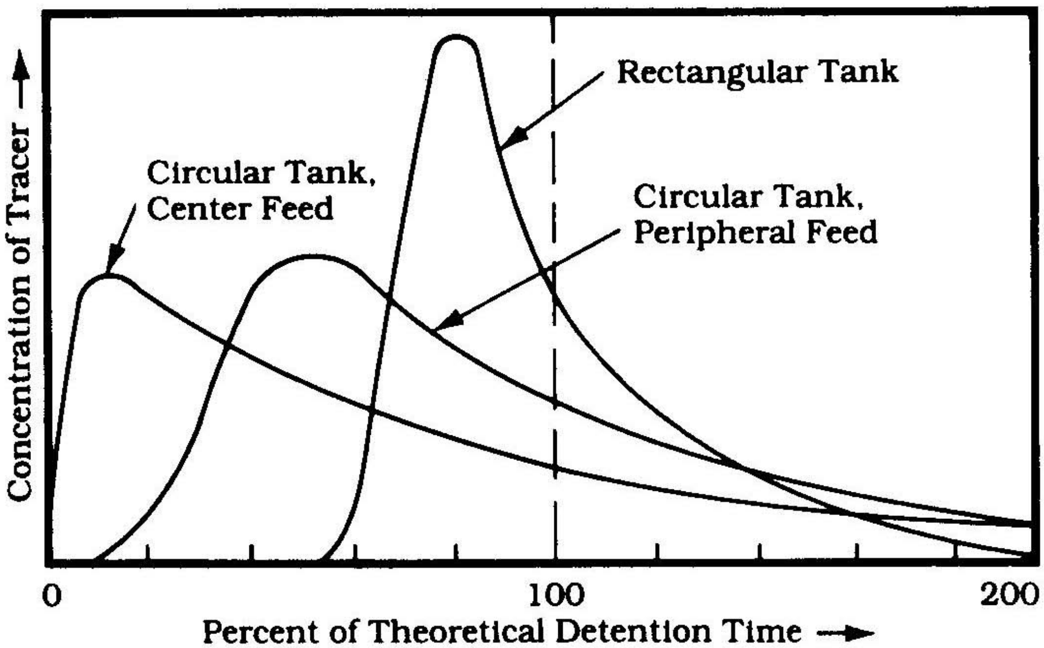


Figure 2. Typical results from a spike tracer test in sedimentation tanks. Plots show tracer concentration in the tank effluent as a function of time after a large mass of tracer is input at $t = 0$, and no tracer is injected thereafter. (From Reynolds and Richards.)

In this handout, we consider the characteristics of the kinds of particles that are typically encountered in drinking water treatment systems, and then assess how those characteristics might affect the design of a settling basin to meet a given treatment objective. Ways to facilitate flocculation and coagulation, and key features of filtration processes, are considered in subsequent units of the course.

Like all water treatment processes, the design and treatment objectives for sedimentation are established at least partly by the applicable regulations. Regulations regarding particulate matter are based on a variety of measurements. In wastewater, the solids are usually quantified based on mass concentration (mg/L) and are often reported in sub-categories that indicate something about their composition or size (*e.g.*, total and volatile suspended solids [TSS and VSS, respectively], filterable solids, etc.). In drinking water systems, the concentrations are usually much lower and are more often reported as turbidities or particle number concentrations in various size ranged.

Turbidity is a measure of the clarity of the water and is directly related to the tendency of the suspension to refract light. It is measured by passing a beam of light through the sample and determining how much light is refracted so that it is detected by a sensor at a 90^{o} angle to the incident beam. Turbidity is not a direct indicator of the total number or mass concentration of particles in the suspension, because the tendency to refract light is sensitive to particle size, shape, and transparency (e.g., refraction is greatest by particles that are of similar dimension as the wavelength of the light). However, turbidity *is* a good measure of what we normally perceive as clarity. Turbidity is reported in units defined historically that have no particular meaning outside of this context. The units are called *nephelometric turbidity units* and are almost always referred to by their initials, as NTUs.

Particle counting devices have been used for decades in research but only recently in water treatment practice. These instruments work by sensing either the amount of light blocked or the increase in electrical resistance of the solution in a small channel that, in theory, allows only one particle at a time to pass in front of the detector. Particle counts have been used in recent years to estimate the removal efficiency of particular types of microorganisms, based on the removal of all particles in the size range of those organisms.

The regulations applicable to generic particles in drinking water are specified in terms of turbidity; other regulations relate to specific types of particles (in particular, certain microorganisms) and are discussed in the unit on disinfection. No regulations apply to the sedimentation step *per se*; rather, the regulations apply to the water exiting the filters which follow sedimentation. Current regulations for treated surface water (based on the Long Term 1 Enhanced Surface Water Treatment Rule [LT1ESWTR], promulgated in 2002) are that the turbidity of "finished" (i.e., fully treated) water must be < 1.0 NTU in all samples and < 0.3 NTU in 95% of the samples in any given month (these regulations are referred to as a "turbidity performance standard," rather than an MCL). The primary benefit of efficient sedimentation is that it reduces the load of particles on the filters and therefore allows the filters to operate for longer periods and with less frictional energy loss between cleaning cycles.

TYPES, SIZES, AND DENSITIES OF PARTICLES IN WATER TREATMENT SYSTEMS

The sizes of particles typically found in drinking water systems and of various materials used to treat water are shown schematically in Figure 3. Although few of these items are expected to be circular, they are typically characterized as having a diameter. The diameter usually refers to either a characteristic length in one of the objects' dimensions, or the "equivalent spherical diameter," defined as the diameter of a sphere that would have the same volume as the object of interest. Note that many of the organisms of interest are in the size range of a few micrometers or less, with viruses being much smaller than bacteria or protozoans (like Giardia and Cryptosporidium).

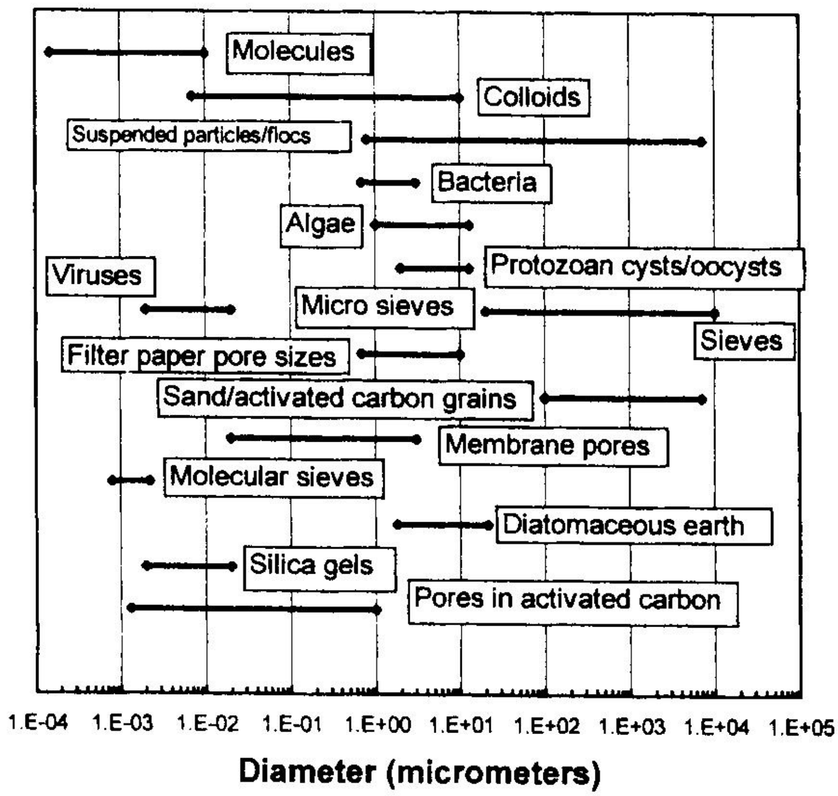


Figure 3. Sizes of some items of interest in water treatment processes. (From Reynolds and Richards.)

SETTLING BEHAVIOR OF IDEAL, WELL-CHARACTERIZED DISCRETE PARTICLES

The efficiency of sedimentation is related most directly to the settling velocity of particles, which in turn is related primarily to their size and density. In the following section, we consider the typical size and density ranges of particles commonly found in water, and how those properties relate to the settling velocity.

The primary factors causing motion of particles in water are collisions with molecules and other particles, fluid motion, and gravity. Collisions with other constituents of the suspension cause *Brownian motion*. The direction of Brownian motion is random and therefore neither helps nor hinders settling. In theory, the net effect of Brownian motion is to transport particles from regions of higher to lower concentration, and thereby generate a more uniform distribution of particles throughout the system. However, the length scale of Brownian motion is orders of magnitude smaller than that of a sedimentation basin, so the effect of Brownian motion on particle concentrations anywhere in a sedimentation tank is negligible. (Brownian motion can play a significant role in enhancing particle collisions, so it has an indirect effect on sedimentation, but we consider that effect separately.)

Fluid motion carries particles in the same direction as the fluid. In a typical sedimentation basin, fluid must be transported from the influent to the effluent, and it might also move in directions perpendicular to this net flow. For sedimentation, vertical movement is of particular interest. Because water is incompressible, any fluid that moves upward in a settling basin must be balanced by movement downward of other fluid. If the settling basin is working well, the particle concentration should increase with depth, so as noted previously, any vertical exchange of fluid carries "dirtier" water toward the top of the water column and cleaner water toward the bottom. Since particles have to settle to the bottom to be removed, this vertical mixing process undoes the beneficial effects of the settling that has already occurred and is therefore undesirable. For this reason, sedimentation basins in which the bulk flow is horizontal are invariably designed to have plug flow. Of course, that ideal is never fully met, but it is a design goal. (Some basins are designed to have bulk flow in the vertical direction; we will consider the reasoning behind that type of design later.)

The third factor causing motion of particles is gravity, or, more correctly, the net effect of gravity and buoyancy. It is this factor that we try to exploit to achieve good sedimentation. In the following discussion, we assume that Brownian motion is negligible compared to the motion caused by gravity and that the particles are settling either in a completely stagnant column of water (as in a batch settling test, with no flow in or out of the system) or in an ideal PFR with horizontal flow (in which case the water is moving relative to an outsider observer, but is stagnant with respect to the water upstream and downstream of it).

The basis for understanding the motion of discrete particles in such systems is a force balance. The fixed vertical forces on any suspended particle are the gravitational force downward (F_G) and the opposing buoyancy force (F_B). In addition, if the particle is moving relative to the surrounding water, a drag force (F_D) operates in the direction opposite from the direction of relative motion; if the particle is settling, the drag force is upward and hence is additive with the buoyancy force.

The magnitude of the drag force is proportional to the product of the cross-sectional area of the particle, the square of the particle's velocity relative to the fluid, and the drag coefficient, C_D. The drag coefficient is a function of the particle Reynolds number, $\text{Re} = \frac{d_p v_p \rho_L}{\mu_L}$, where the subscripts p and L refer to the particle and the liquid, respectively. For spherical particles with Reynolds numbers up to ~10^4, the drag coefficient can be approximated by $C_D \approx \frac{24}{\text{Re}} + \frac{3}{\text{Re}^{0.5}} + 0.34$. This expression represents an empirical fit to experimental data; several other, comparably accurate correlations have been proposed, so you might see other expressions for C_D in other literature. The key point is the trend – that the drag coefficient decreases with increasing Re, and, for a given fluid and particle, C_D is inversely proportional to v_p at low Re:[1]

[1] The dependence on Re and v_p becomes weaker as Re increases. At very large Re (~10^5), C_D becomes independent of Re; however, the constant value is not the one that would be computed using equation given here, because that equation is applicable only up to Re of ~10^4.

Sphere, low Re (less than ~0.1): $$C_D \approx \frac{24}{\text{Re}} = \frac{24\mu_L/(d_p\rho_L)}{v_p} = \frac{\text{constant}}{v_p} \qquad (1)$$

where the final equality applies for a given fluid and particle. The trends for other shapes of particles are similar, although the details vary. These trends are shown for three shapes in the following Figure 4.

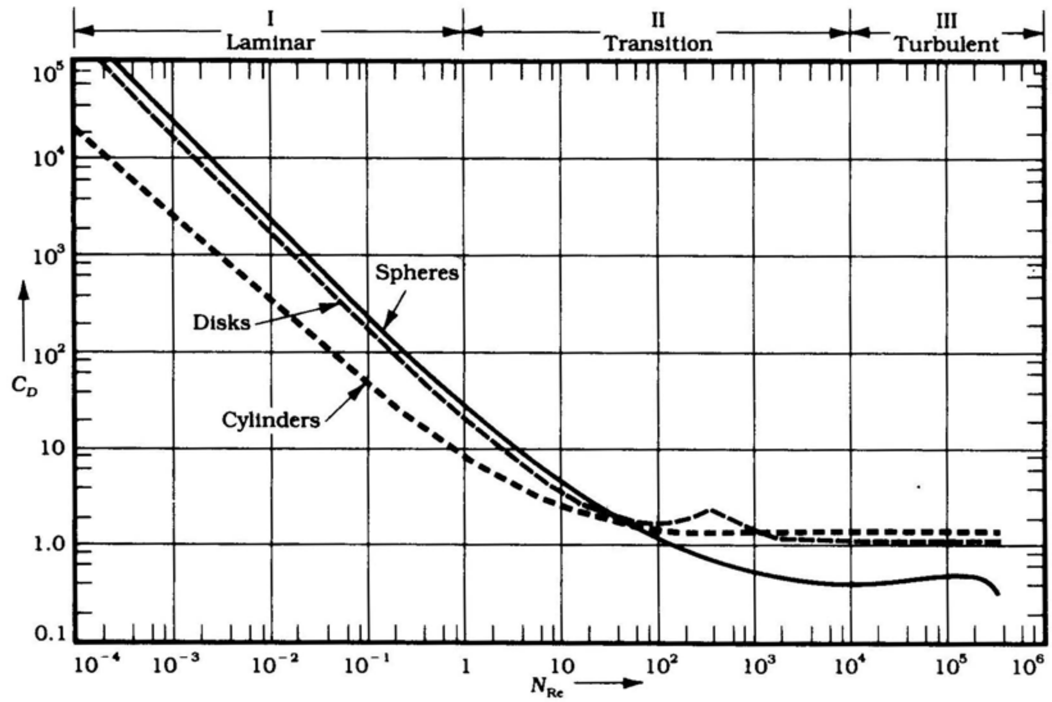


Figure 4. The drag coefficient (C_D) for particles of various shapes settling through water. N_{Re} is the particle Reynolds number, defined as $l_p v \rho_L/\mu_L$, where l_p is a characteristic length of the particle (diameter, for a sphere), v is the velocity of the particle relative to the water, and ρ_L and μ_L are the density and viscosity, respectively, of the water.

For particles of interest in sedimentation, the Reynolds number is always small, though not necessarily < 0.1. We will investigate the range of typical Reynolds numbers after we get a feel for reasonable values of v_p.

The drag force on a particle is given by:

$$F_D = C_D A_p \rho_L \frac{v_p^2}{2} \qquad (2)$$

where A_p is the cross-sectional area of the particle. As a result, if the Reynolds number for a particular, spherical particle is in the range where C_D can be approximated by 24/Re, the drag force on the particle is proportional to v_p:

$$F_D \approx \frac{24\mu_L/\left(d_p\rho_L\right)}{v_p} A_p\rho_L \frac{v_p^2}{2} = \frac{12\mu_L A_p}{d_p} v_p = 3\pi\mu_L d_p v_p \qquad (3)$$

If particles are, initially, uniformly distributed in suspension and then begin to settle, they will have zero velocity at $t = 0$, and therefore experience zero drag. The net force on them at that time will be the difference between the gravitational force and the buoyant force, and this force will lead to a corresponding acceleration:

$$F_{net,t=0} = F_G - F_B = ma \qquad (4)$$

Assuming that the particles are more dense than the fluid, they will settle. However, as they accelerate, the drag force increases, opposing the net downward force and reducing the rate of acceleration. Thus, at all times $t > 0$, the net downward force on the particle is:

$$F_{net,t>0} = F_G - F_B - F_D = ma \qquad (5)$$

$$\frac{\pi}{6} d_p^3 g\left(\rho_p - \rho_L\right) - C_D A_p \rho_L \frac{v_s^2}{2} = \left(\frac{\pi}{6} d_p^3 \rho_p\right)\frac{dv_p}{dt} \qquad (6)$$

where v_p has been rewritten as v_s to indicate that the force balance applies to a particle that is settling, so that only the vertical component of the drag force is relevant. (If the particles are less dense than the fluid, the same balance applies, but the drag force is in the opposite direction, so it would be added to rather than subtracted from F_G.) Over time, the gravitational and buoyant forces remain constant, but the drag force increases. Eventually, the sum of the drag force and the buoyant force equals the gravitational force, so the net force and associated acceleration become zero.

$$\frac{\pi}{6} d_p^3 g\left(\rho_p - \rho_L\right) - C_D A_p \rho_L \frac{v_s^2}{2} = 0 \qquad (7)$$

The particle velocity at this point is called the *terminal settling velocity*, v_{term}. For particles typical of those found in water supplies, v_{term} is reached within a very short time (<<1 s), so the assumption is generally made that particles attain their terminal velocity instantly and that $v_s = v_{term}$ for the whole time that the particles are settling. For many particles, v_{term} is small enough that the particle Reynolds number is <0.1. In that case, if the particle is approximately spherical, the approximation C_D = 24/Re applies, and Equation 7 can be solved to yield:

$$v_{term} = \frac{d_p^2 g\left(\rho_p - \rho_L\right)}{18\mu} \qquad (8)$$

Equation 8 is known as Stokes' Law. Using it, one can compute the terminal velocity of small, spherical particles settling in water. Although in some cases the assumptions of the derivation are clearly not met, Stokes' Law does provide a useful framework for analyzing sedimentation.

With it, if we knew the density, size, and shape of most particles in the suspension, we could determine how fast they would settle, and we could then use this information to design systems to remove the particles. Particle settling velocities computed according to Stokes' Law are shown by the straight lines in the "laminar" region of Figure 5.

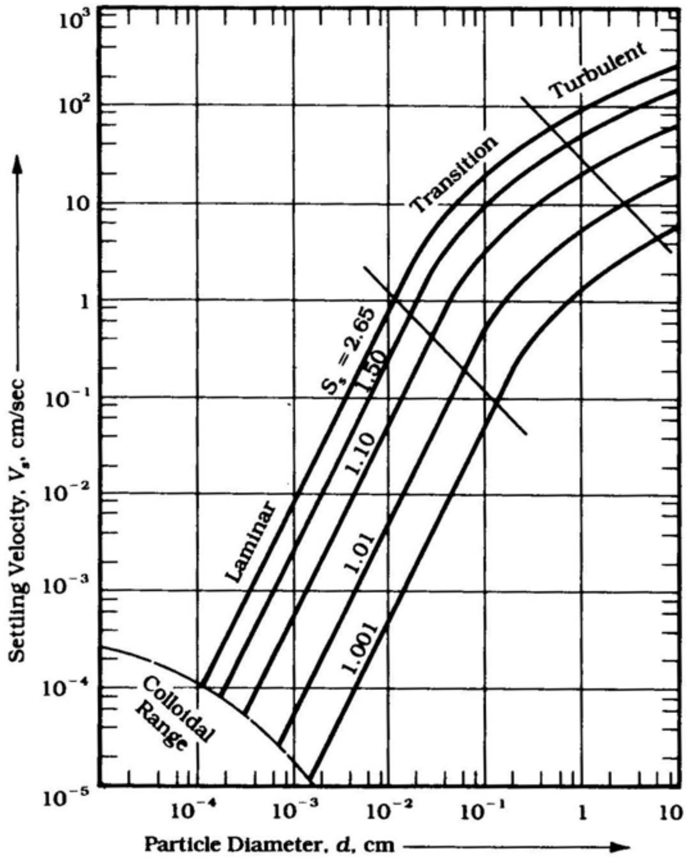


Figure 5. Settling of particles of various diameters in water. The numbers on the different curves correspond to different specific gravities. Stokes Law applies in the linear region; the curves in the other regions are based on more complicated correlations.

CHARACTERIZING SETTLING BEHAVIOR OF A CONTINUOUS DISTRIBUTION OF PARTICLES

The preceding discussion describes how one might utilize information about the physical properties of particles (size, density, etc.) and the water to determine the particles' settling velocities. Hypothetically, one could evaluate this information for all particles in the system to characterize the complete distribution of particle settling velocities. Although the approach describes useful relationships, we virtually never have enough information about individual particles in the system to carry out such an analysis. Rather, what we typically have is a sample of water containing particles with a more or less continuous distribution of sizes and densities. In this section, we consider how we might conduct simple tests to characterize the distribution of settling velocities directly from settling tests, without knowing particle sizes, densities, etc.

One difficulty associated with characterizing these distributions is that the particles sometimes coagulate during the text, so that the particle size distribution is not stable. A similar process occurs in full-scale settling basins: the particle size distribution changes during the time the water is being treated. Such changes are expected in most settling processes, since the particles are usually subjected to upstream processes that are intended to facilitate coagulation, and the coagulation process is not necessarily complete by the time the water enters the sedimentation basin. Sedimentation of particles that do not coagulate as they settle is referred to as *discrete particle settling* or *Type 1 sedimentation*, and sedimentation with simultaneous coagulation is referred to as *flocculant settling* or *Type 2 sedimentation*. Both types of settling can be analyzed by the same procedure, but some simplifications that apply to Type 1 sedimentation make it easier to focus on that category first, so that is how we will proceed.

The terminal velocity distribution of a suspension of particles is typically characterized by conducting a column settling test. In such a test, the well-mixed suspension is placed in a column, and at time $t \leq 0$, all the particles are assumed to be uniformly distributed. Then, at $t = 0$, stirring ceases and we begin measuring particle concentration vs. time at several sampling ports.

Consider what will happen if we have a distribution of Type 1 (non-coagulating) particles in the water. For example, consider a system with four discrete particle groups. All the particles are evenly distributed at $t = 0$, *i.e.*, the concentration of each particle i is $c_{init,i}$ throughout the column initially. Since all particles reach their terminal settling velocity instantly, all the particles with a given settling velocity fall the same distance. Specifically, if the settling velocity of a group of particles i is $v_{s,i}$, the distance they have settled at time t is $v_{s,i}t$.

For instance, imagine the numbers in Figure 6a represent the settling velocities, in cm/min, of four types of particles. If we look at the column 1 min later, the distribution will be as shown in Figure 6b. All the type-1 particles have fallen 1 cm, all the type-2 particles have fallen 2 cm, etc. As a result, the concentration of each type of particle at any time and at any height in the active settling portion of the column is either its initial concentration, $c_{init,i}$, or 0. Correspondingly, if we plot the concentration of total particles versus height in the column, we will get a curve with four steps. At the top of the column the total concentration of particles will be zero, since all the particles have fallen out of this region. The concentration will remain zero for some distance down from the top. However, at some point we will encounter the slowest moving particles (those identified as type-1 in the drawing). At that height, the total concentration of particles in the water will be the initial concentration of type-1 particles. If we continue moving down the column of water, we will later encounter type-2 particles as well. The total concentration of particles will then be the initial concentration of type-1 plus type-2 particles. This process is repeated until eventually we get to a point where all four types of particles are present at their initial concentrations, and the total particle concentration is the same as the initial total concentration; the concentration remains at that value until the bottom of the active settling zone.[2]

[2] In this analysis, we ignore the very bottom of the column, where the particles that have fallen out of the settling zone accumulate.

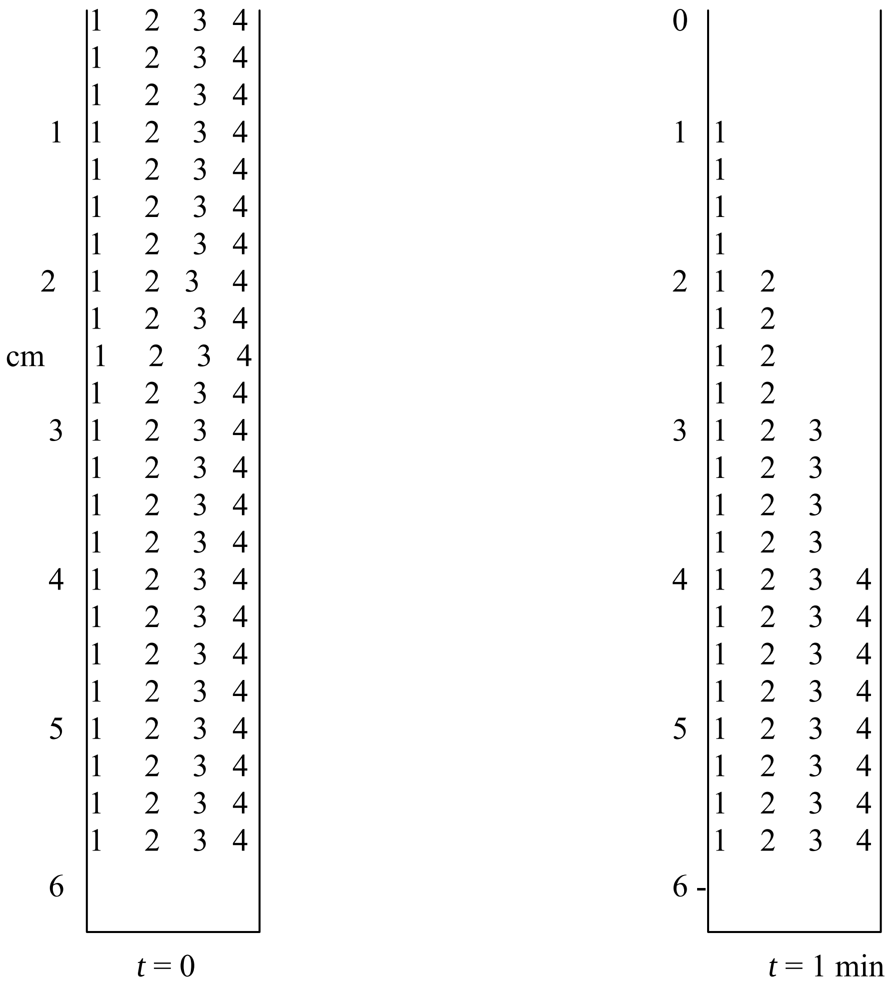


Figure 6. Settling of a suspension containing four types of particles with different terminal velocities.

To make this example a bit more concrete, assume that the initial concentrations of particles were 10,000/L of type-1, 30,000/L of type-2, 40,000/L of type-3, and 20,000/L of type-4. The particle distribution after settling time *t* might be as shown in Figure 7.

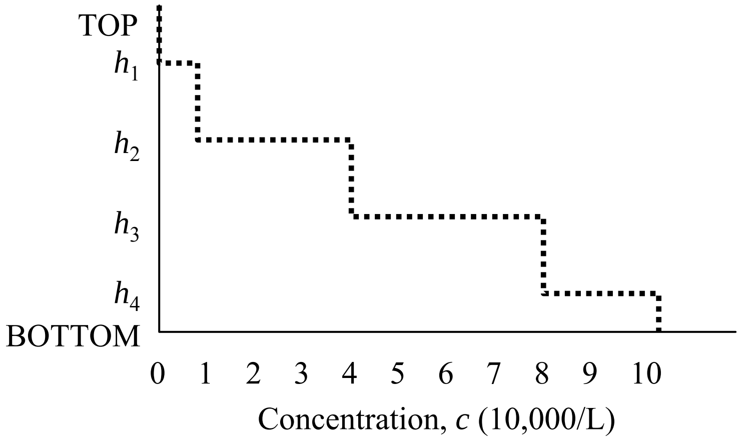


Figure 7. Particle concentration profile for the suspension shown in Figure 6 after some settling time *t*, if the particles have settling velocities of 1, 2, 3, and 4 cm/min, respectively; $h_i = v_{s,i}t$.

If we divide h_1 (the distance fallen by Type 1 particles in time t) by the time of settling (t) and designate this ratio as v_1, we can conclude that 100% of the particles have settling velocities greater than or equal to v_1. This conclusion is based on the fact that no particles remain above h_1 at this time, so all the particles (even those that were right at the top of the column at $t = 0$) must have had terminal velocities that allowed them to settle at least h_1 in time t. Similarly, designating (h_2/t) as v_2, we see that 90% of all the particles in the initial suspension have settling velocities greater than or equal to v_2, and 10% have velocities less than v_2. We could continue this process throughout the column. To generalize the approach: at any height, we could measure the concentration of particles, divide by the initial total concentration of particles, and this fraction would represent the fraction of particles with settling velocity less than h/t, where h is the distance from that point to the top of the column, and t is the time of sampling. Since, in a typical natural or waste water, there will be a wide range of particle sizes and densities, the actual curve will be smooth and continuous rather than a series of steps, as shown in Figure 8 (note that the vertical axis on the diagram represents the fraction of particles with settling velocities *less than* the value on the horizontal axis, so it goes from 0 to 100%).

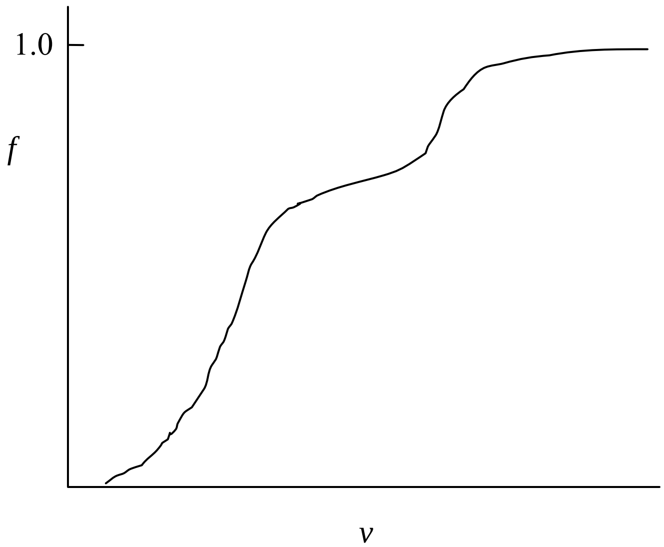


Figure 8. A plausible shape for a plot of *f* vs. *v*, where *f* is the fraction of the particle mass with settling velocity less than *v*.

Note that we can develop this curve at any arbitrary time of settling and any arbitrary depth of settling, and it will be the same. For instance, if we allowed the test to run for twice as long before analyzing samples at various depths, the new values of h_1, h_2, h_3, and h_4 would be twice as great as in the initial test, since all of the particles would have settled through twice as great a distance. Dividing these distances by the total time (twice the initial value) would yield exactly the same distribution of settling velocities.

Now consider how we might plot the same data in a slightly different way. Say we wish to plot an *isopleth* of constant percentage removal on a graph of height vs. time. That is, we wish to connect all the combinations of height and time at which a given percentage removal is achieved. To make the issue more concrete, say we wish to plot the (t, h) data corresponding to 85% removal. If 85% of the particles fall at velocities of at least, say, 1 cm/min, then 85% removal will be achieved at a depth of 1 cm after 1 min, 2 cm after 2 min, etc. Thus, the 85% removal isopleth will start at (0, 0) and will be a straight line with slope equal to 1 cm/min. Similarly, if 70% of the particles fall at a rate greater than 1.5 cm/min, then the 70% isopleth will start at the same point, (0, 0), and will be a straight line of slope 1.5 cm/min. Extending this reasoning, a plot showing six isopleths is provided in Figure 9.

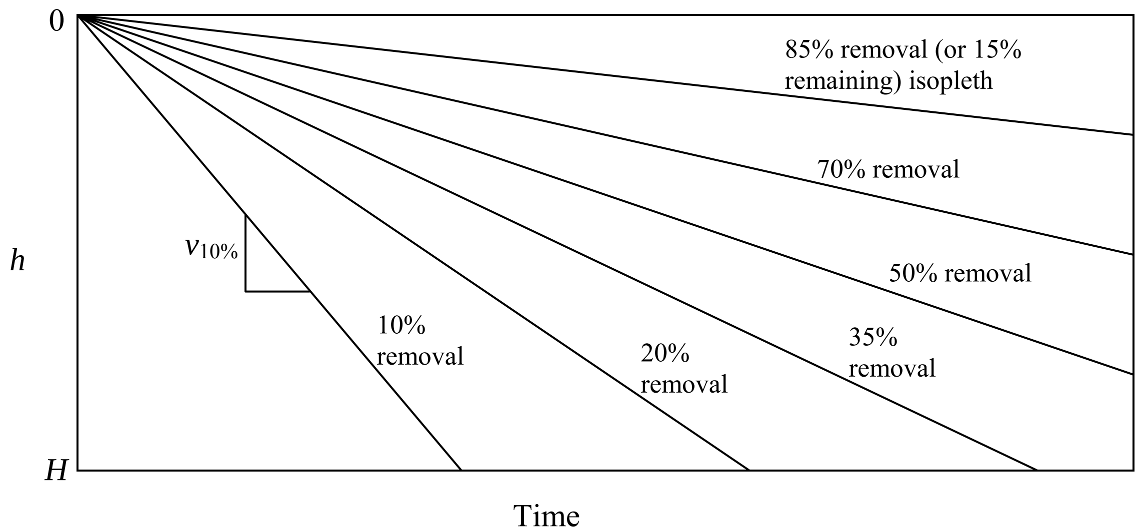


Figure 9. Isopleths of constant percentage removal in a batch test with a Type 1 suspension. The slope of each isopleth corresponds to a velocity, and the specified percentage of the mass of suspended particles has a settling velocity greater than that slope.

As noted, the data in Figures 8 and 9 are redundant, in the following sense. If 70% of the particles have settling velocities greater than 1.5 cm/min, then the point (1.5 cm/min, 0.3) is on the curve in Figure 8. Correspondingly, the slope of the 70% removal (or, equivalently, 30% remaining) isopleth in Figure 9 is 1.5 cm/min. Because 70% of the particles have settling velocities greater than 1.5 cm/min *at all times during the test*, each straight line in Figure 9 is represented by a single point in Figure 8, so Figure 8 is more compact. The implication is that the single curve in Figure 8 applies at all times during the settling test. On the other hand, Figure 9 makes the point more explicitly that the trajectories of individual particles are linear, and the plane separating two different groups of particles in a settling test falls at a steady rate during the test. In the subsequent analysis, we will find both representations of the data to be useful.

Note also that the results shown in Figure 9 apply specifically because we have assumed that particles attain their terminal velocity instantly and do not interact with one another thereafter. Thus, the same results would *not* apply to Type 2 suspensions, in which particles collide and combine as they settle. (In that case, the velocity of a given particle would probably increase when it collided with and stuck to another particle, so the distance fallen would not be directly proportional to the time of settling. We will need to take this particle acceleration into account when we consider settling of Type 2 suspensions.)

DESIGN OF SEDIMENTATION FACILITIES FOR DISCRETE-SETTLING PARTICLES

We next consider how we can use the information generated from a settling column test, such as described above, to design a settling basin. As noted previously, in an ideal settling basin with horizontal flow, the water moves in a perfect plug-flow manner from the inlet of the basin to the outlet. Assume, for the time being, that the basin is rectangular (dimensions $L \times W \times H$). The water enters at one end and travels without mixing through the distance L in time t_d (the hydraulic detention time, equal to V/Q). The horizontal velocity of the water is therefore $v_w = L/t_d$.

Now think about what happens to particles that enter with the water. When the water enters, the concentration of particles at the inlet end of the basin is assumed to be uniform across the width and height of the tank. However, as the water moves through the tank, particles settle out of the top layers of water, and some particles settle all the way to the bottom. Mechanically, the tanks are designed in such a way that the material that reaches the bottom is either vacuumed into a pipe or scraped into a collection bin and subsequently sent to a waste disposal process. Although there is a possibility in a real tank that particles that have reached the bottom of the tank will be resuspended, we assume for now that if a particle falls all the way to the bottom of the tank, it is permanently removed from the suspension. Our goal then, is to determine, for a given distribution of influent particle settling velocities, how we should design the tank so that the desired fraction of the influent particles hits the bottom before the water exits.

If particles reach their terminal velocity instantly, then from the moment they enter the tank they fall vertically with a velocity $v_{s,i}$ (different for each distinct group of particles i), while moving horizontally with the water at velocity v_w. Because, for any individual particle, the horizontal and vertical velocities are constant for the entire time it is in the basin, each particle follows a straight-line trajectory through the tank. Heavier particles follow a steep trajectory, and those that are only slightly more dense than water follow a very shallow trajectory, but regardless of their characteristics, the trajectories of individual particles are linear.

In fact, thinking back to the column settling test, the behavior of the water and particles as they move through the basin is identical to that in the column over time. That is, if we had put a batch of the influent into a column and let the particles settle for a time t_d, the distribution of particles in the column would be identical to the distribution of particles in the effluent from the tank. This is because we have assumed that the flow pattern in the tank is plug flow, and plug flow implies that a column of water entering a tank moves through the tank completely unaffected by the water in front of, behind, or around it.

Because particles have a constant settling velocity while they are in the basin, particles with settling velocity $v_{s,I}$ will fall a distance $h_{s,I}$ equal to either $v_{s,i}t_d$ or H (the total height of the water column) while they are in the basin, whichever is smaller. Knowing how far a particle falls, we can compute the likelihood that it will hit the bottom before it exits the tank, and what fraction of the exiting water column will contain (or not contain) particles with a particular settling velocity.

Consider first particles that enter at the very top of the water column. These particles have the farthest to fall of any influent particle; to be removed, the particles must fall the full height of the water column, H. If the computed, potential fall distance $h_{s,i}$ (*i.e.*, $v_{s,i}t_d$) of a particular particle is $\geq H$, then such a particle will be removed no matter where in the water column it enters. That is, if a settling basin is 2 m deep, and a certain type of particle has a terminal velocity that would cause it to fall 2.2 m in the time the water is in the basin, we would not expect to see any such particles in the effluent from the tank. For a particle to fall a distance $\geq H$ in time t_d, it must have

$v_{s,i} \geq H/t_d$. The velocity H/t_d is called the critical velocity v_{crit}. Thus, we can state that the removal efficiency η_i for particles that have $v_{s,i} \geq H/t_d = v_{crit}$:

$$\text{If } v_{s,i} \geq v_{crit}: \qquad \eta_i = 100\% \tag{9}$$

Since t_d can also be written as $V/Q = (LWH/Q)$, the critical velocity can also be expressed as the ratio of the flow rate to the surface area of the water in the tank:

$$v_{crit} = \frac{H}{V/Q} = \frac{H}{LWH/Q} = \frac{Q}{LW} = \frac{Q}{A} \tag{10}$$

where A is the horizontal surface area (top or bottom) of the tank. Because of the relationship shown in Equation 10, the critical velocity is often referred to as the *overflow rate* (sometimes designated as *O/F*). The overflow rate is expressed using dimensions of flow rate (Q) per unit surface area (A), such as cubic meters per hour per square meter, gallons per minute per square foot, or million gallons per day per square foot. These units (especially the latter two sets) are somewhat non-intuitive ways of describing a velocity; keep in mind that they are simply a different way of describing the critical velocity, H/t_d. Expressing the critical velocity as the ratio of the flow rate to the plan area is convenient at times, because often one must design for a given flow rate which cannot be changed. Use of the overflow rate as the design variable emphasizes that, in such a case, the critical velocity is determined by the horizontal surface area of the tank, and that any shape of tank giving the same surface area will have the same critical velocity.

Next consider a particle with settling velocity less than the critical velocity. If such a particle enters at the top of the water column, it will not have hit the bottom of the tank by the time the water exits the tank, and hence it will not be removed. However, if it enters the tank near the bottom, it may still be removed. What fraction of such particles will in fact hit the bottom before the water exits?

Because a particle i will fall a distance $h_{s,i} = v_{s,i}t_d$ while in the tank, if it enters the tank at a height less than $h_{s,i}$ above the bottom, it will be removed. For instance, a particle that falls 0.6 m in a tank of total height 2 m will be removed if it enters the tank within 0.6 m of the bottom. However, if it enters at a point 0.7 m above the bottom, it will still be 0.1 m above the bottom when the water exits, and will therefore be in the effluent water.

Based on our assumption that the particles are uniformly distributed in the influent, the fraction of the particles which are in the bottom 0.6 m of a 2 m water column at the basin inlet is simply $0.6/2 = 0.3$. Thus, for this example, we would expect 30% of each group of particles in the influent to be in the bottom 0.6 m of the influent water column. Correspondingly, we would expect 30% of the particles that fall 0.6 m while in the tank to hit the bottom of the tank before the water exits. Similarly, we would expect the top 30% of the water column to be devoid of these types of particles, and the bottom 70% to contain the particles at the same concentration as in the unsettled influent. Hence, we would expect the removal efficiency for those types of particles to be 30%. Extending this analysis to all particles, we see that the likelihood that a given type of particle will be removed in a settling basin (assuming it has a fall distance less than

H in the basin) is the ratio of the distance it falls in the tank divided by the total height of the water column, $h_{s,i}/H$. Dividing the numerator and denominator by t_d, we conclude that the likelihood of removal, η_i, is $v_{s,i}/v_{crit}$.

$$\text{If } v_{s,i} < v_{crit}: \qquad \eta_i = \frac{h_{s,i}}{H} = \frac{h_{s,i}/t_d}{H/t_d} = \frac{v_{s,i}}{v_{crit}} \tag{11}$$

Condensing Equations 9 and 11, we can write:

$$\eta_f = \begin{cases} 100\% & \text{if } v_s \geq v_{crit} \\ v_s / v_{crit} & \text{if } v_s < v_{crit} \end{cases} \tag{12}$$

We can now combine our conclusions for both groups of particles which we have considered. If particles have $v_{s,i} > v_{crit}$, their likelihood of removal is 100%, and if particles have $v_{s,i} < v_{crit}$, their likelihood of removal is $v_{s,i}/v_{crit}$. The distribution of particle settling velocities is a characteristic that describes the influent to be treated; for now we will consider it to be beyond our control. The critical settling velocity, being a ratio of the tank height to the hydraulic residence time (or, equivalently, the flow rate to the plan area), is completely under the control of the designer. Thus, the relationships which we have developed between $v_{s,i}$ and v_{crit} provide the key link connecting influent conditions to process performance for individual particles. We next expand the analysis so that we consider the whole distribution of particles in the influent, rather than just the particles with a particular settling velocity.

Several approaches exist for carrying out this expanded analysis. However, they are all related in that they involve determining the removal efficiency for small sub-groups of particles and then carrying out a weighted summation of those values to obtain the overall average removal efficiency. This procedure will work for any choice of how to designate the sub-groups, as long as the sub-groups that are considered account for all the particles in the influent to and effluent from the system. This point is perhaps best understood by an analogy. If we wanted to compute the average height of all Americans, we could find the average height of the population in each state, and then compute a weighted average of those average heights, weighted according to the population of each state. Alternatively, we could determine the average height of all people in the country between 0-10 years old, 10-20 years old, 20-30 years old, etc. We could then take a weighted average of these average heights, weighted according to the population in each decade of ages. The results of these two analyses would be identical, as long as both analyses took into account everyone in the country. It is easy to imagine lots of other ways of computing the global average height, and those ways would also yield the same result, if they considered all people in the country.

Similarly, the average particle removal efficiency by sedimentation can be determined by a variety of approaches, all of which will yield the same result if they consider all the particles in the system. Two or three such approaches are commonly used, and we develop those approaches next. (Two different ways of computing the removal efficiency derive from the same starting point, so they might be considered to be different approaches or two versions of a single approach.)

Approach 1: Averaging based on the f vs. v_s curve. Consider a water sample with an initial particle concentration c_0 equal to 15 mg/L, with the particle settling velocity distribution shown in Figure 10. The values on the abscissa (*x* axis) are velocities (v), and the values on the ordinate (*y* axis) represent the fraction (f) of the particles in the suspension that have terminal settling velocities less than or equal to v. Thus, a fraction f_1 has settling velocities less than v_1. The fraction might be based on either the number of particles or their mass. That is, in the latter case, f would represent the mass of suspended particles that have settling velocities less than v, divided by the mass of all particles in the suspension.

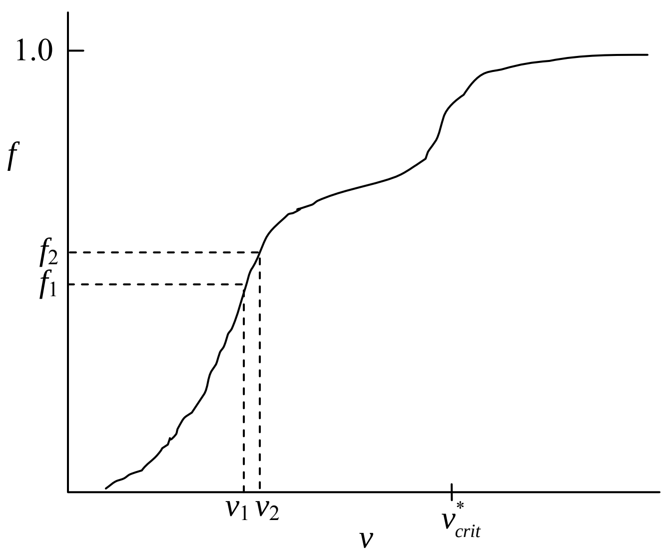


Figure 10. Hypothetical settling velocity profile for the influent to a settling basin. The value on the ordinate, *f*, is the fraction of particles with terminal settling velocities less than or equal to the value of *v* shown on the abscissa.

We can determine the particle removal efficiency for this suspension in a sedimentation basin with a given critical settling velocity v_{crit} $(= Q/A)$, by computing the removal efficiency for a small group of particles, and then extending the analysis to consider all particles. For instance, assume that the values marked f_1 and f_2 in the figure are 0.44 and 0.48, respectively. In that case, the band between the two horizontal broken lines represents 4% of the total concentration of particles in the system. The concentration of particles in this group is $c_0\,\Delta f$, or $(15\text{ mg/L})(0.48-0.44) = 0.60\text{ mg/L}$. The vertical lines drawn from the points where these horizontal lines intersect the settling velocity distribution curve indicate that these particles have settling velocities between v_1 and v_2. Call the average settling velocity of the particles in this group v_{avg} $(= [v_1 + v_2]/2)$.

Say we have tentatively picked a design value of v^*_{crit}, and that for the given 4% of the particles, $v_{avg}/v^*_{crit} = 0.50$. According to Equation 11, for any group of particles with $v_s < v_{crit}$, the removal efficiency is v_s/v_{crit}, so for this group $\eta_i = 0.50$. Thus, (0.50) (0.60 mg/L), or 0.30 mg/L, will be removed. Generalizing this idea, for any group of particles, $c_{removed,i} = \eta_i\left(c_0 \Delta f_i\right)$.

If we divided the whole mixture of particles into small groups, each representing 4% of the particles, and repeated the above process for each group of particles, we could find the concentration removed for each group, and therefore the overall concentration removed. That is, the total concentration of particles removed would be:

$$c_{removed} = \sum_{f_i=0}^{1} \eta_i \left(c_0 \Delta f_i\right) \tag{13}$$

where η_i is the average removal efficiency of particles corresponding to the given Δf_i. The overall efficiency $\eta_{overall}$ is $c_{removed}/c_0$, so, dividing both sides by c_0:

$$\eta_{overall} = \frac{c_{removed}}{c_0} = \sum_{f_i=0}^{1} \eta_i \, \Delta f_i \tag{14}$$

If we make Δf_i differentially small, Equation 14 can be written as:

$$\eta_{overall} = \int_0^{1.0} \eta_i df_i \tag{15}$$

According to Equation 12, η_i is expressed by different functions depending on whether $v_{s,i}$ is less than or greater than v_{crit}. Therefore, it is convenient to split the integral in Equation 15 into two portions – one for each range of f_i where a single expression applies. To do this, we define f^* as the fraction of particles with $v_{s,i} < v_{crit}$; correspondingly, the fraction with $v_{s,i} > v_{crit}$ is $1 - f^*$. Using this definition, we can rewrite Equation 15 as follows:

$$\eta_{overall} = \int_0^{f^*} \eta_i df_i + \int_{f^*}^{1} \eta_i df_i \tag{16}$$

$$= \int_0^{f^*} \frac{v_s}{v_{crit}} df_i + \int_{f^*}^{1} \left(100\%\right) df_i$$

$$= \int_0^{f^*} \frac{v_s}{v_{crit}} df_i + \left(1 - f^*\right) \tag{17}$$

Rewriting Equation 17 without the subscripts *overall* and *i*:

$$\eta = \int_0^{f^*} \frac{v_s}{v_{crit}} df + \left(1 - f^*\right) \tag{18}$$

The subscripts are included in Equations 14 -17 to emphasize that η_i is the removal efficiency of a specific sub-group of particles, as opposed to $\eta_{overall}$ (or just η), which is the removal efficiency for the overall suspension, considering the combined removals of all the different groups of particles.

Equation 18 is the key result that allows us to compute the expected, overall removal efficiency for particles in a settling basin, for a system that meets the assumptions of the derivation (uniform influent distribution, instant attainment of terminal velocity, no flocculation, no resuspension from the bottom of the tank). What it says is that if we pick a value of v_{crit}, we can look at the settling velocity distribution and determine the corresponding value of f^*, the fraction of the particles that settles slower than v_{crit}. Each group of particles represented by the range $0 \le f < f^*$ has some average settling velocity v_s and will be removed with an efficiency equal to v_s/v_{crit}. In addition, the particles that settle faster than v_{crit} are represented by the range $f^* \le f \le 1.0$; these particles will be removed with 100% efficiency. By knowing how many particles are in each group, we can calculate the overall removal efficiency.

The information about how many particles are in each group (*i.e.*, how many particles have settling velocities in a small range of values) comes from the settling velocity distribution curve. We could do the analysis numerically, one discrete group at a time, or use the above integral to evaluate the overall result. As is shown next, the integral can be evaluated graphically quite easily, so that is an approach that is commonly taken.

Consider a rectangular space on the settling velocity distribution graph bounded on the abscissa by the values $v = 0$ and $v = v_{crit}$, and on the ordinate by $f = 0$ and $f = 1$. This area is the sum of areas *I*, *II*, and *III* in Figure 11. The vertical line $v = v_{crit}$ cuts the settling velocity distribution curve at a value of $f = f^*$. The area of the complete rectangle is $(v_{crit})(1.0)$, or v_{crit}.

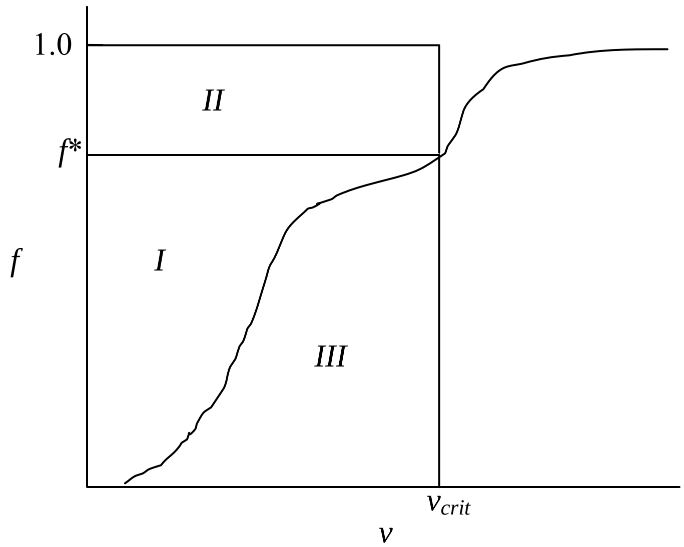


Figure 11. Graphical analysis of a settling curve to compute η.

Now consider the portion of this space which is above the curve. This area can be divided into two parts. The first part is the rectangle above f^* (labeled as area *II* on the diagram) and has an area $(1-f^*)v_{crit}$. The area above the curve but below f^* (*i.e.*, area *I*) does not have a simple shape, but it can be represented mathematically by $\int_{y=0}^{y=f^*} x\,dy$, or, for the specific parameters of interest, by $\int_0^{f^*} v\,df$. The sum of areas *I* and *II* is thus:

$$\text{Area}\left(I+II\right)=\int_0^{f^*} v df+\left(1-f^*\right)v_{crit}$$

Recall that the area of the original rectangle, was v_{crit}. Thus, if we divide area $(I+II)$ by the total area of the rectangle, we find:

$$\frac{\text{Area }\left(I+II\right)}{\text{Area }\left(I+II+III\right)}=\frac{\int_{f=0}^{f^*} v\,df}{v_{crit}}+\left(1-f^*\right)$$

$$=\int_{f=0}^{f^*}\frac{v}{v_{crit}}df+\left(1-f^*\right) \qquad (19)$$

This is the expression we derived earlier for η (Equation 17), so:

$$\eta = \frac{\text{Area } (I + II)}{\text{Area } (I + II + III)} \tag{20}$$

Correspondingly, the fraction of the original particle concentration remaining in the suspension exiting the settling basin is $1 - \eta$, so:

$$\text{Fraction remaining} = 1 - \eta = \frac{\text{Area } III}{\text{Area } (I + II + III)} \tag{21}$$

These results tell us that by comparing two easily measured areas on the settling velocity distribution graph, we can compute the expected particle removal efficiency for any given v_{crit}. Furthermore, if we find that the expected removal efficiency is unacceptable for a given choice of v_{crit}, we can estimate the efficiency for another value of v_{crit} using the same graph simply by moving the value of v_{crit} along the *x* axis.

Note that η increases as v_{crit} decreases. That is, if the desired η is 0.8, and the first guess gives $\eta = 0.6$, the line representing v_{crit} must be moved to a lower v_{crit} to increase η. This makes sense since $v_{crit} = H/t_d$. Reducing v_{crit} allows a larger fraction of the particles to have settling velocities greater than v_{crit}, thereby increasing the number of particles removed. v_{crit} can be decreased by decreasing H at constant t_d or increasing t_d at constant H. Keep in mind that decreasing v_{crit} corresponds to increasing the surface area of the basin, for a given flow rate.

To summarize, to design a system to meet a given removal criteria, we can pick a tentative value for v_{crit}, measure the appropriate areas on the *f* vs. *v* graph to estimate the corresponding removal efficiency, and then adjust v_{crit} to larger or smaller values as needed to satisfy the criteria. This procedure is demonstrated in the following examples.

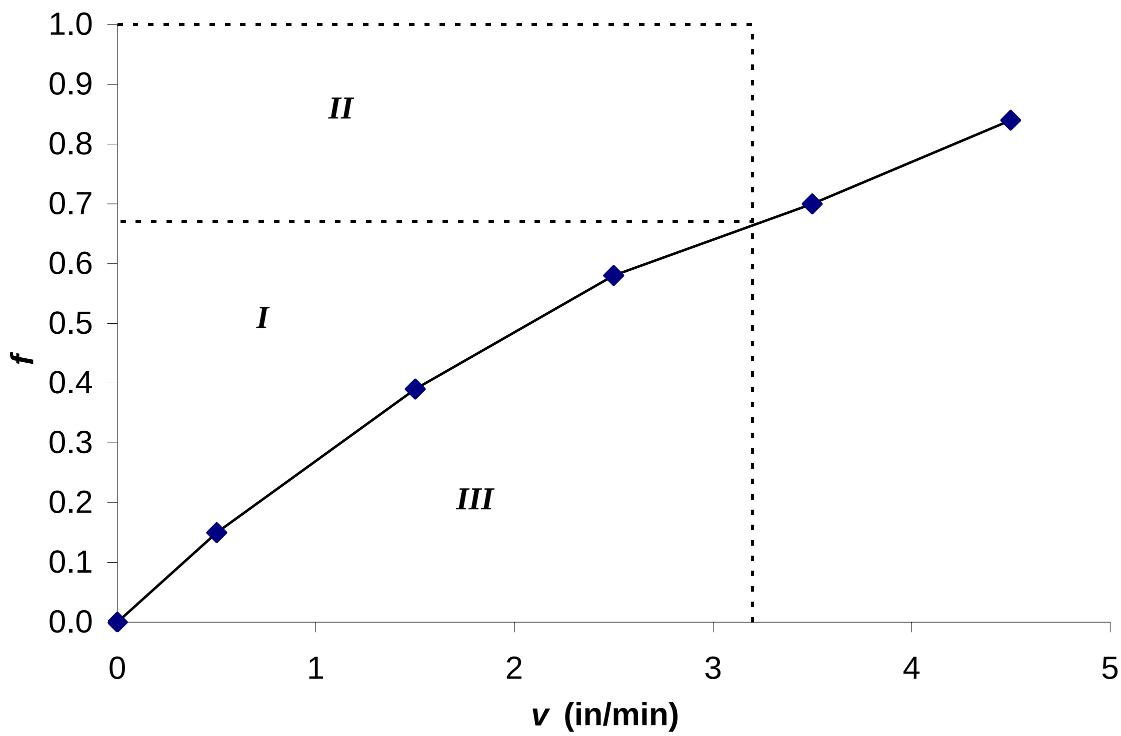
Example 1. Given the following settling velocity analysis for the influent suspended matter to a sedimentation basin, find the overall particle removal efficiency that can be expected. The overflow rate is 2 gpm/ft^2.

v	*f*
0.5 in/min	0.15
1.5 in/min	0.39
2.5 in/min	0.58
3.5 in/min	0.70
4.5 in/min	0.84

Solution. The critical settling velocity is the same as the overflow rate. Converting the given value into the same units as are used to express v_s:

$$v_{crit} = \left(2\,\frac{\text{gpm}}{\text{ft}^3}\right)\left(\frac{1\text{ ft}^3}{7.48\text{ gal}}\right)\left(\frac{12\text{ in}}{\text{ft}}\right) = 3.2\,\frac{\text{in}}{\text{min}}$$

The settling data and v_{crit} are shown on the graph below.



As shown by Equation 21, the overall particle removal efficiency can be evaluated by considering the following areas marked on the graph.

$$\text{Area } III = \int_0^{f^*} v_s\, df$$

$$\text{Area}\left(I + II + III\right) = \left(v_{crit}\right)\left(1.0\right) = v_{crit}$$

From the plot, $f^* = 0.67$. Also, a numerical integration of the area below the curve (Area *III*) yields a value of 1.24 in/min. The fraction of the particle mass expected to be present in the effluent for this v_{crit} is therefore:

$$1 - \eta = \frac{\text{Area } III}{\text{Area }\left(I + II + III\right)} = \frac{1.24\text{ in/min}}{3.2\text{ in/min}} = 0.39$$

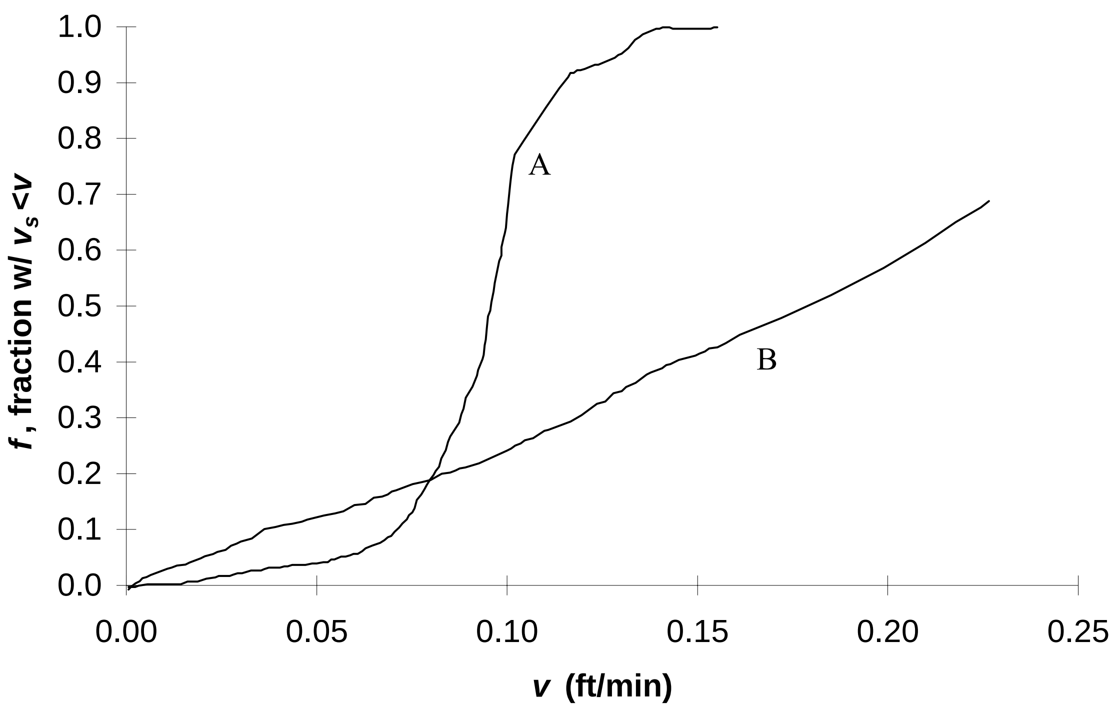
The expected fractional removal is $1 - 0.39$, or 0.61.

Example 2. Two water sources are under consideration for use as a community's water supply. The terminal settling velocities of the particles in the two waters are described by the diagrams below. The total particle concentration is about the same in both waters, but (as shown by the diagram) their settling properties are quite different. The particles in both water sources are non-flocculating.

The existing water treatment plant has a sedimentation basin with dimensions 50 x 25 x 10 ft (L x W x D). The plant must treat an average of 1 MGD (million gallons/day), but under worst case conditions, the demand for water may increase to 2.1 MGD.

(a) What are the hydraulic residence time (hr) and the overflow rate (gal/day-ft^2) of the basin on an average day?

(b) Which water supply will be most efficiently treated by the sedimentation basin on an average day? on the worst (highest demand) day?



Solution. (a) The hydraulic residence time is simply the ratio of volume to flow rate, and the overflow rate is the ratio of the flow rate to the plan area:

$$t_d = \frac{V}{Q} = \frac{(50 \text{ ft})(25 \text{ ft})(10 \text{ ft})}{(10^6 \text{ gal/d})(1 \text{ ft}^3/7.48 \text{ gal})(1 \text{ day/24 hr})} = 2.24 \text{ hr}$$

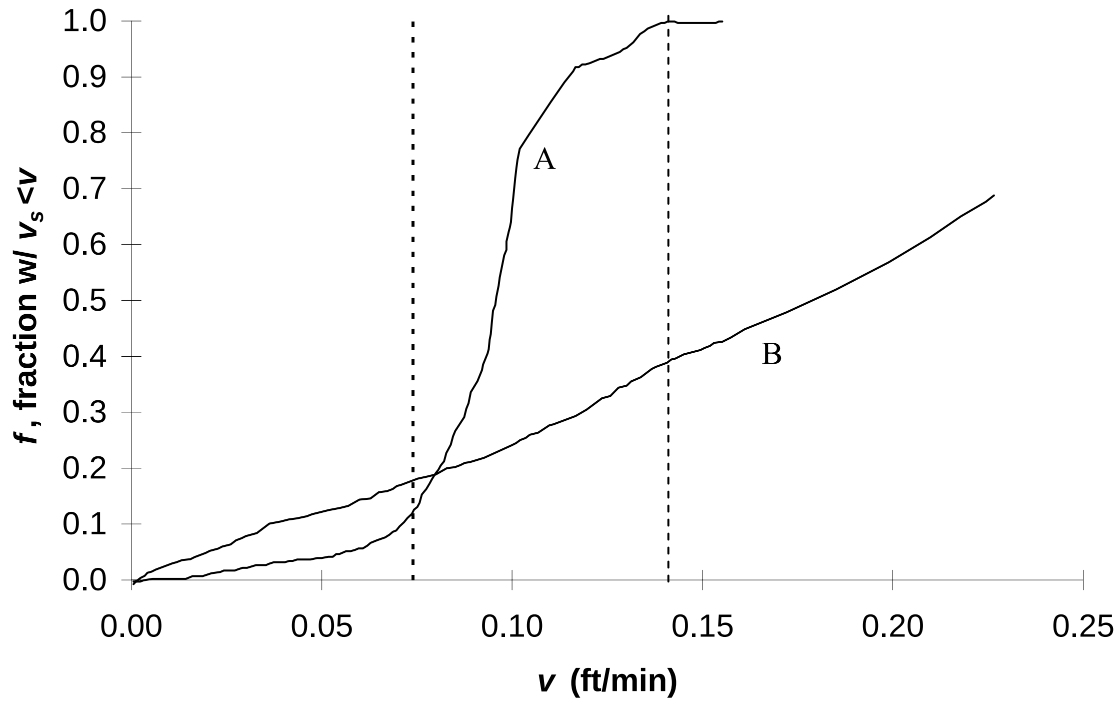
$$\text{O/F} = \frac{Q}{A} = \frac{10^6 \text{ gal/d}}{(50 \text{ ft})(25 \text{ ft})} = 800 \frac{\text{gal}}{\text{day-ft}^2}$$

(b) The critical velocity is the same thing as the overflow rate. Calculating v_{crit} in units of ft/min for both the average and highest demand days, we obtain:

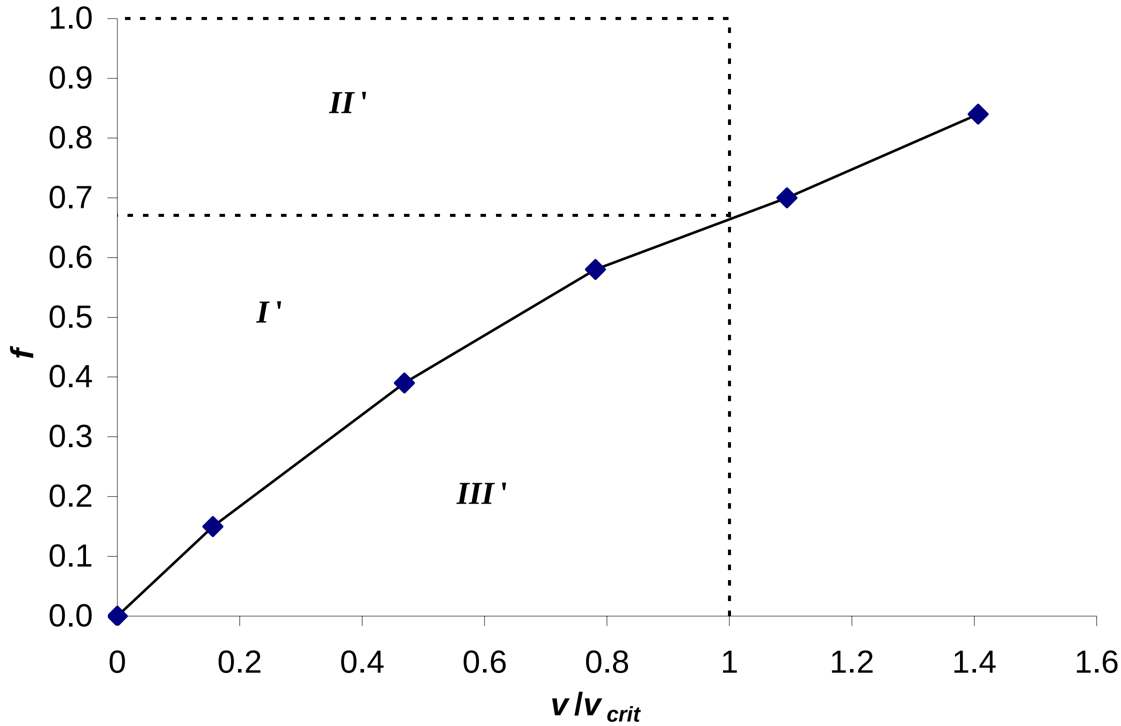
$$v_{crit,\ avg.\ day} = \frac{10\text{ ft}}{(2.24\text{ hr})(60\text{ min/hr})} = 0.074\frac{\text{ft}}{\text{min}}$$

$$v_{crit,\ worst\ day} = \left(0.074\frac{\text{ft}}{\text{min}}\right)2.1 = 0.16\frac{\text{ft}}{\text{min}}$$

The values of v_{crit} are indicated by the dashed lines on the diagram below. Comparing the areas above the curves over the range $0 < v < v_{crit}$, it is clear that the removal efficiency would be better for supply A on the average day (v_{crit} = 0.074 ft/min), but on the high demand day, (v_{crit} = 0.16 ft/min), the particle removal efficiency would be greater for supply B.



The same analysis as above is sometimes shown in a slightly altered form. Since v_{crit} is a constant, we can divide all values on the x axis by it, to normalize the settling curve. The rectangle of interest is then bounded by the area $0 < (v_s/v_{crit}) < 1$ on the x-axis and $0 < f < 1$ on the y axis, so it has a total area of 1.0. For instance, a plot of f vs. v/v_{crit} for the suspension characterized in Example 1 would look as follows.



In this case, the areas above and below the curve give, respectively, the overall removal efficiency and the overall fraction remaining, directly:

$$\eta = \int_{f=0}^{f^*} \frac{v_s}{v_{crit}} df + 1 - f^* \tag{22}$$

$$= \frac{\text{Area } I' + \text{Area } II'}{\text{Area } I' + \text{Area } II' + \text{Area } III'} = \frac{\text{Area } I' + \text{Area } II'}{1} = \text{Area } I' + \text{Area } II'$$

$$1 - \eta = \text{Area } III' \tag{23}$$

An important result of the analysis is that, for a given flow rate of a given influent suspension, the removal efficiency η for non-flocculating particles is independent of basin depth. That is, the design variable that completely determines the removal efficiency is the overflow rate, which can be expressed as Q/A, v_{crit}, or H/t_d. If Q is fixed, then specifying A (the surface area of the settling basin) is equivalent to specifying the overflow rate and hence fixing η.

The fact that, for a given flow rate, η is determined entirely by A leads to the conclusion that the depth of the basin is irrelevant to its performance. The counter-intuitive result can be explained by considering what happens if we double the depth of a basin in which a non-flocculating suspension is being settled. If the length and width of the basin and the flow rate through it are held constant, then the volume of the basin doubles, as does the residence time of the water in the basin ($t_d = 2V/2Q = V/Q$). Since the particles are assumed to reach their terminal velocity instantaneously upon entering the tank, they all fall exactly twice as far (since they will be in the tank twice as long) as in the first case. On the other hand, the increased distance to the bottom will exactly compensate for this. That is, if a group of particles falls $(1/3)H_1$ in the first case, its removal efficiency will be 33%. In the second case, these particles will fall $(2/3)H_1$, but since $H_2 = 2\,H_1$, they will fall $(1/3)H_2$ and still have 33% removal efficiency. Thus, as was predicted by the design equation, removal efficiency is independent of depth. By the same reasoning, removal efficiency is also independent of the shape of the basin. That is, a tank with a given surface area will have the same removal efficiency for a given flow regardless of whether the tank is circular, square, or rectangular with any aspect ratio, as long as the assumption of plug flow is valid.

Given the conclusion that removal efficiency is independent of depth, it would seem to make sense to design very shallow sedimentation basins. Such basins would be less expensive to construct than deeper basins and would seem to have no drawbacks with respect to performance. According to the reasoning above, the residence time in these basins would be very short, but since the particles would have very short distances to fall, the removal efficiency would be maintained. This reasoning is valid as long as the assumptions of the model are met. However, note that one consequence of shortening the residence time while holding the surface dimensions constant is that the linear velocity of water in the system increases. Recall that two of the assumptions of the derivation were that the water followed a plug flow pattern and that when particles struck the bottom of the tank, they were permanently removed. If the tank becomes too shallow, these assumptions will be violated. Specifically, the increased velocity will increase the Reynolds number of the flow and thereby increase mixing, causing the assumption of plug flow to be violated. In addition, the drag forces along the bottom will generate significant shear that will cause particles that have already reached the bottom of the tank to be re-entrained. Thus, in a very shallow basin, particles might be just as likely to strike the bottom as in a deeper basin, but the assumption that they will stay there is likely to be violated.

Thus, even though our theoretical analysis suggests that we could reduce the depth of a settling basin for a Type 1 suspension without severe consequences, in reality, doing so is not very practical. On the other hand, the fact that the performance of such systems depends only on Q/A has led to the development of some very clever approaches to enhance removal efficiency without increasing the basin size. For example, if a false bottom is placed in the basin at one-half the full depth, then the flow rate and the cross-sectional area remain the same, so the horizontal velocity through the basin is unaltered. Nevertheless, the maximum depth that any particle must fall before it hits a surface and is removed is reduced by half. Equivalently, we might say that the total surface area devoted to particle capture in the basinhas been doubled. In either case, the net effect is that the overflow rate, and therefore the critical velocity, has been reduced by half. The false bottom can be sloped in the hope that particles will slide down toward the collection bin, it might be scraped by a continuously moving blade, or the surface itself might be a slowly moving conveyor belt. Regardless of the exact form, the net effect is that the overflow rate is reduced while the linear velocity of the water through the system is unaltered.

Another approach that takes advantage of the same idea is to fill the basin with steeply sloped surfaces, as shown in Figure 12. Once again, the placement of the structures has a negligible impact on the volume of the basin, so the residence time remains the same. However, as in the system where one or more false bottoms are installed, this modification decreases the distance the particles must fall before they strike a surface, from where they are conveyed to the sludge collection hopper.

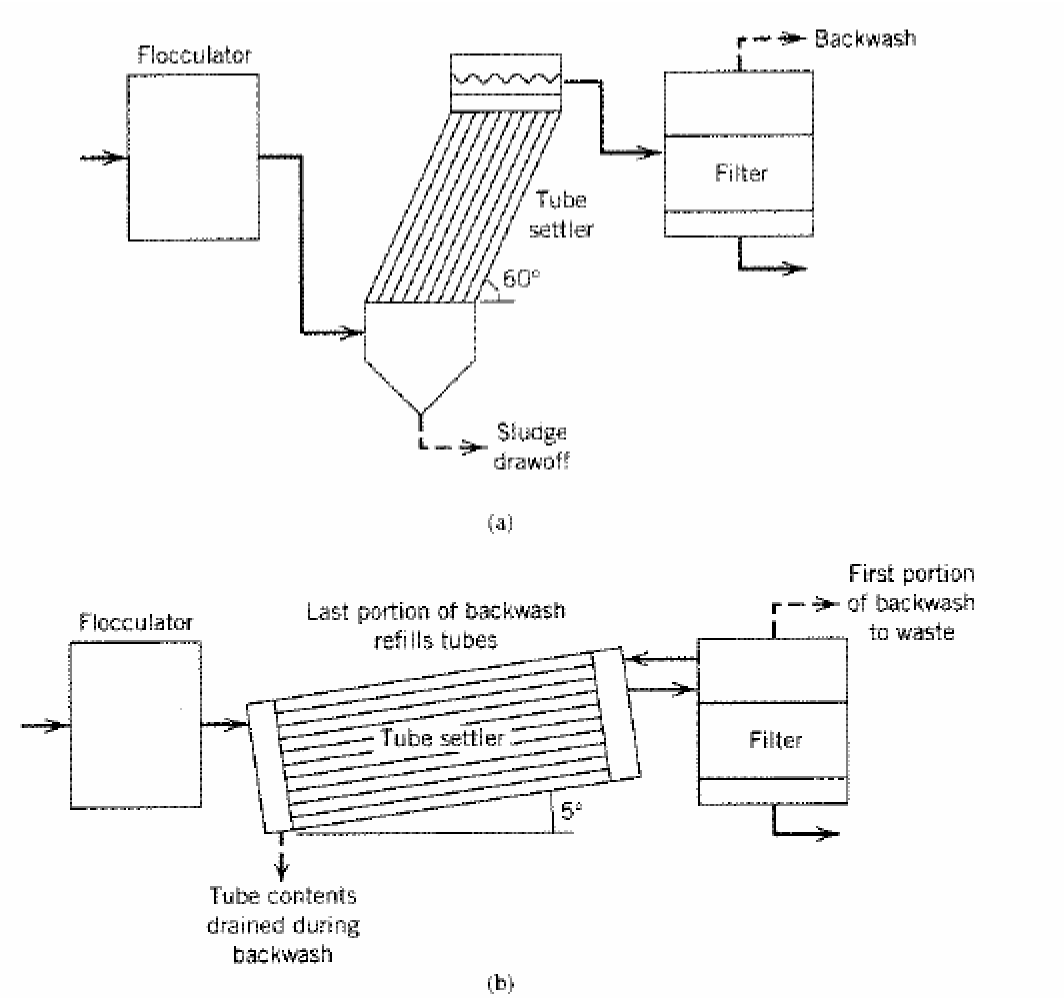


Figure 12. "Tube settlers," in which inclined tubes are placed in sedimentation basins to provide lots of surface area onto which particles can fall and thereby be removed from the suspension. (From Droste.)

Example 3. An industrial waste with a flow of 0.4 m^3/s contains plastic spheres that are approximately spherical, with a 0.2-mm diameter and a density of 0.87 g/cm^3. The waste is to be treated in a flotation basin, where it is hoped that the spheres will rise to the surface so that they can be continuously removed by a skimmer. The dimensions of the basin are: L = 20 m; W = 8 m; H = 2.5 m, and the water is at 20°C.

(a) What are the magnitude and direction of the drag force on a particle rising at its terminal velocity?

(b) What is the overflow rate of the basin in cm/s?

(c) What fraction of the beads in the influent is likely to be removed in the basin?

Solution. (a) The direction of the drag force is opposite the direction of movement. Since the particle is rising, the drag force is directed downward. The magnitude of the drag force can be computed a few ways. The easiest is to note that when the particle is moving at its terminal velocity, the drag force is equal in magnitude and opposite in direction to the net gravitational force (gravity − buoyancy). Thus the net drag force is:

$$F_D = F_G - F_B = V_{bead}\left(\rho_{bead} - \rho_w\right)g$$

$$= -\left(\frac{1}{6}\pi\left(0.01\text{ cm}\right)^3\right)\left\{\left(0.87 - 0.98\right)\frac{\text{g}}{\text{cm}^3}\right\}\left(980\frac{\text{cm}}{\text{s}^2}\right)$$

$$= 4.5\text{x}10^{-4}\ \frac{\text{g-cm}}{\text{s}^2} = 4.5\text{x}10^{-4}\ \text{dyne}$$

(b) The overflow rate is: Q/A = (0.4 m^3/s)/(20 x 8 m^2)

$$\frac{Q}{A} = \frac{0.4\text{ m}^3\text{/s}}{20\text{m x 8 m}} = 2.5\text{x}10^{-3}\ \frac{\text{m}}{\text{s}} = 0.25\frac{\text{cm}}{\text{s}}$$

(c) $$v_{term} = \frac{g\left(\rho_{bead} - \rho_w\right)d^2}{18\mu}$$

$$= \frac{\left(980\text{ cm/s}^2\right)\left(0.87 - 0.98\text{ g/cm}^3\right)\left(0.02\text{ cm}\right)^2}{18\left(0.0089\text{ g/cm-s}\right)}$$

$$= -0.27\text{ cm/s (upward motion)}$$

Since $|v_{term}| > Q/A$, the beads are 100% removed in the process.

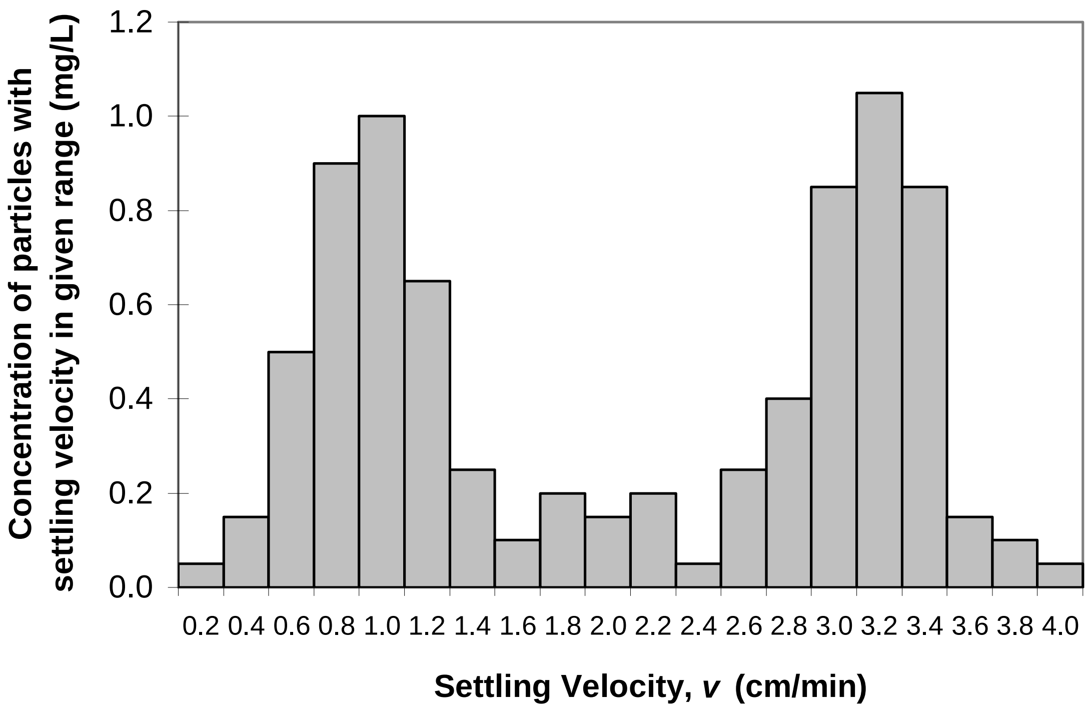
Example 4. Shown below is a histogram describing the settling velocities of non-flocculating particles in a water supply. The number under each bar indicates the value of v_s at the end of range, e.g., the first bar represents particles with settling velocities between 0 and 0.2 cm/min. As you can see, the distribution is bi-model, *i.e.*, it has two distinct peaks. If a smooth curve is drawn through the data, the areas under the curve are such that about 50% of the suspended solids concentration is included in each group.

(a) Sketch a plot of f (the fraction of particles with settling velocity [v_s] less than v) vs. v for this system.

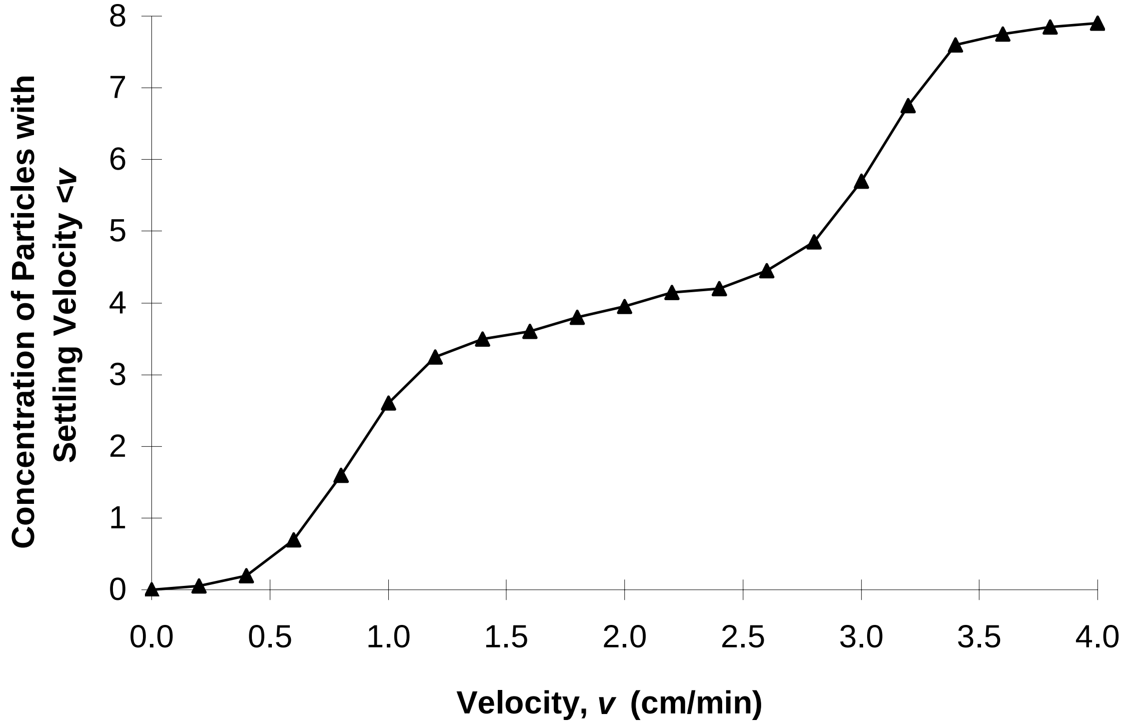
(b) If the particles are non-flocculating, and the water passes through a sedimentation basin with a critical settling velocity of 2.8 cm/min, sketch two curves that might reasonably describe the settling velocity distribution of the particles in the effluent: one that has the same axes as the plot shown below, and another that shows a cumulative distribution, i.e., the fraction with settling velocity less than v, vs. v.

(c) If you designed a settling basin with v_{crit} = 2.0 cm/min, would significantly less than half, about half, or significantly more than half of the suspended solids be removed? Explain.

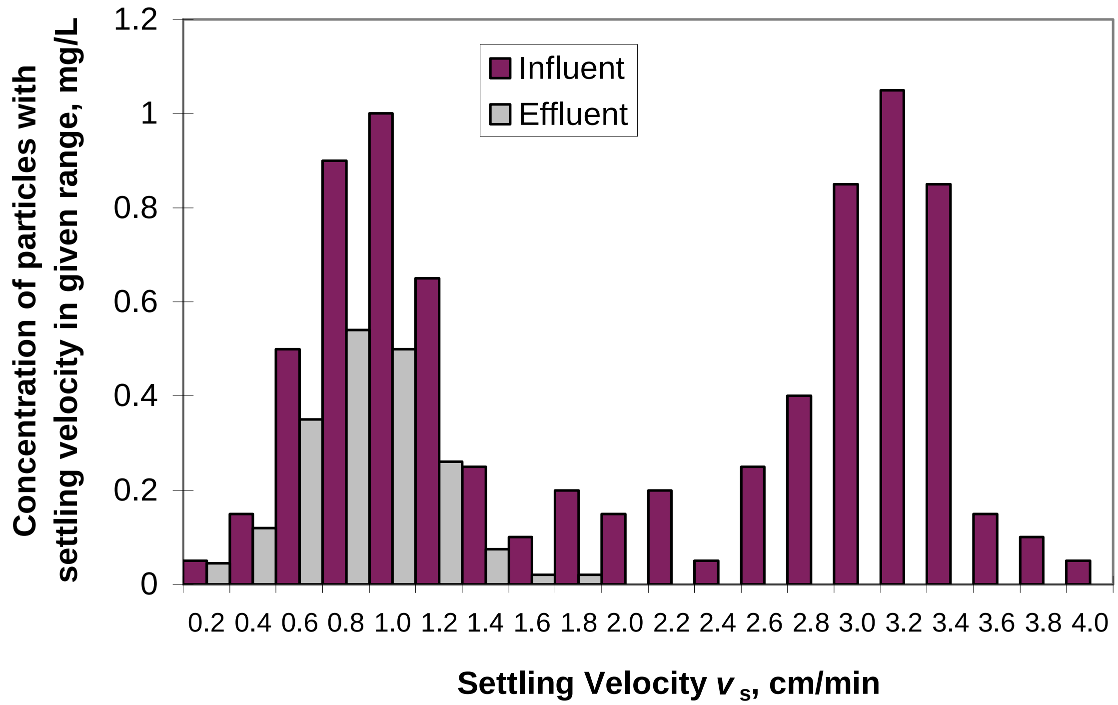
(d) For the basin in part *c*, would overall removal efficiency be improved if it turned out that the group of larger particles could flocculate? Why or why not?



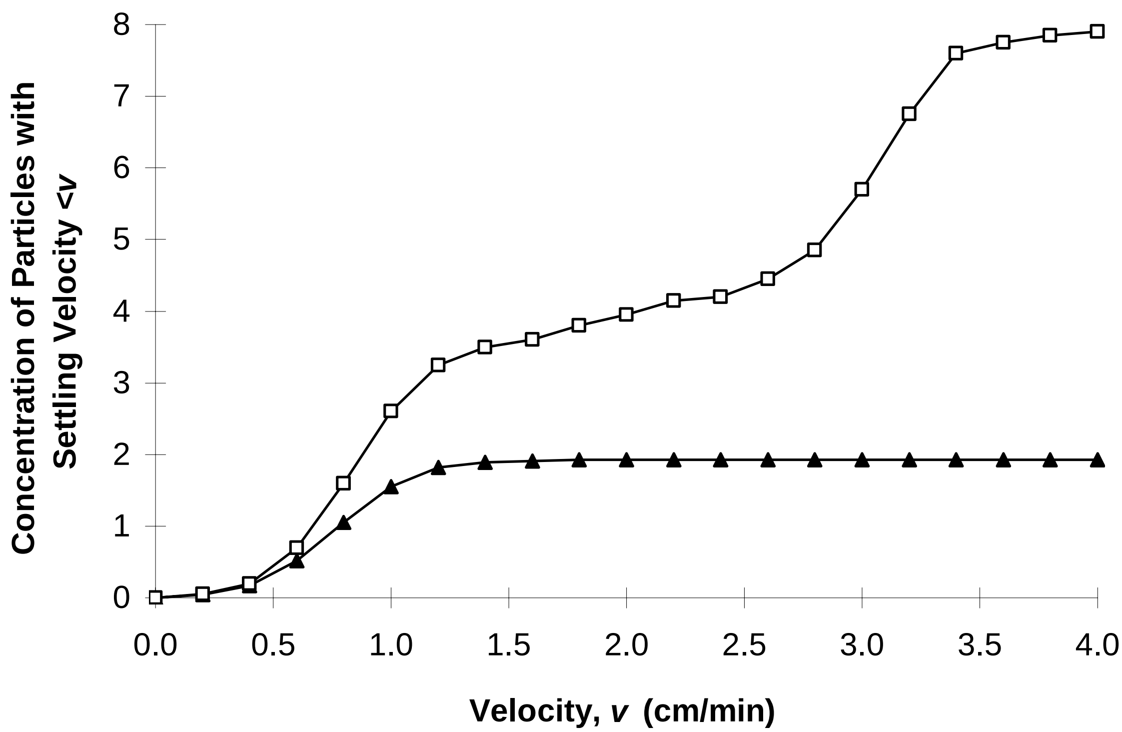
Solution. (a) The plot is shown below. It is steep over a given range of *v* if there are a lot of particles in that range, i.e., in the ranges corresponding to the peaks in the histogram, and shallow in other ranges.



(b) If v_{crit} is 2.8 cm/min, all particles with v_s greater than 2.8 cm/min will be removed in the basin, so 100% of the particles in the effluent will have velocities less than this value. Those with velocities substantially less will be removed only slightly, while those with velocities only slightly less than v_{crit} will be removed almost (but not quite) completely. The selective removal of rapidly settling particles is very apparent when the inlet and outlet distributions are plotted together; these two data sets are shown below in the form of both a histogram and an f vs. v plot.



Concentration of particles with settling velocity in given range, mg/L
1.2
1
0.8
0.6
0.4
0.2
0
Influent
Effluent
0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0 2.2 2.4 2.6 2.8 3.0 3.2 3.4 3.6 3.8 4.0
Settling Velocity v s, cm/min



Concentration of Particles with Settling Velocity <v
8
7
6
5
4
3
2
1
0
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0
Velocity, v (cm/min)

Approach 2: Averaging based on the height at which particles cross the effluent wall. The idea behind computing the overall fractional removal based on small (or differential) increments in f is that the range from $f = 0$ to $f = 1$ considers all the particles in the system. By considering all those particles one small group at a time, determining the removal efficiency for each group, and then carrying out a weighted summation (weighted according to the number or mass of particles in the group), we obtain an overall average removal efficiency. As noted previously, the same result can be obtained by dividing the particle population in different ways, as long as we still consider all the particles that might arrive at the effluent wall. We next consider a different approach for carrying out this analysis.

In this alternative approach, we divide the particles according to the maximum height at which we expect them to be present when the water leaves the settling basin. Recognizing that a PFR is just like a stagnant column on a conveyor belt, we expect the vertical concentration profile at the outlet of an ideal settling basin with horizontal flow to be identical to that in a column with the same height as the settling basin, if the settling time t in the column is the same as the residence time t_d in the settling basin.

In a column test, a certain fraction of the particles have settling velocities greater than some value v_1; to make the example explicit, assume that this fraction is 35%. In a stagnant column, the level at which the removal efficiency is 35% falls at a steady velocity of $v_{35\%}$, as was shown in Figure 9. Correspondingly, if that column were put on a conveyor belt, the height corresponding to 35% removal would fall linearly as the column moved. Extrapolating this result to a settling basin with plug flow, we conclude that any isopleth of constant fractional removal for a Type 1 suspension will be linear, as illustrated in Figure 13 for the same suspension as was characterized in Figure 9 for a batch settling test.

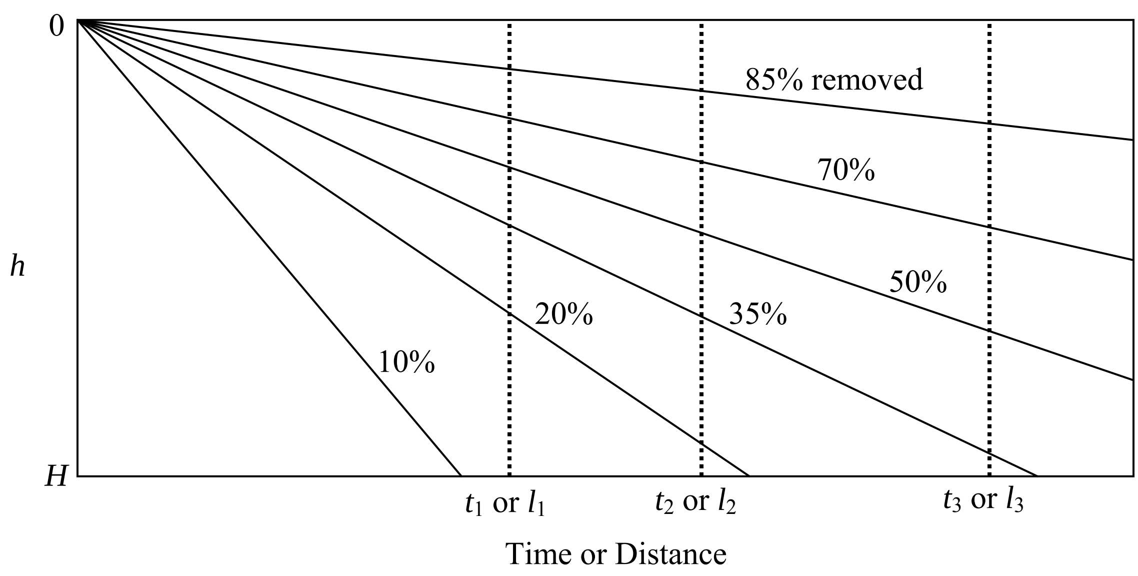


Figure 13. Isopleths of constant removal efficiency in an ideal settling basin, for a Type 1 suspension.

In the figure, the percentages are percent removed at the given location and time. The slopes of the lines can be interpreted as settling velocities, *i.e.*, 35% of the particles have settling velocities greater than the slope of the 35% line. The isopleths are straight because, for a Type 1 suspension, each particle settles at its characteristic settling velocity forever. Thus, for Type 1 suspensions, the percentage of the particles that have velocities greater than any given value is the same, no matter how much time has elapsed, or how far the particles have already fallen. (This situation contrasts with that for Type 2 suspensions, where settling velocities increase over time, due to particle growth; the isopleths for Type 2 suspensions are therefore curved.)

To summarize, for discrete settling particles:

- The velocity of each particle is constant during the entire time it is in the basin
- The trajectory of each particle is a straight line while it is in the water column
- Isopleths of constant removal percentages are straight lines emanating from zero at the top of the water column at the inlet. If the distance in the direction of bulk flow is divided by the horizontal velocity of the fluid, it is converted to the travel time from the inlet to that location. In that case, the slopes of the lines of constant removal percentage can be interpreted as velocities. The isopleth of x% removal corresponds to a velocity v such that x% of the particles have $v_s > v$.

Now consider the concentration profile at the effluent wall for the example system shown in Figure 13. Assume for now that this wall is at distance l_3 from the inlet, so that the residence time in the basin is t_3. The figure is shown again below (Figure 14), with some additional values of h marked for reference.

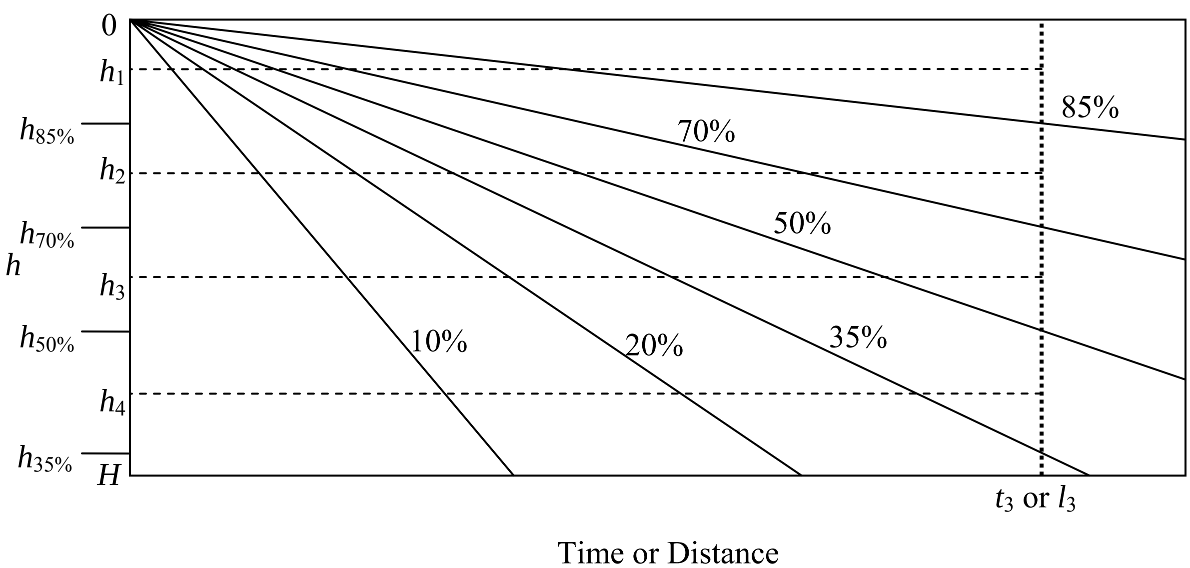


Figure 14. Hypothetical settling basin, with t_d equal to t_3 and several heights of interest marked.

By interpolation between the isopleths for 20% and 35% removal, we might estimate that the removal efficiency at the bottom of the water column at the outlet is 33%. Slightly above the bottom, at depth $h_{35\%}$, the removal efficiency is 35%. We therefore approximate the average removal throughout in the vertical region between depths H and $h_{35\%}$ as 34%. That region accounts for a fraction of the total water depth equal to $(H - h_{35\%})/H$. Similarly, the next higher vertical region represents a fraction of the total depth equal to $(h_{50\%} - h_{35\%})/H$, and the average removal efficiency in that region is $(50\% + 35\%)/2$. We can continue this process until we reach the top of the water column. The overall removal is then computed as the weighted sum of the removal efficiencies along the entire vertical length of the wall, with the fractional heights as the weighting factors, *i.e.*:

$$\eta = \frac{0.33+0.35}{2}\frac{H-h_{35\%}}{H} + \frac{0.50+0.35}{2}\frac{h_{50\%}-h_{35\%}}{H} + \frac{0.70+0.50}{2}\frac{h_{70\%}-h_{50\%}}{H} + \frac{0.85+0.70}{2}\frac{h_{85\%}-h_{70\%}}{H} + \frac{1.00+0.85}{2}\frac{h_{100\%}-h_{85\%}}{H} \tag{24}$$

Each term in the above summation equals the product of the average removal efficiency corresponding to a vertical segment of the water column at the basin effluent and the depth of that segment. Generalizing this process, we can write:

$$\eta = \sum_0^H \eta_i \frac{\Delta h}{H} = \frac{\sum_0^H \eta_i \Delta h}{H} \tag{25}$$

where i now represents a specific segment of depth along the effluent wall (as opposed to a specific subset of settling velocities, as in the preceding analysis), and η_i is the average particle removal efficiency in that segment. If the segments are shrunk to differential lengths, we obtain the following integral expression for η:

$$\eta = \frac{\int_0^H \eta_i dh}{H} \tag{26}$$

The integral in Equation 26 can be evaluated graphically by considering the particle concentration profile along the effluent wall. Recall that, for a group of discrete particles, the profile is a series of steps, as shown in Figure 7, and that for a continuous distribution of particle settling velocities, this step function becomes a smooth curve, such as shown in Figure 15.

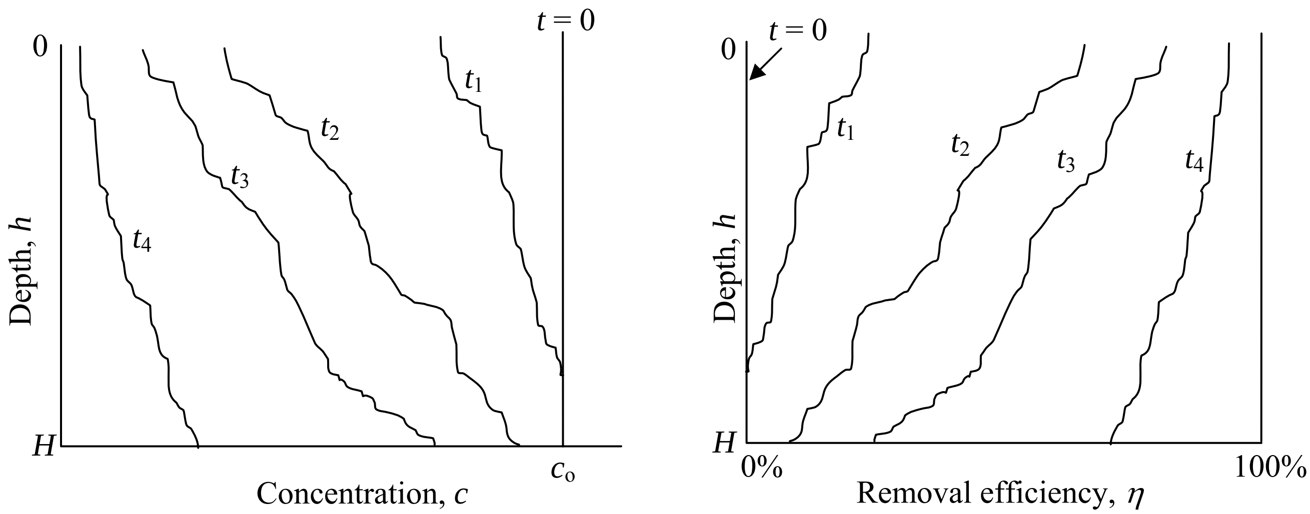


Figure 15. Typical profiles for concentration and removal efficiency expected at the effluent wall of a settling basin at various times; $0 < t_1 < t_2 < t_3 < t_4$.

The graphical analysis is shown in Figure 16 for the time designated t_3 in Figure 15. Area I equals $\int_0^H \eta_i dh$, and the combined area $(I + II)$ equals H, so:

$$\frac{\text{Area } I}{\text{Area } (I + II)} = \frac{\int_0^H \eta_i dh}{H} = \eta_{overall} \tag{27}$$

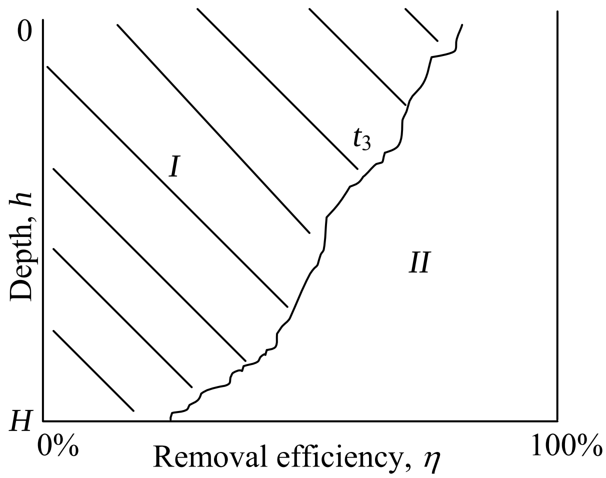


Figure 16. Evaluation of η based on graphical analysis of the particle concentration profile at the effluent wall.

Alternatively, Area I in Figure 16 can be represented as follows:

$$\text{Area } I = \int_0^1 h d\eta \tag{28}$$

Therefore, dividing through by Area ($I + II$):

$$\eta_{overall} = \frac{\text{Area } I}{\text{Area}\left(I + II\right)} = \frac{\int_0^1 h d\eta}{H} = \int_0^1 \frac{h}{H} d\eta \tag{29}$$

or, in discrete form:

$$\eta_{overall} = \sum_{\eta=0}^{1.0} \frac{h_i}{H} \Delta\eta_i \tag{30}$$

In this case, $\Delta\eta_i$ is an interval of removal efficiencies at the effluent wall, and h_i is the average height of that interval (*i.e.*, the height in the middle of the interval). Conceptually, h_i/H can be thought of as the fraction of the column from which groups of the influent particles have been completely removed, and $\Delta\eta_i$ is the indicator of how large each group is. When this approach is used to analyze the system characterized in Figure 14, the relevant calculation is:

$$\eta_{overall} = \frac{H}{H}\left(0.33-0\right)+\frac{h_4}{H}\left(0.50-0.33\right)+\frac{h_3}{H}\left(0.70-0.50\right)+\frac{h_2}{H}\left(0.85-0.70\right)+\frac{h_1}{H}\left(1.00-0.85\right) \qquad (31a)$$

$$= 0.33+\frac{h_4}{H}\left(0.17\right)+\frac{h_3}{H}\left(0.20\right)+\frac{h_2}{H}\left(0.15\right)+\frac{h_1}{H}\left(0.15\right) \qquad (31b)$$

Equations 25 and 26 are, respectively, the discrete and differential equations for computing η based on local removal efficiencies at various heights along the effluent wall, using a weighting factor that corresponds to the fraction of the wall height over which that removal efficiency applies. Similarly, Equations 29 and 30 are, respectively, the differential and discrete equations for computing η based the distance that particles fall while they are in the sedimentation basin, using a weighting factor that corresponds to the fraction of the total influent concentration that falls the specified distance. These two sets of equations represent the second and third approaches for estimating overall removal in a sedimentation basin (with Equations 14, 15, and 20 comprising the first approach). If the data were perfect, all of these equations would yield exactly the same result for η of a given influent in a given basin.

DESIGN OF SEDIMENTATION FACILITIES FOR FLOCCULATING PARTICLES

If particles collide and coagulate as they settle, then their trajectories through the sedimentation basin are likely to be curvilinear, bending "downward" over time. That is, the particles are likely to accelerate as a result of their growth, so that their velocities are continually increasing with increasing time in the basin. Despite this difference from the situation for Type 1 settling (for which the particle trajectories are linear), it is still true that a sample taken at a port at depth h in a column test, after having settled for a time t, will contain only those particles that have settled a distance less than h during time t; by definition, if they had settled any more than that, they wouldn't be in the sample. However, in the test with Type 2 settling, the particles have presumably had a range of settling velocities during the test; thus, we cannot ascribe a single velocity to any single particle or group of particles. However, we can say that the particles that remain in the sample at depth h after time t have had an *average* settling velocity less than or equal to h/t during the test.

Because the average settling velocity of the whole collection of particles increases over time in Type 2 settling, the f vs. v curve for such a suspension steadily shifts down and to the right as the duration of the test increases. For example, imagine that 40% of the particles in a suspension had average settling velocities less than 0.60 cm/min in a 30-minute settling test (i.e., $f = 0.40$ for $v = 0.60$ cm/min at $t = 30$ min). We would draw this conclusion if a sample taken at a depth of 18 cm at the end of the test contained a particle concentration equal to 40% of the original concentration. If the test were allowed to run for 60 minutes, the average settling velocity of the particles would increase. Say, for instance, that at the end of such a test, only 30% of the particles were present at a depth of 36 cm. Like the sample described above, v for this sample is 0.60 cm/min, but now, at $t = 60$ min, f is only 0.3; thus, the value of f at a given v decreased when the test duration increased. Correspondingly, 40% of the particles might remain in suspension at a depth of 40 cm after 60 min. This data point would indicate that $f = 0.40$ for $v = 0.67$ cm/min at $t = 60$ min; this indicates that the value of v at a given f increased when the test duration increased. Because such shifts to lower f and higher v would occur for all the points on the curve, the f vs. v curve can be described as shifting downward and to the right with increasing settling time.

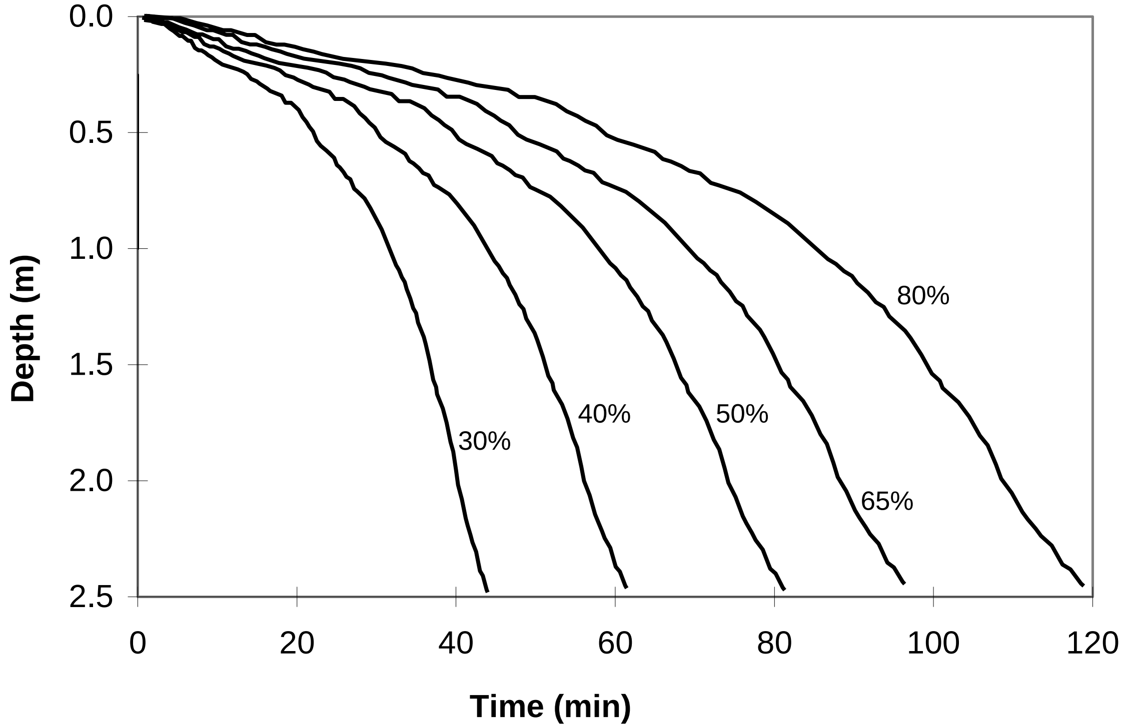
The net effect the of the situation described above is that, in Type 2 settling, specifying the critical velocity of a sedimentation basin is insufficient for determining the removal efficiency that will be achieved in that basin. For example, imagine two basins receiving the same flow rate of the same influent and having the same surface dimensions (L and W), but with one basin having twice the depth of the other. Because Q is the same in the two basins, the deeper basin has twice the hydraulic residence time, and both basins have the same critical velocity (H/t_d in the shallower basin, $2H/2t_d$ in the deeper one). If the influent suspension were characterized by Type 1 settling, the basins would achieve equal degrees of sedimentation. However, because the particles undergo Type 2 settling, doubling the depth and the residence time causes the particles to fall *more* than twice as far in the deeper basin as in the shallower one (i.e., the average settling velocities are greater in the deeper basin). As a result, the deeper basin achieves more particle removal than the first basin. Thus, specification of the critical velocity is necessary but not sufficient to establish the particle removal efficiency with Type 2 settling; one must also specify either t_d or H.

Historically, the analyses presented in the preceding section as the second and third approaches have been used almost exclusively for analysis of Type 2 settling, whereas the first approach has been used for analysis of Type 1 settling. However, in truth, all three approaches are applicable to either non-interacting or flocculating particles. This point is demonstrated in the following example.

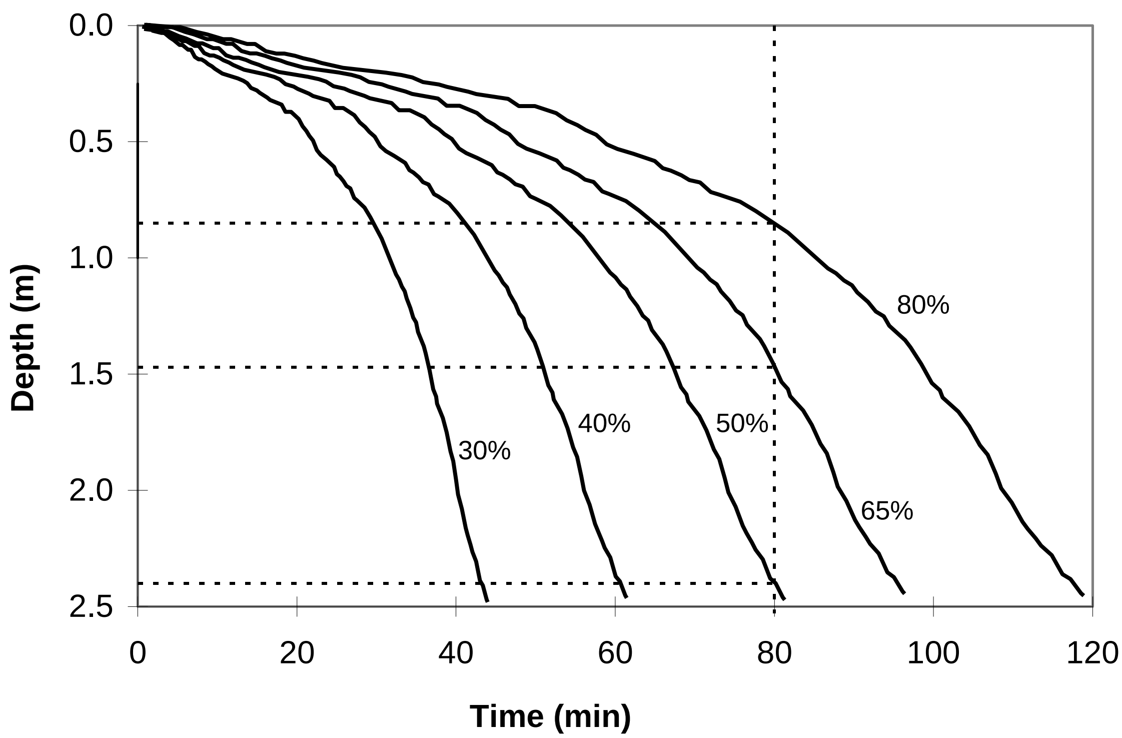
Example 5. Consider the model settling data in the figure below, derived from a settling test in a stagnant water column; the percentages shown are the percentage removal of particles at the given height and time, compared to the initial concentration in the settling column.

(a) Compute the expected particle removal efficiency in a 2.5 m-deep basin for a residence time of 80 min. Compare the result obtained using Equation 26 with that using Equation 30.

(b) Plot curves of f vs. v at 40 and 80 min, and compute the expected particle removal efficiency for an ideal settling basin with $t_d = 80$ min. Compare the result with those obtained in part *a*.



Solution. (a) The settling diagram is repeated below, with a vertical line added at t_d and horizontal lines at locations where that vertical line intersects the isopleths of constant removal percentage.



We could use either approach described previously to determine the overall removal efficiency. The following discussion uses Approach 2, in the form of Equation 25:

$$\eta = \sum_0^H \eta_i \frac{\Delta h}{H} = \frac{\sum_0^H \eta_i \Delta h}{H} \tag{25}$$

The isopleths of 80%, 65%, and 50% removal pass through depths of 0.85, 1.47, and 2.40 m, respectively, at $t = 80$ min. In addition, we estimate that the 49% removal isopleth passes through the point (80 min, 2.5 m), and we assume that the removal at the very top of the basin ($h = 0$) is 100%. Therefore, considering increments of height from the top, we estimate that:

- The average value of η between the top ($h = 0$) and the 80% isopleth ($h = 0.85$ m) is 90%;
- The average value of η between $h = 0.85$ m and 1.47 m is (80% + 65%)/2, or 72.5%;
- The average value of η between $h = 1.47$ m and 2.40 m is (65% + 50%)/2, or 57.5%;
- The average value of η between $h = 2.40$ m and 2.50 m is (50% + 49%)/2, or 49.5%.

Applying Equation 25 to the data set:

$$\eta = (90\%)\frac{0.85\text{ m} - 0.0\text{ m}}{2.5\text{ m}} + (72.5\%)\frac{1.47\text{ m} - 0.85\text{ m}}{2.5\text{ m}} + (57.5\%)\frac{2.40\text{ m} - 1.47\text{ m}}{2.5\text{ m}} + (49.5\%)\frac{2.50\text{ m} - 2.40\text{ m}}{2.5\text{ m}}$$

$$= 72.0\%$$

As an alternative, we could analyze the same data based on Equation 30. In this case, the logic would be as follows:

> As noted above, 49% of the solids fall $\geq$2.5 m in 80 min. All of these particles will be removed in the basin;
>
> 50% of the particle mass falls ≥2.40 m in 80 min, so (50 – 49)% = 1% falls a distance $2.4 \leq h \leq 2.5$ m. We assume that, on average, these particles fall 2.45 m;
>
> 65% of the particle mass falls $\geq$1.47 m, so (65 – 50)% = 15% fall a distance of $1.47 \leq h \leq 2.45$ m, or an average of 1.96 m;
>
> 80% of the particle mass falls $\geq$0.85 m, so (80 – 65)% = 15% falls a distance of $0.85 \leq h \leq 1.47$ m, or an average of 1.16 m;
>
> 100% of the particle mass falls $\geq$0 m, and so (100 – 80)% = 20% falls a distance of $0.00 \leq h \leq 0.85$ m, or an average of 0.43 m.

According to Equation 30, a reasonable estimate of overall efficiency is thus:

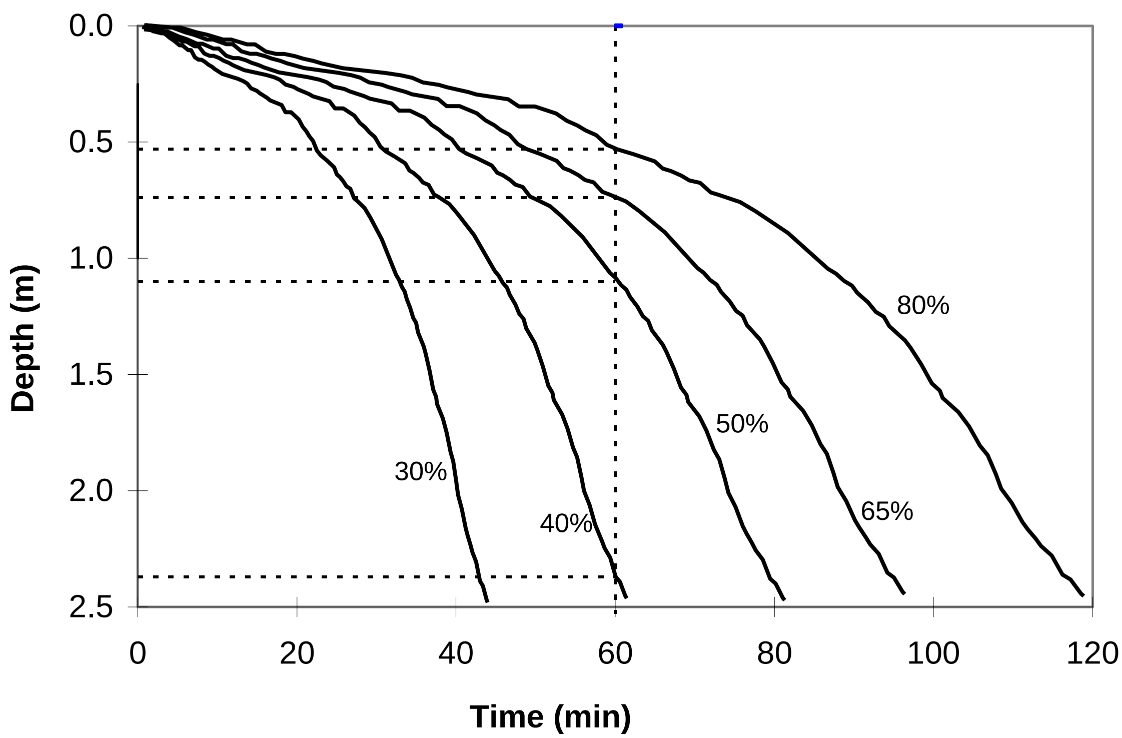
$$\eta = \sum_{\eta=0}^{1.0} \frac{h_i}{H} \Delta\eta_i \qquad (30)$$

$$= \left(\frac{2.50\text{m}}{2.50\text{m}}\right)(49\% - 0\%) + \left(\frac{2.45\text{m}}{2.50\text{m}}\right)(50\% - 49\%) + \left(\frac{1.96\text{m}}{2.50\text{m}}\right)(65\% - 50\%) +$$

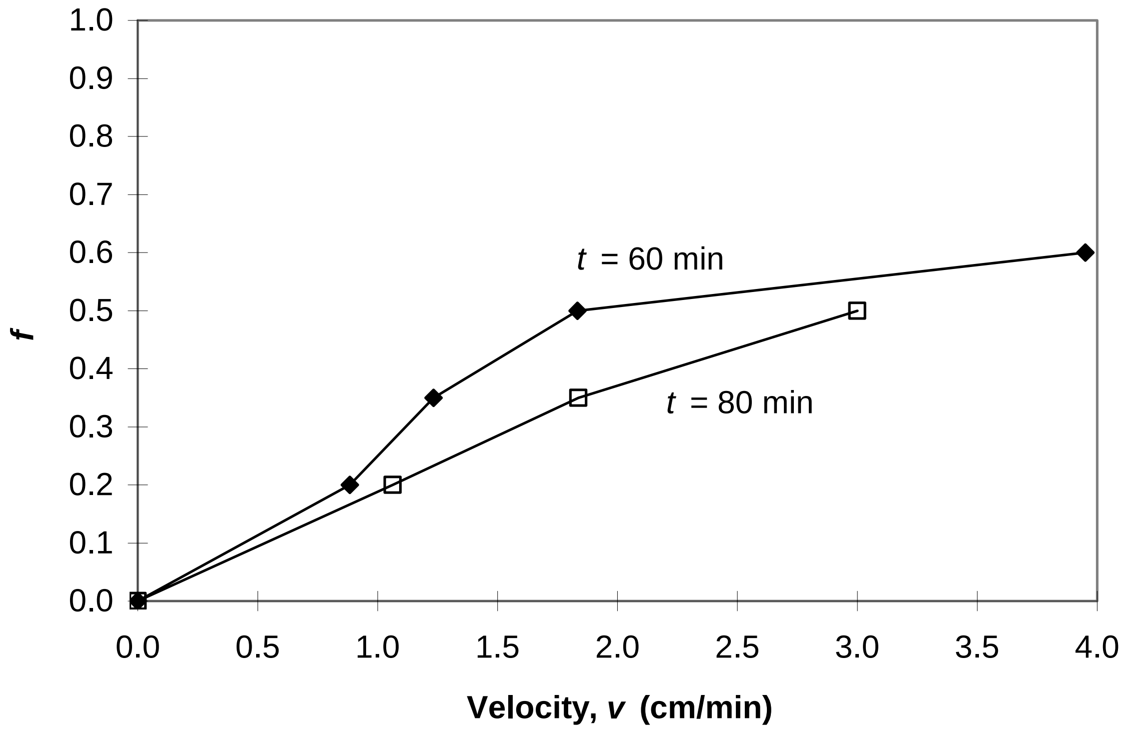
$$\left(\frac{1.16\text{m}}{2.50\text{m}}\right)(80\% - 65\%) + \left(\frac{0.43\text{m}}{2.50\text{m}}\right)(100\% - 80\%)$$

$$= 72.1\%$$

(b) The curves of f vs. v at each time can be drawn based on the information from the settling curves. For example, at $t = 80$ min, we found that the 80% removal isopleth was at a depth of 0.85 m. The implication is that the particles that remained at that depth (20% of the influent particles) had average settling velocities less than 0.85 m / 80 min, or 1.06 cm/min during that 80 min period. Similar calculations have been made for the other isopleths at 80 min and all the isopleths at $t = 40$ min, and are summarized below. (The data at $t = 40$ min are shown on the following graph.) The f vs. v curves are then plotted. As expected, the fraction of the particles that have settling velocities less than any given v decreases as t increases from 40 to 80 min.



t (min)	Isopleth (f value)	h (m)	$v = h/t$ (cm/min)
60	80% ($f = 0.20$)	0.53	0.88
60	65% ($f = 0.35$)	0.74	1.23
60	50% ($f = 0.50$)	1.10	1.83
60	40% ($f = 0.60$)	2.37	3.95
80	80% ($f = 0.20$)	0.85	1.06
80	65% ($f = 0.35$)	1.47	1.84
80	50% ($f = 0.50$)	2.40	3.00



The value of v_{crit} based on the 80 min settling time is:

$$v_{crit} = \frac{2.5\text{ m}}{80\text{ min}} = 0.031\frac{\text{m}}{\text{min}} = 3.1\frac{\text{cm}}{\text{min}}$$

After the water has passed through the settling basin, which has a residence time of 80 min, the plot of f vs. v corresponding to the 80-min data provides the same information as a plot of f vs. v for a Type 1 suspension. The only difference is that, if the particles are non-interacting (i.e., if the suspension is Type 1), the values of v are the constant velocities at which particles have settled during the whole time they are in the basin; by contrast, if the particles are flocculant (Type 2 suspension), the values of v respresent the *average* settling velocities of particles during their time in the basin, including early times when they probably settled more slowly and later times when they settled more rapidly. Regardless, the analysis of the f vs. v curve to estimate η is the same. The value of η can therefore be determined by comparing the area above the curve (for $t = 80$ min) to the area of the whole rectangle, within the limits $0 < v < v_{crit}$ and $0 < f < 1$.

In the example system, the largest value of v for which data are available is 3.0 cm/min, so we need to extrapolate the data slightly to extend it to $v_{crit} = 3.1$ cm/s. Doing that and using the trapezoidal rule to estimate the area under the f vs. v curve, we find:

$$\begin{aligned}\begin{pmatrix}\text{Area under}\\ f \text{ vs. } v \text{ curve}\end{pmatrix} &= \left(\frac{1}{2}\right)\left(1.06\frac{\text{cm}}{\text{min}}\right)(0.2) + \left([1.84-1.06]\frac{\text{cm}}{\text{min}}\right)\left(\frac{0.20+0.35}{2}\right)\\ &\quad + \left([3.00-1.84]\frac{\text{cm}}{\text{min}}\right)\left(\frac{0.35+0.50}{2}\right) + \left([3.10-3.00]\frac{\text{cm}}{\text{min}}\right)\left(\frac{0.50+0.52}{2}\right)\\ &= 0.865\frac{\text{cm}}{\text{min}}\end{aligned}$$

$$\begin{pmatrix}\text{Area of complete}\\ f \text{ vs. } v \text{ rectangle}\end{pmatrix} = v_{crit} * 1.0 = 3.10\frac{\text{cm}}{\text{min}}$$

$$\eta = \frac{\begin{pmatrix}\text{Area above}\\ f \text{ vs. } v \text{ curve}\end{pmatrix}}{\begin{pmatrix}\text{Area of complete}\\ f \text{ vs. } v \text{ rectangle}\end{pmatrix}} = 1 - \frac{\begin{pmatrix}\text{Area under}\\ f \text{ vs. } v \text{ curve}\end{pmatrix}}{\begin{pmatrix}\text{Area of complete}\\ f \text{ vs. } v \text{ rectangle}\end{pmatrix}} = 1 - \frac{0.865\frac{\text{cm}}{\text{min}}}{3.10\frac{\text{cm}}{\text{min}}} = 1 - 0.279 = 0.721 = 72.1\%$$

As we would anticipate, the result from this analysis is identical to that from the preceding analyses (because they are all using different approaches to analyze the same thing).