FLOW EQUALIZATION

Flow equalization is the process of mitigating changes in flow rate through a portion of a system by providing storage to hold water when it is arriving too rapidly, and to supply additional water when it is arriving less rapidly than desired. In many water treatment systems, it is useful to implement flow equalization downstream of the plant, because the downstream demand is not steady, but in general, the treatment processes work more efficiently if the flow rate through them is steady. In treating wastewater, the rate at which the waste arrives at the treatment process might vary dramatically during the day, so it is convenient to equalize the flow before feeding it to the various treatment steps. In either case, the engineering issue is deciding how large an equalization basin is required to allow a steady, the treatment processes to operate with a steady, average flow.

Consider, for example, an equalization basin (i.e., a reservoir) downstream of a drinking water treatment plant. The flow rate out of the basin varies in a predictable way, but we wish to treat the raw water at a steady, average rate. How big a basin is required so that we can store the "excess" treated water until it is needed later? We can answer this question by writing a mass balance on water, with the basin as the control volume. Water is neither generated nor destroyed by any chemical reactions taking place in the basin, so $r = 0$ in the mass balance. Also, the "concentration" of water in the tank is constant and equal to the density of water. The required values of Q_{out} are known as a function of time, and we intend to operate with Q_{in} equal to Q_{avg} at all times. The mass balance can therefore be written as follows:

$$\begin{pmatrix} \text{Rate of change} \\ \text{of amount of} \\ \text{water in the system} \end{pmatrix} = \begin{pmatrix} \text{Net rate } [\text{in } - \text{ out}] \\ \text{at which water enters} \\ \text{the system by flow} \end{pmatrix} + \cancel{\begin{pmatrix} \text{Rate at which} \\ \text{water is generated} \\ \text{by reaction} \end{pmatrix}}$$

$$\cancel{\rho} \frac{dV(t)}{dt} = \cancel{\rho} \left(Q_{\text{avg}} - Q_{\text{out}}(t) \right)$$

$$\frac{dV(t)}{dt} \approx \frac{\Delta V(t)}{\Delta t} = Q_{\text{avg}} - Q_{\text{out}}(t)$$

The preceding equation simply says that the rate of change in the volume of water stored in the basin over any short time period Δt equals the difference between the influent (average) and effluent flow rates during that period. Rearranging slightly:

$$\Delta V(t) = \left(Q_{\text{avg}} - Q_{\text{out}}(t) \right) \Delta t$$

To use the equation to determine the required storage volume, we compute the change in the amount of water in the reservoir over time, start at some time that we designate as $t = 0$. We don't know much water is in the basin at $t = 0$, so we just call that volume $V(0)$ and deal with it later. We can then use the known values of Q_{avg} and $Q(t)$ to compute the change in the volume of water in the basin between $t = 0$ and any future time; i.e., we can compute $V(t) - V(0)$ for all future t. If $V(t) - V(0) < 0$, then the implication is that more water has been withdrawn from the tank since $t = 0$ than has been provided to it. For this to be possible, there must have been a volume of water in the tank at $t = 0$ equal to at least $V(0) - V(t)$. Correspondingly, by finding the *maximum* negative value $V(t) - V(0)$, we can identify the *minimum* amount of water that must be in the basin at $t = 0$ to assure that the system meets the future demand for water.

By the same token, if $V(t) - V(0) > 0$, then the cumulative input of water to the basin since $t = 0$ has been greater than the cumulative removal; i.e., this has been a period during which more water was treated than was needed. We need to store this water for future use, so the available, empty volume in the basin at $t = 0$ must be large enough to accept this water. In this case, the *maximum* positive value of $V(t) - V(0)$ indicates the *minimum* amount of storage that must be available at $t = 0$ to hold all the "excess" water that will be treated and then held until it is demanded.

The required volume of the basin equals the sum of the amount of water that must be present at $t = 0$ (the *maximum* negative value $V(t) - V(0)$) and the amount of empty space that must be available at that time for future use (*maximum* positive value of $V(t) - V(0)$). This sum can easily be determined either with a spreadsheet or by plotting $V(t) - V(0)$ and determining the vertical gap between its minimum and maximum values.

Example. The water demand from a community during a typical day is summarized below, in half-hour increments. If the treatment plant supplying the water is to be operated at a steady rate, how large an equalization basin should be constructed?

t (h)	$Q(t)$ (m^3/min)	T (h)	$Q(t)$ (m^3/min)	t (h)	$Q(t)$ (m^3/min)
0	8	8	18	16	14.8
0.5	6	8.5	17	16.5	15.3
1	5	9	14.5	17	16.6
1.5	3	9.5	14	17.5	17.5
2	2	10	13.5	18	17.5
2.5	2.2	10.5	13.5	18.5	18.5
3	1.8	11	14.5	19	19.5
3.5	2	11.5	15	19.5	16
4	2.4	12	15.5	20	13
4.5	3	12.5	16	20.5	14
5	4.4	13	15	21	13
5.5	6	13.5	13.5	21.5	12
6	10	14	13	22	11
6.5	13	14.5	13.2	22.5	11
7	22	15	14	23	9.5
7.5	22.5	15.5	14.3	23.5	9
				24	8

Solution. The average flow rate during the whole day (Q_{avg}) can be computed based on the given flow rates, and turns out to be 12.0 m^3/min. (Note that $t = 0$ is essentially the same time as $t = 24$, so those values should be used only once in the averaging.) The average flow during each half-hour interval ($Q_{\text{avg},i}(t)$) can also be computed, as the mean of the values at the ends of the interval; these flow rates are plotted in Figure 1.

The difference $Q_{\text{avg}} - Q_{\text{avg},i}(t)$ is the "excess" production rate. That is, it is the rate at which clean water is being produced above that which is needed at that instant; these values are plotted in Figure 2. Correspondingly, the excess volume of water produced during any interval Δt is $\{Q_{\text{avg}} - Q_{\text{avg},i}(t)\}\Delta t$. If this value is positive, it represents the volume of water that must be put into storage for later use; if it is negative, it is the volume of water that must be removed from storage during the interval to meet the demand.

We can then compute the cumulative excess volume of water produced since the start of the cycle (which, for this example, is midnight). To do this, we define the cumulative excess volume at midnight as zero, and we compute the cumulative excess volume at each subsequent time as the value at the previous time plus the excess production during the preceding interval. The results, shown in Figure 3, indicate that the cumulative storage requirement has a maximum of 2297.5 m^3 at Hour 6.5 (6:30am) and a minimum of –218.5 m^3 at Hour 21.5 (9:30pm). The interpretation is that, at the beginning of the cycle, there would have to be at least 2297.5 m^3 of available (empty) space in the reservoir to hold all the excess water that will be produced over the upcoming period, and there would have to be at least 218.5 m^3 of water stored in the reservoir to supply the volume to be needed later in the day. The total reservoir volume would have to be the sum of these values, or 3216 m^3.

Figure 4 shows the volume of water in the basin as a function of time. The trend follows that in Figure 3 perfectly, but the vertical axis is adjusted by 218.5 m^3, so that it shows the minimum volume stored as zero at 9:30pm. In reality, one would want to provide a safety factor by having the reservoir be somewhat larger than the minimum volume computed above, in which case the minimum volume stored would always be somewhat larger than zero.

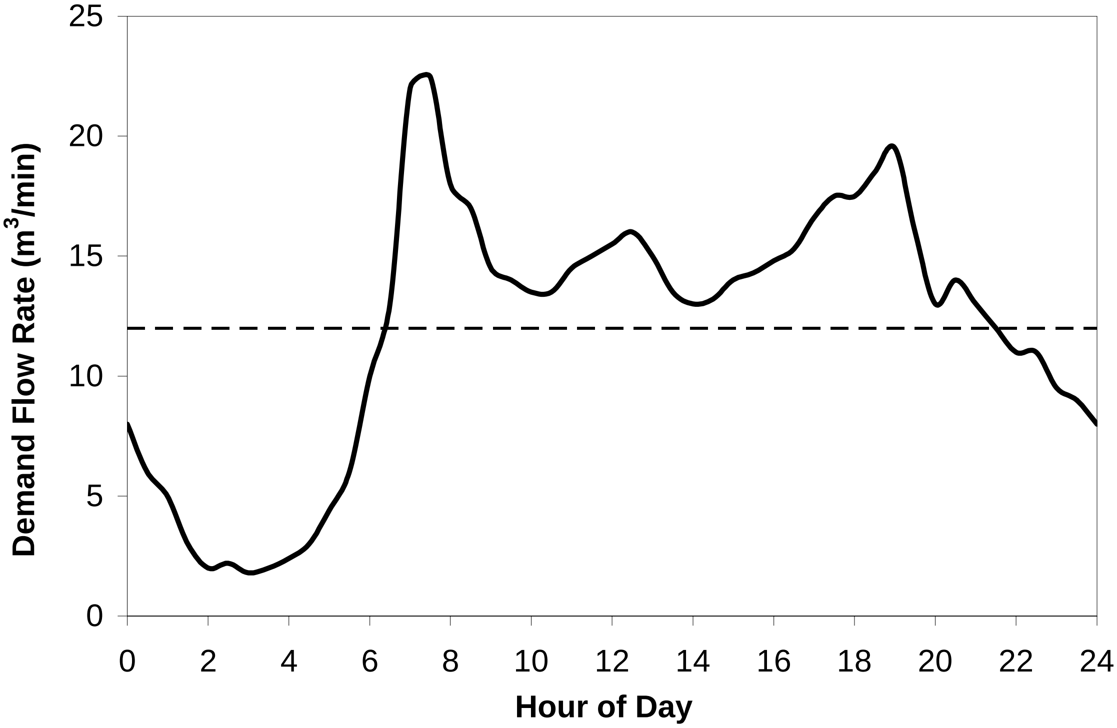

Figure 1. Changes in the demand rate for water during a typical day.

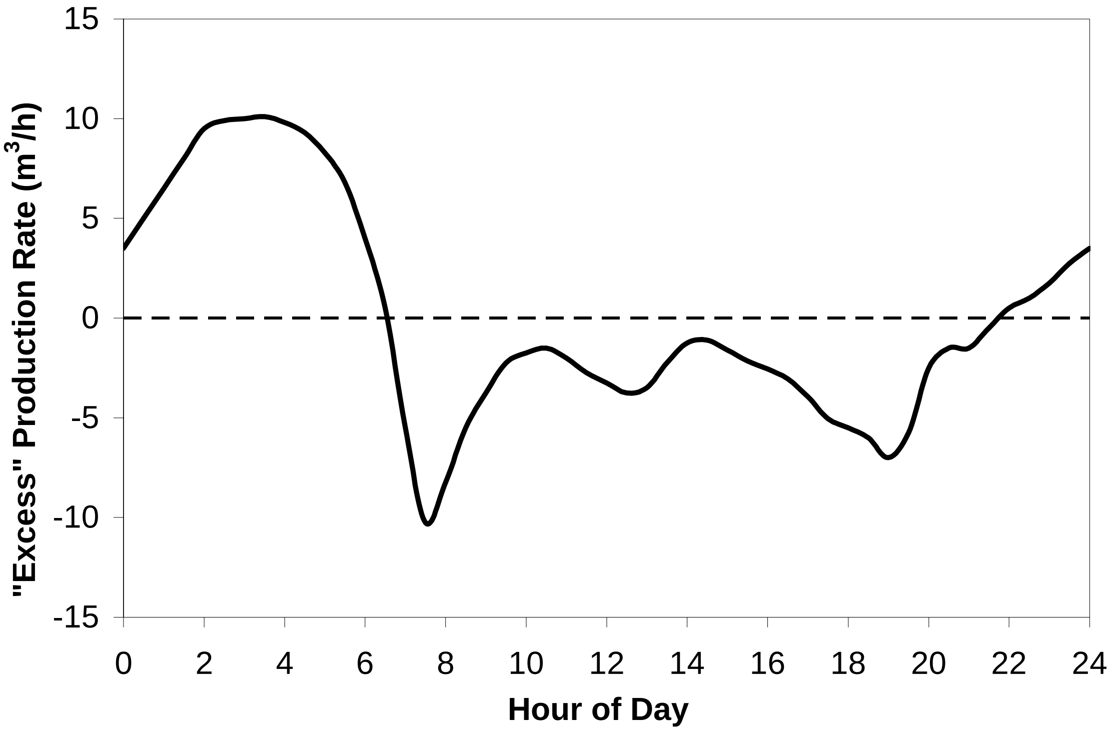

Figure 2. Water production rate in excess of the demand.

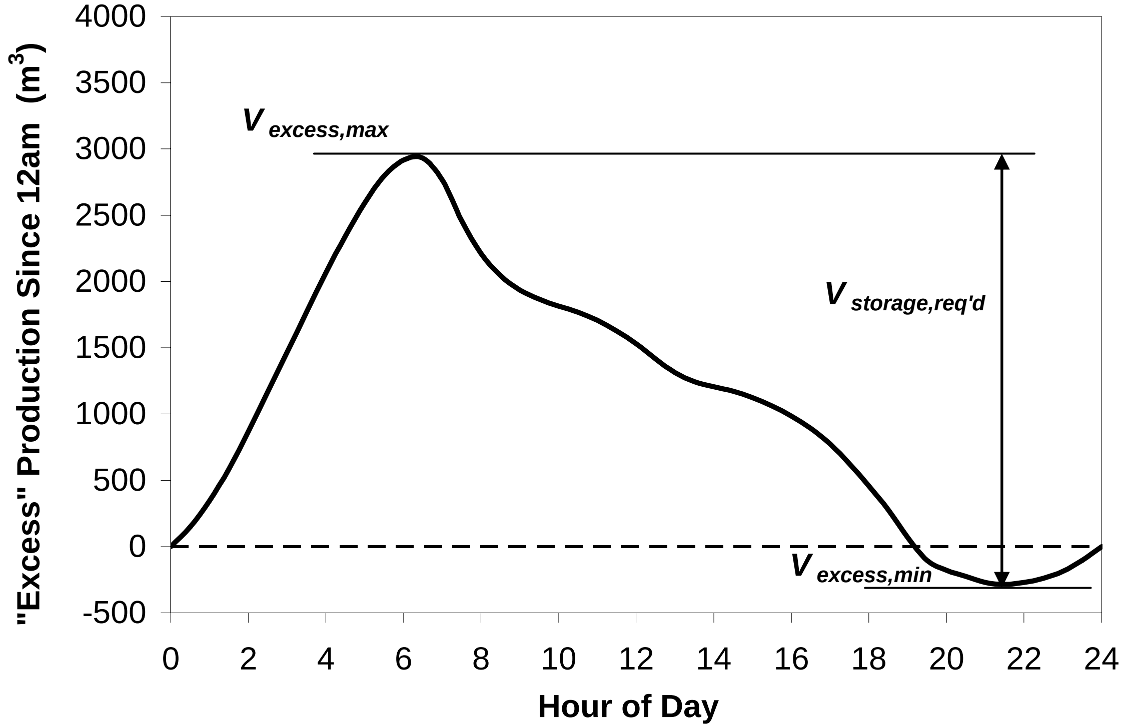

Figure 3. The cumulative volume of water treated above that which has been needed, since midnight.

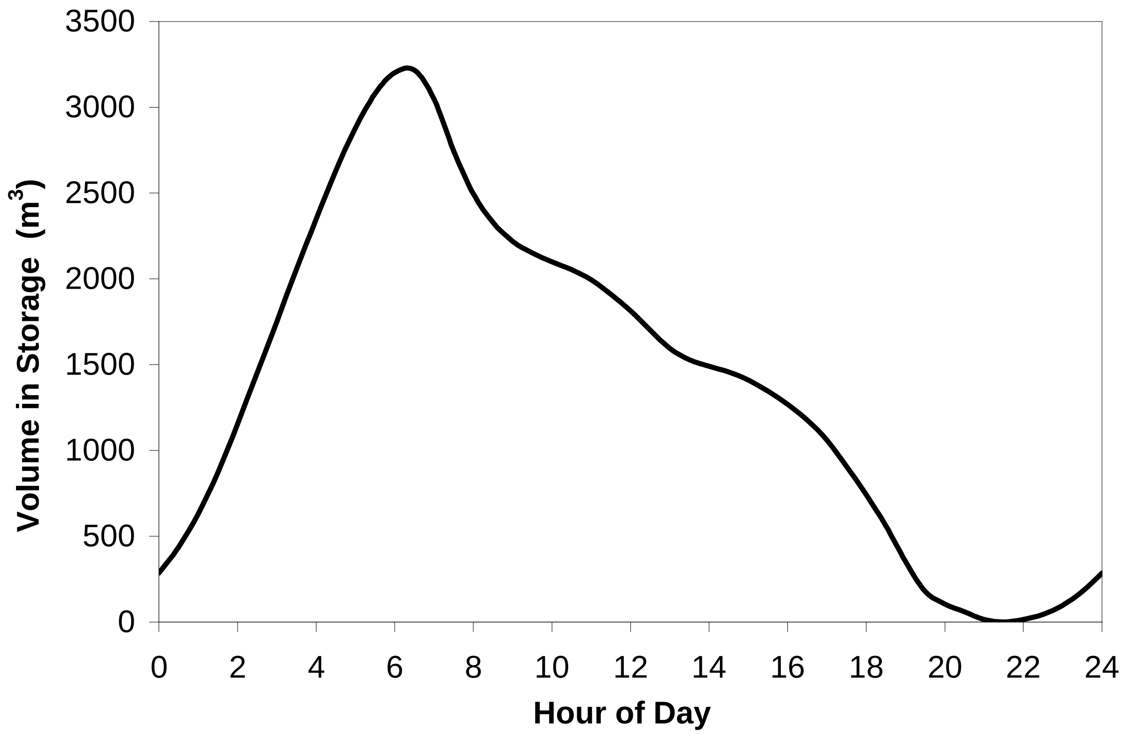

Figure 4. Volume of water stored in the reservoir at different times during the day.