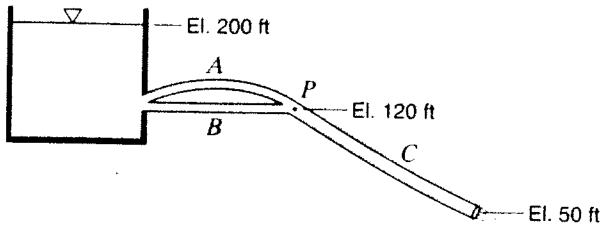
SAMPLE PROBLEM 8.18 Three pipes A, B, and C are interconnected as in Fig. S8.18. The pipe characteristics are as follows:

Pipe	D (in)	L (ft)	f
A	6	2000	0.020
B	4	1600	0.032
C	8	4000	0.024



Find the rate at which water will flow in each pipe. Find also the pressure at point P. All pipe lengths are much greater than 1000 diameters, so neglect minor losses.

Solution

Eq. (5.28):
$$0 + 200 + 0 - 0.020\frac{2000}{6/12}\frac{V_A^2}{2g} - 0.024\frac{4000}{8/12}\frac{V_C^2}{2g} = 0 + 50 + \frac{V_C^2}{2g}$$

i.e.,
$$150 = 80\frac{V_A^2}{2g} + 145\frac{V_C^2}{2g} \qquad (1)$$

Continuity:
$$6^2V_A + 4^2V_B = 8^2V_C$$

i.e.,
$$36V_A + 16V_B = 64V_C \qquad (2)$$

Also,
$$h_{fA} = h_{fB} = 0.020\frac{2000}{6/12}\frac{V_A^2}{2g} = 0.032\frac{1600}{4/12}\frac{V_B^2}{2g}$$

i.e.,
$$80V_A^2 = 153.6V_B^2, \quad V_B = 0.722V_A$$

Substituting into (2):
$$36V_A + 16(0.722V_A) = 64V_C$$
$$47.5V_A = 64V_C, \quad V_A = 1.346V_C$$

Substituting into (1):
$$150 = 80\frac{(1.346V_C)^2}{2g} + 145\frac{V_C^2}{2g} = 289.9\frac{V_C^2}{2g}$$
$$V_C^2 = 2(32.2)150/289.9 = 33.3$$
$$V_C = 5.77 \text{ fps}, \quad Q_C = A_CV_C = (0.349)5.77 = 2.01 \text{ cfs} \qquad \textit{ANS}$$
$$V_A = 1.346V_C = 7.77 \text{ fps}, \quad Q_A = (0.1963)7.77 = 1.526 \text{ cfs} \qquad \textit{ANS}$$
$$V_B = 0.722V_A = 5.61 \text{ fps}$$
$$Q_B = A_BV_B = (0.0873)5.61 = 0.489 \text{ cfs} \qquad \textit{ANS}$$

As a check, note that $Q_A + Q_B = Q_C$ is satisfied.

To find the pressure at P:

Eq. (5.28):
$$0 + 200 + 0 - 80\frac{V_A^2}{2g} = \frac{p_P}{\gamma} + 120$$
$$\frac{p_P}{\gamma} = 80 - 80\frac{(7.77)^2}{2(32.2)} = 5.01 \text{ ft}$$

Note: This example uses small arrows (↻ or ↺) to indicate the direction of flow, rather than + and −

SAMPLE PROBLEM 8.19 If the flow into and out of a two-loop pipe system are as shown in Fig. S8.19, determine the flow in each pipe using only a basic scientific calculator. The K values for each pipe were calculated from the pipe and minor loss characteristics and from an assumed value of f, and $n = 2$.

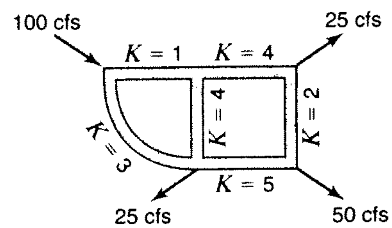
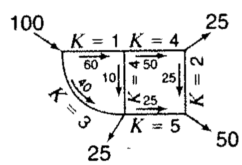


Figure S8.19

Solution

As a first step, assume a flow in each pipe such that continuity holds at all junctions. Take clockwise flows as positive. Calculate ΔQ for each loop, make corrections to the assumed Qs, and repeat several times until the ΔQs are quite small.



First approximation

Left loop

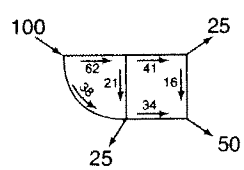
$h_L = KQ_0^n$	$n\lvert KQ_0^{n-1}\rvert$
$1 \times 60^2 = 3{,}600$ ↻	$1 \times 2 \times 60 = 120$
$4 \times 10^2 = 400$ ↻	$4 \times 2 \times 10 = 80$
$3 \times 40^2 = 4{,}800$ ↺	$3 \times 2 \times 40 = 240$
800 ↺	440

$$\Delta Q_1 = \frac{-(-800)}{440} \approx 2 ↻$$

Right loop

$h_L = KQ_0^n$	$n\lvert KQ_0^{n-1}\rvert$
$4 \times 50^2 = 10{,}000$ ↻	$4 \times 2 \times 50 = 400$
$2 \times 25^2 = 1{,}250$ ↻	$2 \times 2 \times 25 = 100$
$4 \times 10^2 = 400$ ↺	$4 \times 2 \times 10 = 80$
$5 \times 25^2 = 3{,}125$ ↺	$5 \times 2 \times 25 = 250$
7,725 ↻	830

$$\Delta Q_1 = \frac{-(+7725)}{830} \approx 9 ↺$$



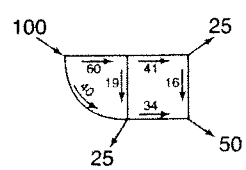
After first correction

$h_L = KQ_0^n$	$n\lvert KQ_0^{n-1}\rvert$
$1 \times 62^2 = 3{,}844$ ↻	$1 \times 2 \times 62 = 124$
$4 \times 21^2 = 1{,}764$ ↻	$4 \times 2 \times 21 = 168$
$3 \times 38^2 = 4{,}332$ ↺	$3 \times 2 \times 38 = 228$
1,276 ↻	520

$$\Delta Q_2 = \frac{-(+1276)}{520} \approx 2 ↺$$

$h_L = KQ_0^n$	$n\lvert KQ_0^{n-1}\rvert$
$4 \times 41^2 = 6{,}724$ ↻	$4 \times 2 \times 41 = 328$
$2 \times 16^2 = 512$ ↻	$2 \times 2 \times 16 = 64$
$4 \times 21^2 = 1{,}764$ ↺	$4 \times 2 \times 21 = 168$
$5 \times 34^2 = 5{,}780$ ↺	$5 \times 2 \times 34 = 340$
308 ↺	900

$$\Delta Q_2 = \frac{-(-308)}{900} \approx 0$$



After second correction

Further corrections can be made if greater accuracy is desired.

15.11 *At its BEP, a given pump develops 30 ft of head when operating at 720 rpm. The flow rate is 4.2 cfs. Find the head and flow rate for a homologous pump operating at 600 rpm if its diameter is 90% that of the given pump.*

BG

Eq. 15.4: $Q \propto nD^3$, $Q = 4.2(600/720)0.9^3 = 2.55$ cfs ◀

Eq. 15.5: $h \propto n^2D^2$, $h = 30(600/720)^2 0.9^2 = 16.88$ ft ◀

15.4.1 *A model centrifugal pump has a scale ratio of 1:15. The model when tested at 3600 rpm, delivered 0.10 m³/s of water at a head of 40 m with an efficiency of 80 percent. Assuming the prototype has an efficiency of 88 percent, what will be its speed, capacity, and power requirement at a head of 50 m?*

SI

Eq. 15.5: $h_m = K_h D_m^2 n_m^2$; $40 = K_h(D_p/15)^2 3600^2$; $K_h = 40(225)/[D_p^2 3600^2] = 0.000\,694/D_p^2$

Eq. 15.5: $h_p = K_h D_p^2 n^2 = 50 = (0.000\,694/D_p^2)D_p^2 n_p^2$; $n_p^2 = 72{,}046$; $n_p = 268$ rpm ◀

From Eq. 15.4: $Q \propto D^3 n$; $Q_p = Q_m(15)^3(268/3600) = 251Q_m$; $Q_p = 251(0.10) = 25.1$ m³/s ◀

From Eq. 15.2: $P_p = \gamma Q_p h_p/\eta = 9.81(25.1)50/0.88 = 14\,000$ kW ◀

15.4.2 *A 19½-in-diameter centrifugal-pump impeller discharges 20 cfs at a head of 100 ft when running at 1200 rpm. (a) If its efficiency is 85 percent, what is the brake horsepower, i.e., what is the horsepower input to the shaft of the pump? (b) If the same pump were run at 1500 rpm, what would be h, Q, and the brake horsepower for homologous conditions?*

BG

(*a*) Eq. 15.2: $bhp = \gamma Qh/(550\eta) = 62.4(20)100/[500(0.85)] = 267$ bhp ◀

(*b*) Eq. 15.5: $h = 100(1500/1200)^2 = 156.3$ ft ◀

Eq. 15.4: $Q = 20(1500/1200) = 25$ cfs ◀

Eq. 15.6: $bhp = 267(1500/1200)^3 = 521$ bhp ◀

Sample Problem 15.4 The pump whose characteristic curve is shown in Fig. 15.6 when operating at 1450 rpm is used to pump water from reservoir A to reservoir B through an 18-in-diameter pipe ($f = 0.032$) 500 ft long. Neglecting minor losses, find the flow rate for the following conditions: (*a*) reservoir water-surface elevations are identical; (*b*) water-surface elevation of reservoir B is 20 ft higher than that in reservoir A; (*c*) water-surface elevation of reservoir B is 65 ft higher than that in reservoir A. Assume f does not change with flow rate. Also find the efficiency by referring to Fig. 15.8.

Solution

The pump characteristic curve is given by Fig. 15.6. Note that this curve is the same as Curve 1 of Fig. 15.8.

Energy equation: $$0 + h - h_L = \Delta z$$

from which we find the equation of the system characteristic:

$$h = \Delta z + h_L = \Delta z + f\frac{L}{D}\frac{V^2}{2g}$$

$$= \Delta z + (0.032)\frac{500}{1.5}\frac{Q^2}{[\pi(1.5)^2/4]^2 2(32.2)}$$

$$= \Delta z + 0.0530Q^2 \qquad (Q \text{ expressed in cfs})$$

Coordinates of system curves:

Q, cfs	Q, gpm	(*a*) h_L, ft	(*b*) $20 + h_L$	(*c*) $65 + h_L$
0	0	0.0	20.0	65.0
10	4480	5.3	25.3	70.3
20	8970	21.2	41.2	86.2
30	13,450	47.8	67.8	(112.8)[7]

[7] For plotting purposes only. The pump cannot develop that much head when operating at 1450 rpm.

By plotting these system curves on Fig. 15.8 we find h and Q at the points of intersection with Curve 1. To find pump efficiency we note the location of the points of intersection and interpolate between contours of equal efficiency.

(*a*) $h \approx 45$ ft and $Q \approx 13{,}200$ gpm, $\eta \approx 71\%$
(*b*) $h \approx 55$ ft and $Q \approx 11{,}700$ gpm, $\eta \approx 81\%$ *ANS*
(*c*) $h \approx 73$ ft and $Q \approx 5800$ gpm, $\eta \approx 60\%$

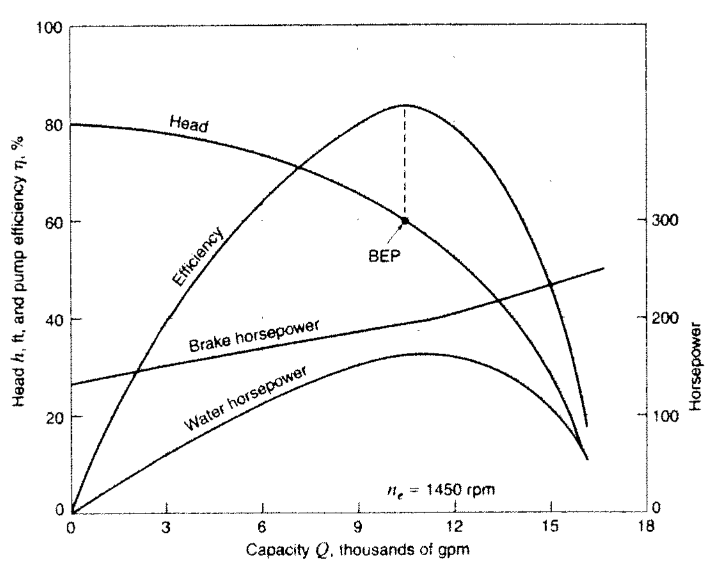


Figure 15.6

15.7.1 *Repeat Sample Prob. 15.4 for the case where the length of the 18-inch diameter pipe is 1000 ft.*
Sample Prob. 15.4: $f = 0.032$, $L = 500$ ft, $D = 18$ in, resulting in $h = \Delta z + 0.0530Q^2$.

BG

Refer to the solution to Sample Prob. 15.4 (textbook page 661).

This 1000 ft pipe is twice as long as the 500 ft pipe of Sample Prob. 15.4.

$\therefore$ here, $h = \Delta z + 2(0.0530)Q^2 = \Delta z + 0.1061Q^2$ where Q is in cfs. Coordinates of system curves:

Q (cfs)	Q (gpm)	(*a*) $h = h_f$ (ft)	(*b*) $20 + h_f$	(*c*) $65 + h_f$
0	0	0	20	65
10	4490	10.6	30.6	75.6
20	8980	42.4	62.4	107.4
30	13,460	95.6	115.6	160.6

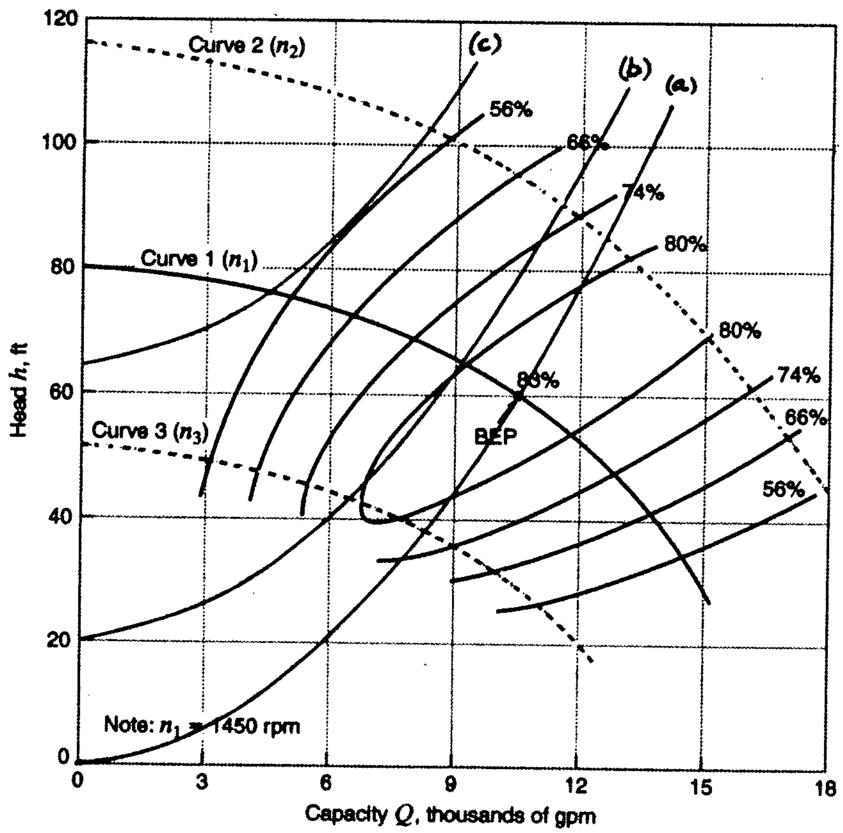


Figure 15.8, modified

By plotting the system curves on Fig. 15.8 we find the points of intersection with Curve 1:

(*a*) $h \approx 60$ ft and $Q \approx 10{,}500$ gpm ◄

(*b*) $h \approx 65$ ft and $Q \approx 9200$ gpm ◄

(*c*) $h \approx 76$ ft and $Q \approx 4500$ gpm ◄

From the same figure, approx. pump efficiencies are: 83%, 79%, and 52% respectively ◄