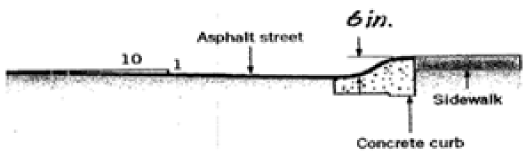
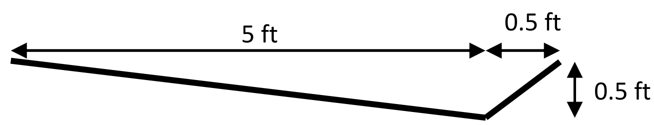
CEE 345, Part 2, Spring 2012, Final Exam Solutions

If you do not know a value that you believe you need to solve a problem, make a reasonable assumption for that value, state the assumption clearly, and proceed. For any of the problems, if you get to a point where you believe an iterative solution is required, just carry out one iteration and then indicate what guess you would make for the next iteration.

1. Rainwater flows down a street whose cross-section is shown below. The schematic characterizes the driving surface, where the slope is 1:10, plus the curb section, where the surface rises to meet the sidewalk. The street has a downhill slope of 2^o in the direction of travel (into the page, for the 2-D schematic shown). For modeling purposes, you decide to simplify the geometry and represent the two sections together as a triangle (shown on the right), and to assume that both sections have a Manning coefficient of 0.014. What is the maximum flowrate that can be supported without allowing water to overflow the curb onto the sidewalk?





Answer. The flow rate of the water when it fills the channel formed by the street and curb can be found from the Manning equation for BG units as:

$$Q = VA = \frac{1}{n} R_h^{2/3} S_o^{1/2} A$$

The values of n and S_o are given, and we can find R_h and A from geometry:

$$P_{wetted} = \sqrt{(5\text{ ft})^2 + (0.5\text{ ft})^2} + \sqrt{(0.5\text{ ft})^2 + (0.5\text{ ft})^2} = 5.73\text{ ft}$$

$$A = 0.5(5\text{ ft})(0.5\text{ ft}) + 0.5(0.5\text{ ft})(0.5\text{ ft}) = 1.375\text{ ft}^2$$

$$R_h = \frac{A}{P_{wetted}} = \frac{1.375\text{ ft}^2}{5.73\text{ ft}} = 0.240\text{ ft}$$

The street has a slope of 2^o, so S_o is $\tan(2^o)$, or 0.035. The flow rate is therefore:

$$Q = \frac{1.49}{n} R_h^{2/3} S_o^{1/2} A = \frac{1.49}{0.014}(0.240)^{2/3}(0.035)^{1/2}(1.375) = 10.56\frac{\text{ft}^3}{\text{s}}$$

2. Water in a 5-ft-wide, rectangular, horizontal channel has a flowrate of $Q = 30$ ft^3/s and a depth of $y_1 = 2.5$ ft. Downstream, the channel has a sill where the bottom elevation increases by 0.6 ft. What is the maximum frictional headloss that can occur between the upstream point and the end of the sill without causing the water to 'dam up'?

Answer. The flow per unit width, upstream velocity, and specific energy can be computed directly from the given information:

$$q = \frac{Q}{b} = \frac{30 \text{ ft}^3/\text{s}}{5 \text{ ft}} = 6\frac{\text{ft}^2}{\text{s}}$$

$$V_1 = \frac{Q}{by_1} = \frac{30 \text{ ft}^3/\text{s}}{(5 \text{ ft})(2.5 \text{ ft})} = 2.40 \text{ ft/s}$$

$$E_1 = y_1 + \frac{V_1^2}{2g} = 2.5 \text{ ft} + \frac{(2.4 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} = 2.589 \text{ ft}$$

The maximum allowable headloss is the headloss that causes the conditions on the top of the sill to just become critical. The depth and specific energy under these conditions can be found as:

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{\left[6 \text{ ft}^2/\text{s}\right]^2}{32.2 \text{ ft/s}^2}\right)^{1/3} = 1.038 \text{ ft}$$

$$E_c = 1.5y_c = 1.5(1.038 \text{ ft}) = 1.557 \text{ ft}$$

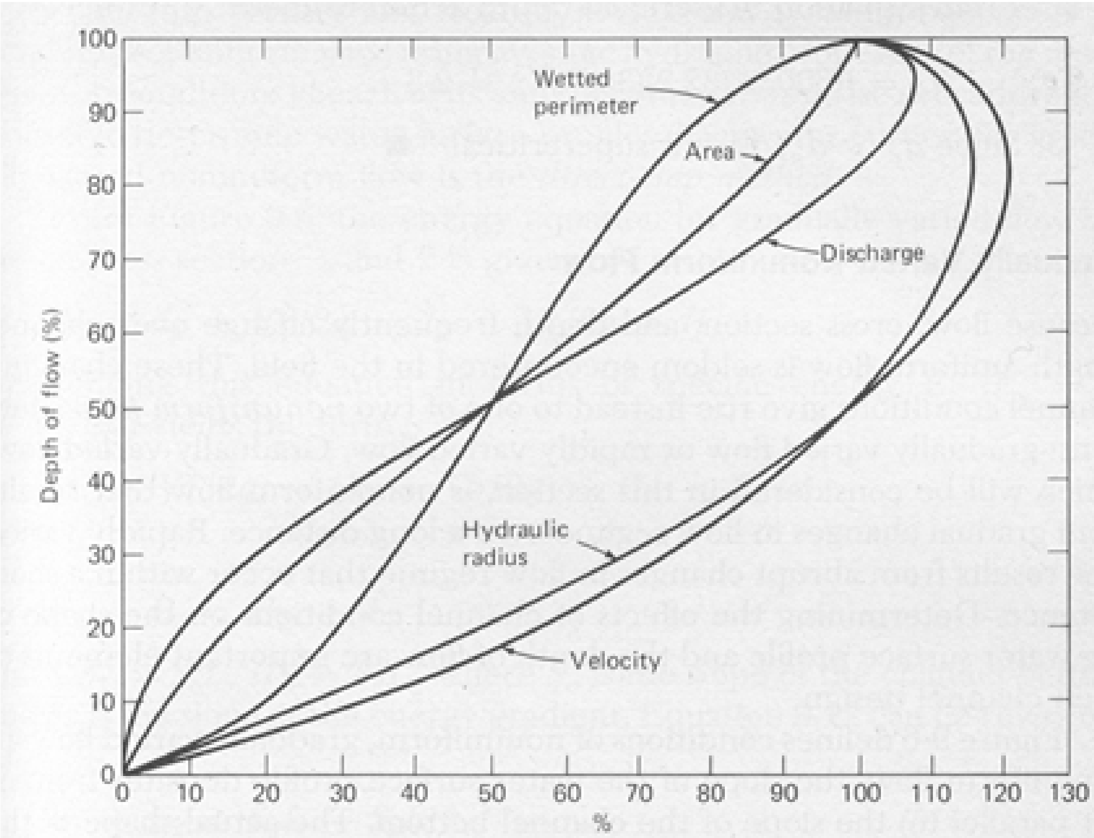
The specific energy on the sill equals the specific energy upstream minus the losses of specific energy due to the change in elevation of the channel bottom (Δz) and the headloss, so:

$$E_2 = E_c = E_1 - \Delta z - h_L$$

$$h_L = E_1 - \Delta z - E_c = (2.589 - 0.6 - 1.557)\text{ft} = 0.433 \text{ ft}$$

3. (a) What is the specific energy of water flowing at a depth of 1.5 m in a 2-m-diameter pipe on a 1% slope, if the Manning roughness coefficient is 0.020?

   (b) **Not enough info given. Question not graded.**



Answer. (a) The velocity when the pipe is full can be found from the Manning equation (SI units), noting that the hydraulic radius of a full pipe is *D*/4:

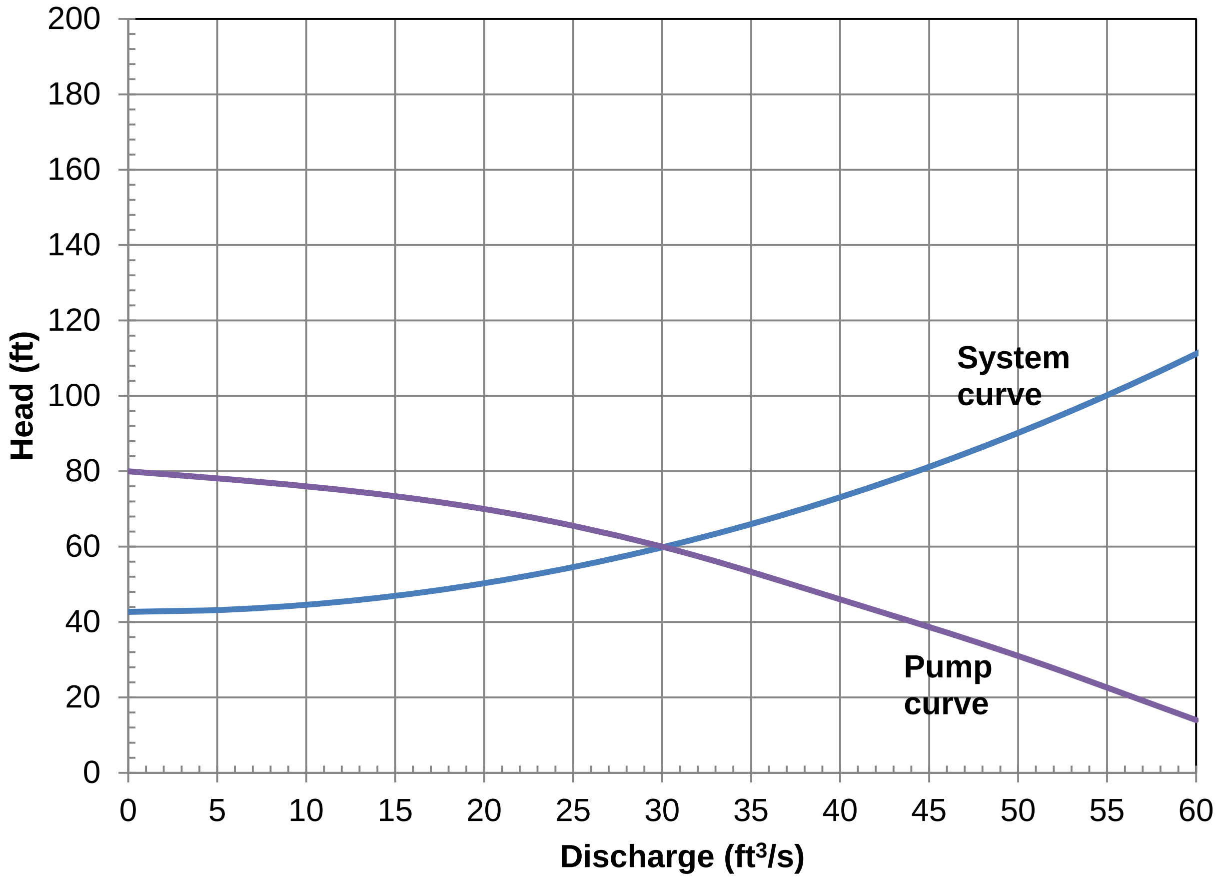
$$V_{full} = \frac{1}{n} R_h^{2/3} S_o^{1/2} = \frac{1}{0.020}\left(\frac{2.0}{4}\right)^{2/3}(0.01)^{1/2} = 3.15\frac{\text{m}}{\text{s}}$$

According to the figure showing the hydraulic sections of a circle, the fluid velocity when a pipe is three-quarters full is approximately 114% of the velocity when it is completely full. The velocity and specific energy under the specified conditions are therefore:

$$V = 1.14\left(3.15\frac{\text{m}}{\text{s}}\right) = 3.59\frac{\text{m}}{\text{s}}$$

$$E = y + \frac{V^2}{2g} = 1.50\text{ m} + \frac{(3.15\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} = 2.16\text{ m}$$

4. A distribution system is currently being serviced by two identical pumps operating in parallel. The system curve and the composite pump curve (considering both pumps together) are shown in the following diagram. The current operation is not meeting the community needs, and an engineer suggests that, by re-plumbing the system so that the pumps are arranged in series, both the water flow rate and the head added by the pumps can be increased. By drawing (roughly) additional curves on the diagram to characterize the system if the re-plumbing is carried out, determine whether you agree or disagree with the engineer's prediction. Explain your logic.



Answer. The system curve is independent of the pumps used and therefore will not change. We can determine the pump curve for each of the two existing pumps by noting that, for pumps operating in parallel, the TDH is the same for both pumps, and the total discharge is the sum of the flow rates through the two pumps. Because the two pipes in the current scenario are identical, this means that, at each value of the TDH, the pump curve for each of the two pumps has one-half the discharge shown for the existing, composite curve. When two such pumps are combined in series, the new composite pump curve has double the TDH at each value of the discharge.

Based on this logic, the pump curves for a single pump and for the two pumps in series are shown in the following diagram. The new operating point indicates that the system will operate with both lower TDH and lower discharge than when the pumps are in parallel, so the engineer was incorrect.