CEE 345, Part 2, Winter 2012, Final Exam Solutions (Open Channel Flow)

1. (a) (8) List and briefly describe the forces that must be considered in an analysis of flow in a trapezoidal channel with a slope of 0.006. (One or two sentences should be enough for each force.) Identify the location where each force acts, and its direction.

   The forces include pressure upstream and downstream, which act in the downstream and upstream directions, respectively; the gravitational force, which acts vertically downward but has a component in the direction of flow; and the shear force, which acts along all solid/fluid boundaries and is in the opposite direction from flow.

   (b) (7) Which forces can be ignored if the flow is uniform, and why can they be ignored?

   The pressure forces can be ignored in uniform flow, because they are of equal magnitude (γh_c) and in opposite directions.

2. (5) When considering open channel flow, what is the difference between the 'normal' depth and the 'critical depth'?

   The normal depth is the depth corresponding to uniform flow; at this depth, the slope of the energy line equals the slope of the channel bottom. The critical depth is the depth at which the specific energy is minimized for the given q and is the depth that separates sub-critical from super-critical flow.

3. (5) For flow in a pipe to be laminar, the criterion is that the Reynolds number be less than about 2000, but for open channel flow, the criterion is that the Reynolds must be less than about 500. What is the major reason for this difference?

   The numerator of the Reynolds number includes a term for the 'characteristic length' of the system under consideration. For pipe flow, this characteristic length is defined as the pipe diameter, whereas for open channel flow, it is defined as the hydraulic radius. For a pipe, $D = 4R_h$, and that accounts for the factor of four difference in the Re value where flow changes from laminar to transitional. Thus, the actual fluid "conditions" are very nearly identical at the transition point in the two types of flow; only the definition of how Re is computed differs.

4. (5) Water in an approximately rectangular channel has a velocity head equal to its depth. Is the flow sub-critical, critical, or super-critical? Explain your reasoning.

   When the depth is the critical depth, the velocity head ($V^2/2g$) equals $0.5\,y$; if the velocity head is greater or less than $0.5\,y$, the flow is super-critical or sub-critical, respectively. Under the conditions specified, the velocity head is greater than $0.5\,y$, so the flow is super-critical.

5. (5) (a) Flow in a horizontal open channel approaches a sill under super-critical conditions. Assuming that friction is negligible, will the velocity of the water increase, decrease, or not change as it passes over the sill? Explain your reasoning.

   When the water passes over the sill, its specific energy declines. For super-critical flow, the system is characterized by the lower leg of the y vs. E curve, which has an increasing value of y for decreasing values of E. Therefore, flow over the sill leads to an increase in y and, based on continuity, a decrease in velocity.

   (5) (b) Will the change in velocity be greater, smaller, or the same, if the sill generates significant frictional resistance? Again, explain your reasoning.

   Frictional resistance will cause the decline in specific energy to be even larger, so the decline in velocity will be bigger in this case than in the absence of friction. (Note that simply stating that friction causes the water to lose energy and therefore to slow down is not a sufficient answer, because in some cases [if the flow is sub-critical], a loss in energy would cause the velocity to increase. A correct explanation must link the change in velocity to the condition of super-critical flow.)

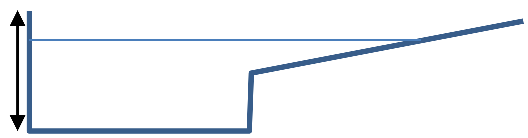
6. (5) Water flows 7 mm deep in a 0.5-m-wide rectangular flume. What would be the percentage error in the velocity of uniform flow if the flow were assumed to be 'wide and shallow'? Would the computed velocity be larger or smaller than without the assumption?

   If the flow is wide and shallow, the friction contributed by the side slopes is ignored, so that R_h *is approximated as being equal to y. If the side slopes are included in the calculation of* R_h, R_h *is* $by/(b+2y)$*. In the scenario of interest,* $b = 500$ *mm and* $y = 7$ *mm, so the value of* R_h *is estimated to be 7 mm if the assumption of wide-and-shallow flow is made, and 6.81 mm if the simplification is not applied. According to the Manning equation, V is proportional to* $R_h^{2/3}$*, so the computed V with the assumption would be a factor of* $(7/6.81)^{2/3}$ *times as large as in the absence of the assumption. This quantity equals 1.019. The computed velocity would therefore be 1.9% higher with the assumption than without it. Thus, treating the flow as wide and shallow leads to an increase in the computed value of V. This result makes sense, since the assumption is equivalent to ignoring a portion of the wetted perimeter and therefore ignoring part of the frictional resistance to flow.*

7. (5) What is the main advantage of using a V-shaped weir compared to a horizontal weir for flow measurement?

   In a V-shaped weir, the cross-section for flow is narrow at the bottom and becomes wider as the depth of flow increases. As a result, this type of weir leads to a much larger response (ΔH) *to a change in flow rate than a rectangular weir does. In particular, it is much easier to detect and accurately measure low flows with a V-shaped weir. This result is also apparent from the fact that the flow rate is proportional to* $H^{5/2}$ *for a V-shaped weir and proportional to* $H^{3/2}$ *for a horizontal weir.*

8. (15) For a channel with the cross-section shown below, what bed slope is required to have a uniform flow of 16 m^3/s when the depth of flow is 1.50 m, if the Manning friction coefficient is 0.015?



The cross-sectional area of flow is the sum of the wetted areas of the wetted rectangular and the triangular areas. The area of the wetted rectangle is 1.5m x 3m, or 4.5m^2. The base and height of the full triangle (not just the wetted portion) are 5m and 1.0m, respectively. Because the triangular section is wetted to 50% of its height, the base and height of the wetted portions are each 50% of those values. Because the area of a triangle is 0.5 x base x height, the wetted area associated with the triangular section is 0.5 x 5m x 1m, or 0.625m^2. The total wetted area, considering both parts of the channel, is therefore 5.125 m^2.

Similarly, the wetted length of the sloped portion of the channel (the hypotenuse of the wetted triangle) is 50% of the full length of that sloped segment, or $0.5\sqrt{1^2+5^2}$ *m ,which equals 2.55 m. The wetted perimeter is therefore (1.5 + 3 + 1 + 2.55) m, or 8.05 m, and the hydraulic radius is:*

$$R_h = \frac{A_{flow}}{P_{wetted}} = \frac{5.125 \text{ m}^2}{8.05 \text{ m}} = 0.637 \text{ m}$$

Rearranging the Manning equation and inserting known values, we find the bed slope as:

$$V = \frac{1}{n} R_h^{2/3} S_o^{1/2}$$

$$Q = \frac{1}{n} R_h^{2/3} S_o^{1/2} A$$

$$S_o = \left(\frac{nQ}{R_h^{2/3} A} \right)^2 = \left(\frac{(0.015)(16)}{(0.637)^{2/3}(5.125)} \right)^2 = 4.00\text{x}10^{-3}$$

9. (10) Water in a horizontal channel is flowing at a velocity of 2.6 m/s and a depth of 0.4 m. If the channel width is 6.0 m, how much power would be dissipated in a hydraulic jump?

We can find the flow rate per unit channel width and the specific energy of the water upstream of the jump as follows:

$$q = V_{up} y_{up} = (2.6 \text{ m/s})(0.4 \text{ m}) = 1.04 \text{ m}^2/\text{s}$$

$$E_{up} = y_{up} + \frac{V_{up}^2}{2g} = 0.4\text{ m} + \frac{(2.6\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 0.745\text{ m}$$

Because the flow rate and channel width are the same downstream as upstream, q remains the same. The water depth, velocity, and specific energy after a hydraulic jump can therefore be found as

$$y_{down} = \frac{y_{up}}{2}\left(\sqrt{1+8\frac{q^2}{gy_{up}^3}}-1\right) = \frac{0.4\text{ m}}{2}\left(\sqrt{1+8\frac{(1.04\text{ m}^2/\text{s})^2}{(9.81\text{ m/s}^2)(0.4\text{ m})^3}}-1\right) = 0.569\text{ m}$$

$$V_{down} = \frac{V_{up}y_{up}}{y_{down}} = \frac{(2.6\text{ m/s})(0.4\text{ m})}{0.569\text{ m}} = 1.83\text{ m/s}$$

$$E_{down} = y_{down} + \frac{V_{down}^2}{2g} = 0.569\text{ m} + \frac{(1.83\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 0.739\text{ m}$$

The energy loss across the hydraulic jump is $E_{up} - E_{down}$, i.e.:

$$h_{L,max} = E_{up} - E_{min} = 0.745\text{ m} - 0.739\text{ m} = 0.006\text{ m}$$

$$P = \gamma Q h_L = \left(9810\frac{\text{N}}{\text{m}^3}\right)\left[(2.6\text{ m/s})(0.4\text{ m})(6.0\text{ m})\right](0.0053\text{ m}) = 324\text{ W}$$

10. (25) Water is flowing with a depth of 2.5 ft and a flow rate of 270 ft^3/s in a 20-ft-wide rectangular channel. The channel then narrows to a width of 15 ft as the bottom drops by 0.2 ft. Assume that the frictional headloss is negligible during this transition.

(a) (4) Is the upstream flow sub-critical, critical, or super-critical?

The critical depth can be found as follows:

$$q_1 = \frac{Q}{b_1} = \frac{270\text{ ft}^3/\text{s}}{20\text{ ft}} = 13.5\frac{\text{ft}^2}{\text{s}}$$

$$y_{c,1} = \left(\frac{q_1^2}{g}\right)^{1/3} = \left(\frac{(13.5\text{ ft}^2/\text{s})^2}{32.2\text{ ft/s}^2}\right)^{1/3} = 1.78\text{ ft}$$

The actual water depth (2.5 ft) is greater than $y_{c,1}$, so the flow is sub-critical.

(b) (7) What are the critical depth and the minimum specific energy needed for steady flow after the channel geometry has changed?

The critical depth can be computed using the same equation as in part a, but for the new geometry. The overall flow rate is the same as before the change in geometry, so the new value of q and the corresponding y_c are*:*

$$q_2 = \frac{Q}{b_2} = \frac{270\ \text{ft}^3/\text{s}}{15\ \text{ft}} = 18\frac{\text{ft}^2}{\text{s}}$$

$$y_{c,2} = \left(\frac{q_2^2}{g}\right)^{1/3} = \left(\frac{\left(18\ \text{ft}^2/\text{s}\right)^2}{32.2\ \text{ft/s}^2}\right)^{1/3} = 2.16\ \text{ft}$$

The minimum specific energy is 1.5 times $y_{c,2}$, or:

$$E_{min,2} = 1.5y_{c,2} = (1.5)(2.16\ \text{ft}) = 3.24\ \text{ft}$$

(c) (7) Will there be 'damming action' (i.e., an increase in the water level) upstream of the transition?

If the water has more than the minimum amount of specific energy needed to support the given flow rate when the channel narrows, then no damming action will be observed. On the other hand, if the water does not have that amount of specific energy, damming action will cause water to build up behind the location where the stream narrows, thereby increasing the upstream specific energy. We can find the upstream specific energy as follows:

$$E_1 = y_1 + \frac{q_1^2}{2g}y_1^{-2} = 2.5\ \text{ft} + \frac{\left(13.5\ \text{ft}^2/\text{s}\right)^2}{2\left(32.2\ \text{ft/s}^2\right)}\left(\frac{1}{2.5\ \text{ft}}\right)^2 = 2.95\ \text{ft}$$

Because we are assuming that frictional energy loss is negligible, the change in specific energy is entirely attributable to the change in elevation of the channel bottom, so:

$$E_2 = E_1 - \Delta z_{channel} = 2.95\ \text{ft} - (-0.2\ \text{ft}) = 3.15\ \text{ft}$$

Because this computed value of E_2 is less than the minimum required, damming action will occur.

(d) (7) What will the water depth at the location where the channel has narrowed to its final width?

Because damming occurs, the downstream flow would be critical, and the depth would be the critical depth, as determined in part b (2.16 ft).