**CEE 345, Spring 2012, Midterm Exam Solutions: Pipe Networks and Pumps**

1. The frictional headloss is given by the D‑W equation as . Although it is true that *f* declines with increasing *V*, the velocity head increases in proportion to *V*2. The dependence of *f* on *V* is such that *f* varies with *V**n*, where *n* has a value between 1 (for laminar flow) and 0 (for fully turbulent flow). Therefore, the product *f*\**V*2 is proportional to *Vm*, where *m* varies between 1 and 2. As a result, *hL* always increases when *V* increases.
2. (a) Under fully turbulent flow conditions, *f* is independent of Re and therefore independent of *V* for flow in a given pipe. Consequently, the D‑W equation predicts that the headloss is proportional to *V*2. The H‑W equation predicts that the headloss is proportional to *V*1.85. Therefore, for fully turbulent flow, the D‑W equation predicts a stronger dependence of *hL*/*l* on *V* than the H‑W equation does.

(b) For laminar flow conditions, *f* equals 64/Re and is therefore proportional to *V*1. Under these conditions, the D‑W equation predicts that *hL*/*l* varies with *V*1. The H‑W equation predicts the same dependence of *hL/l* on *V* (i.e., *hL*/*l* is proportional to *V*1.85) for any value of Re. Therefore, if the flow is laminar, the H‑W equation predicts a stronger dependence of *hL*/*l* on *V* than the D‑W equation does (although the H‑W equation was never intended to be used for laminar flow, so its use under those conditions would be inappropriate).

1. 

From the Moody diagram, under fully turbulent flow conditions (high Re), a friction factor of 0.033 corresponds to a relative roughness of 0.0065. The relative roughness is **/*D*, so



If the headloss is one-third of a velocity head for a travel distance of 1 m, it is 333.3 velocity heads for a travel distance of a km. That is:



Because the pipe is horizontal, the fluid’s elevation head remains constant, and because the pipe has a fixed diameter, the velocity (and therefore the velocity head) remains constant. As a result, the headloss is manifested entirely as a pressure loss. The headloss can be converted to an equivalent pressure by multiplying by the specific weight of water, so:



1. (a) The total headloss is the sum of the frictional headloss in the pipe (i.e., the major headloss) and the minor headlosses associated with the two bends and the loss of velocity when the water enters the tank. The total headloss is:



The fraction of the total headloss attributed to the minor headlosses is therefore:



1. The power lost in the two elbows, and therefore the power that could be saved by eliminating the elbows, is:



1. The expression for headloss for each pipe individually can be written as *hL*= *KQn*, with *n*= 1.5, and *K* for each pipe equal to:



The value of *K* for a pipe that is equivalent to a group of real pipes in parallel is , so, for the combination of pipes A and B:



Pipes C and D are identical to pipes A and B, so *Keq*,C+D is also 0.276. The two pairs of pipes are in series with one another, and the *K* value of a pipe that is equivalent to other pipes in series is the sum of the *K* values of those pipes, so:



Finally, we can use this value in conjunction with the definition of *K* to find the equivalent length:





Because all parameters are specified to have SI units, the length of the equivalent pipe is 35.4 m.

1. The system diagram indicates that the booster pump will be installed in series with the existing pump. Therefore, all the flow will have to go through both pumps, and both pumps will add head to the water. Correspondingly, the total head added to the water (TDH*tot*) will be the sum of the heads added by the two pumps. Based on this logic, for any given flow rate *Q* through the pumps, TDH*tot* will be TDH*orig*+TDH*boost* for that *Q*. This summation can be carried out graphically.

For the pump and system to operate under steady conditions, TDH*tot* will have to equal the headloss in the system, as characterized by the system curve. That equality is satisfied under the conditions where the TDH*tot* curve intersects the system curve. The value of *Q* at the operating point indicates the flow rate through both pumps and the piping. The head added by the original pump is the value of TDH*orig* at that *Q*, and the head added by the booster pump is the value of TDH*boost* at that *Q*. As seen on the following diagram, this condition occurs around a *Q* of 80cfs, at which TDH*orig*= 68 ft, and TDH*boost*= 42 ft, and TDH*tot*= 110 ft.

