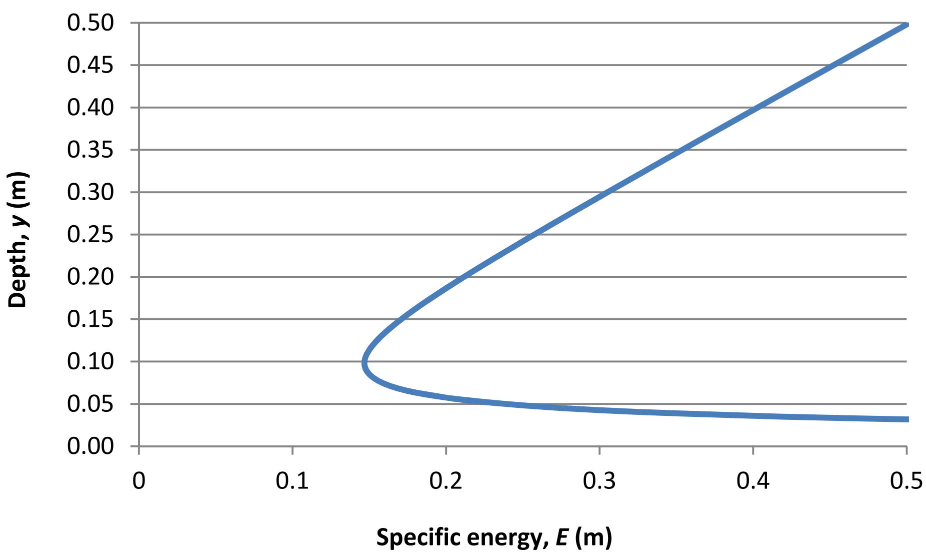
CEE 345 Part 2, Assignment #4, Due 6/1, 11:30am

1. a) The measured velocities, computed as L/t, were 7.66, 7.66, and 7.80 m/s, yielding an average value of 0.777 m/s. The measured depths were 13.09 and 11.60 cm, for an average of 12.34 cm, so the average value of q was:

$$q_{avg} = V_{avg} y_{avg} = (0.777 \text{ m/s})(0.123 \text{ m}) = 9.60\text{x}10^{-2} \frac{\text{m}^2}{\text{s}}$$

b) A plot of y vs. E can be prepared by choosing various values of y and computing E for each value as $y + q^2/(2gy^2)$. The plot is shown below. The measured depths under the test conditions were 11.60 and 13.09 cm, both of which are (barely) in the sub-critical range.



c) For the computed, average q:

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left[\frac{\left(9.60\text{x}10^{-2} \frac{\text{m}^2}{\text{s}}\right)^2}{9.81 \text{ m/s}^2}\right]^{1/3} = 0.0981 \text{ m} = 9.81 \text{ cm}$$

$$V_c = \frac{q}{y_c} = \frac{9.60\text{x}10^{-2} \frac{\text{m}^2}{\text{s}}}{0.0981 \text{ m}} = 0.978 \frac{\text{m}}{\text{s}}$$

$$E_{min} = \frac{3}{2} y_c = \frac{3}{2}(9.81 \text{ cm}) = 14.7 \text{ cm}$$

d) V and E at the two measuring points can be computed as:

$$V_{up} = \frac{q}{y_{up}} = \frac{9.60\text{x}10^{-2}\ \frac{\text{m}^2}{\text{s}}}{0.01309\ \text{m}} = 0.733\frac{\text{m}}{\text{s}} \qquad E_{up} = 0.01309\ \text{cm} + \frac{(0.733\ \text{m/s})^2}{2(9.81\ \text{m}^2/\text{s})} = 0.158\ \text{m}$$

$$V_{down} = \frac{q}{y_{down}} = \frac{9.60\text{x}10^{-2}\ \frac{\text{m}^2}{\text{s}}}{0.0116\ \text{m}} = 0.827\frac{\text{m}}{\text{s}} \qquad E_{down} = 0.0116\ \text{cm} + \frac{(0.827\ \text{m/s})^2}{2(9.81\ \text{m}^2/\text{s})} = 0.151\ \text{m}$$

e) If the flow channel bottom is horizontal, the headloss between the two points is simply the decline in E, which is:

$$h_L = E_{up} - E_{down} = 0.158\ \text{m} - 0.151\ \text{m} = 0.007\ \text{m}$$

f) The Manning equation can be rearranged to a form to solve for n as:

$$n = \frac{1}{V} R_h^{2/3} S_\text{o}^{1/2}$$

where all the parameters must be in SI units. The Manning equation was developed for uniform flow, which we do not have in this case. However, we are assuming that the flow is close enough to uniform that the equation can be used to estimate the headloss. In uniform flow, the headloss (equal to $S_f l$) equals the change in the elevation of the channel bottom, $S_\text{o} l$. Therefore, given our approximation, we can equate the computed friction slope with the hypothetical bottom slope (i.e., $S_f \approx S_\text{o}$), and write:

$$n = \frac{1}{V_{avg}} R_{h,avg}^{2/3} S_{f,avg}^{1/2} = \frac{1.49}{V_{avg}} R_{h,avg}^{2/3} \left(\frac{h_L}{l}\right)^{1/2}$$

All the terms on the right side of this equation other than $R_{h,avg}$ are known, and $R_{h,avg}$ can be computed as:

$$R_{h,avg} = \frac{A_{wetted}}{P_{wetted}} = \frac{by_{avg}}{b + 2y_{avg}} = \frac{(0.314\ \text{m})(0.123\ \text{m})}{0.314\ \text{m} + 2(0.123\ \text{m})} = 0.069\ \text{m}$$

$$n = \frac{1}{0.777}(0.069)^{2/3}\left(\frac{0.007}{3.60}\right)^{1/2} = 0.0095$$

No value is given in your textbook for the Manning n for smooth plastic, but a value of 0.010 is given in Table 10.1 for glass. The computed estimate is consistent with that value.

2. a) The specific energy and Froude number at each point of interest can be computed from:

$$E = y + \frac{q^2}{2g} y^{-2} \qquad \text{Fr} = \frac{q}{\sqrt{gy^3}}$$

The value of q at all the points of interest is the same as in question #1 (9.60×10^{-2} m^2/s), and y can be determined from the measured elevation differences at the top and bottom of the water. The results are:

Location:	1	2	3	4	5
Depth, *y* (cm)	24.05	12.28	9.71	6.85	5.42
Specific energy, *E* (m)	0.249	0.154	0.147	0.169	0.214
Froude number, Fr	0.260	0.712	1.013	1.710	2.429

Based on the values of the Froude number, the flow is subcritical as the water approaches and rises up the sill, becomes critical over the sill, and then becomes super-critical as it reaches the end of the sill and starts flowing downhill, consistent with expectations and observations.

b) The specific energy at location #1 is 0.249 m, and the water depth is 24.05 cm. The alternate depth can be found using Solver to identify the depth in the super-critical range where E is again 0.249 m. We can see from the y vs. E plot that this depth is close to 5 cm, so we can use that as our initial guess. The result is that the alternative depth is 0.048 m, or 4.8 cm.

c) The distance between locations #1 and #5 can be computed based on geometry. Points #1 and #5 were 2 cm upstream and downstream of the sill, and the length of the horizontal section of the sill is given as 46.5 cm, and the lengths of the sloped sections can be computed using the Pythagorean theorem as 30.5 and 28.4 cm, respectively. The total distance traveled was therefore:

$$l \text{ (cm)} = (2 + 30.5 + 46.5 + 28.4 + 2)\text{cm} = 109.4 \text{ cm} = 1.094 \text{ m}$$

The average values of the hydraulic radius and velocity over the flat part of the sill can be computed from the average depth between points #2 and #4, which is 9.6 cm:

$$R_{h,avg} = \frac{by_{avg}}{b + 2y_{avg}} = \frac{(0.314 \text{ m})(0.096 \text{ m})}{0.314 \text{ m} + 2(0.096 \text{ m})} = 0.060 \text{ m}$$

$$V_{avg} = \frac{q}{y_{avg}} = \frac{0.096 \text{ m}^2/\text{s}}{0.096 \text{ m}} = 1.0 \frac{\text{m}}{\text{s}}$$

The predicted headloss for the estimated travel distance at the average velocity can then be computed as:

$$S_f = \left(\frac{nV_{avg}}{R_{h,avg}^{2/3}} \right)^2$$

$$h_L = S_f l = \left(\frac{nV_{avg}}{R_{h,avg}^{2/3}} \right)^2 l$$

$$= \left(\frac{[0.0095][1.0]}{[0.060]^{2/3}} \right)^2 (1.094) = 0.0042$$

Because the calculation is in SI units, the headloss is in meters, and the predicted headloss is 0.42 cm. The actual headloss between points #1 and #5 is the difference in specific energy between those points, which is 0.035 m, or 3.5 cm. The difference between the two values is because the Manning equation accounts for only the friction associated with uniform flow over the channel surface, and in this case a substantial amount of energy is lost in the transitions from sub- to super-critical flow. In addition, the headloss is predicted to be proportional to V^2, so h_L increases dramatically as V increases. By carrying out the computation using V_{avg}, we underestimate the effects of the high velocities more than we overestimate the effect of the low velocities, which also contributes to the underestimate in h_L.

3. a) Data collected when the sill was absent and a hydraulic jump was induced in the channel are summarized below.

	Upstream	Downstream
Water surface (cm)	25.14	36.59
Channel bottom (cm)	20.23	21.52

b) The measured upstream and downstream water depths are 4.91 and 15.07, respectively. According to theory, the depth downstream of a hydraulic jump can be computed as:

$$y_2 = \frac{y_1}{2} \left(\sqrt{1 + 8\frac{q^2}{gy_1^3}} - 1 \right)$$

$$= \frac{4.91 \text{ cm}}{2} \left(\sqrt{1 + 8\frac{\left(0.096 \text{ m}^2/\text{s}\right)^2}{\left(9.81 \text{ m/s}^2\right)\left(0.0491 \text{ m}\right)^3}} - 1 \right) = 17.3 \text{ cm}$$

The measured value is about 13% lower than the theoretical value. Given the limited accuracy of the measurements, the two values seem acceptably close to one another.

c) The specific energy upstream and downstream of the jump can be computed from:

$$E = y + \frac{q^2}{2g} y^{-2}$$

The results are that E_{up} =0.244 m, and E_{down} =0.171 m. The difference between these two values, or 0.073 m, is the headloss across the jump. The corresponding power dissipation is:

$$\begin{aligned} P &= \gamma Q \Delta h \\ &= \left(9789 \frac{\text{N}}{\text{m}^3}\right)\left[\left(9.60\text{x}10^{-2}\ \frac{\text{m}^2}{\text{s}}\right)(0.314\ \text{m})\right](0.073\ \text{m}) \\ &= 21.5\ \text{W} \end{aligned}$$