CEE 345 Part 2, Assignment #3 Solutions

1. The velocity in the channel can be computed as:

$$V = \frac{Q}{by} = \frac{10\ \text{m}^3/\text{s}}{(3\ \text{m})(2\ \text{m})} = 1.667\ \text{m/s}$$

The Froude number is therefore:

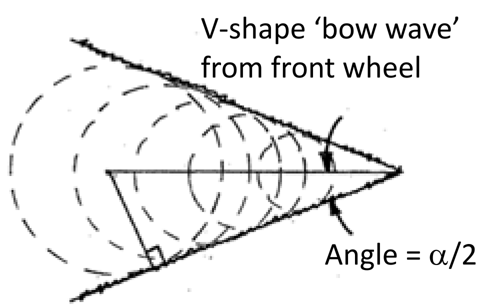
$$\text{Fr} = \frac{V}{c} = \frac{V}{\sqrt{gy}} = \frac{1.667\ \text{m/s}}{\sqrt{(9.81\ \text{m/s}^2)(2\ \text{m})}} = 0.376$$

Because the Froude number is <1, the flow is sub-critical. The critical depth can be computed as:

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{[Q/b]^2}{g}\right)^{1/3}$$

$$= \left\{\frac{\left[(10\ \text{m}^3/\text{s})/(3\ \text{m})\right]^2}{9.81\ \text{m/s}^2}\right\}^{1/3} = 1.04\ \text{m}$$

2. Over a short time period Δt, the bicyclist moves forward a distance $V_{bike}\Delta t$, while the wave generated by the wheel moves outward in all directions a distance $c\Delta t$ from the point where the wheel was at the beginning of the time step. As shown in the following diagram, the outer edge of the "bow wave" is defined by a line that starts at the bicyclist's current position and is tangent to the waves that the wheel has recently generated.



As a result, the original location of the wheel (which is also the origin of the wave), the location of the wheel after Δt, and the edge of the bow wave form a right triangle with the property:

$$\sin\frac{\alpha}{2} = \frac{V_{bike}}{c} = \frac{V_{bike}}{\sqrt{gy}}$$

For the given scenario, $y = 3$ in. and $\alpha = 40^{\circ}$, so:

$$V_{bike} = \sqrt{gy}\,\sin\frac{\alpha}{2}$$

$$= \sqrt{(32.2\text{ ft/s}^2)(0.25\text{ ft})}\,\sin\frac{40^{\circ}}{2}$$

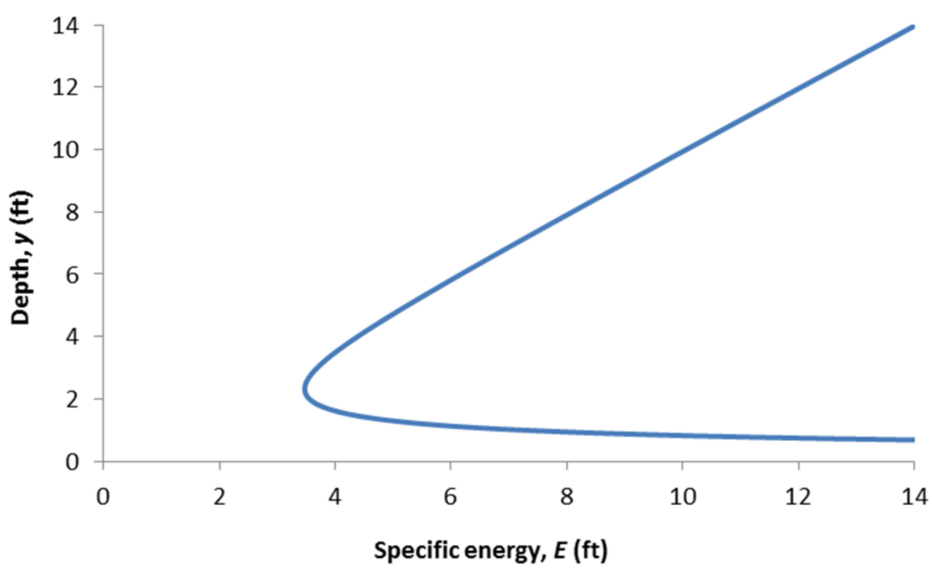
$$= 8.30\frac{\text{ft}}{\text{s}}$$

3. The Pitot tube measurement indicates that the total head of the water is 4.5 ft when the datum level is the channel bottom. This quantity is, by definition, also the specific energy of the water (i.e., it is the total head of the water above and beyond that accounted for by the terrain). We can therefore write:

$$E = 4.5\text{ ft} = y + \frac{q^2}{2g}y^{-2}$$

In the current problem, q is given as 20 cfs/ft, so we can solve for the depth, y. Three values of y satisfy the above equation, but only two are positive and therefore realistic. Using Solver, these values of y are found to be 1.42 ft/s and 4.14 ft/s.

Specific energy diagram the generated by inserting various values of y in the right-hand expression in the above equation and solving for E. The diagram is shown below.



4. (a) We can find the upstream specific energy as follows:

$$q_1 = \frac{Q}{b_1} = \frac{25\ \text{ft}^3/\text{s}}{4\ \text{ft}} = 6.25\frac{\text{ft}^2}{\text{s}}$$

$$E_1 = y_1 + \frac{q_1^2}{2g}y_1^{-2} = 2\ \text{ft} + \frac{\left(6.25\ \text{ft}^2/\text{s}\right)^2}{2\left(32.2\ \text{ft/s}^2\right)}\frac{1}{\left(2\ \text{ft}\right)^2} = 2.15\ \text{ft}$$

When the channel narrows, Q will remain the same, but b decreases to 3 ft, so q is:

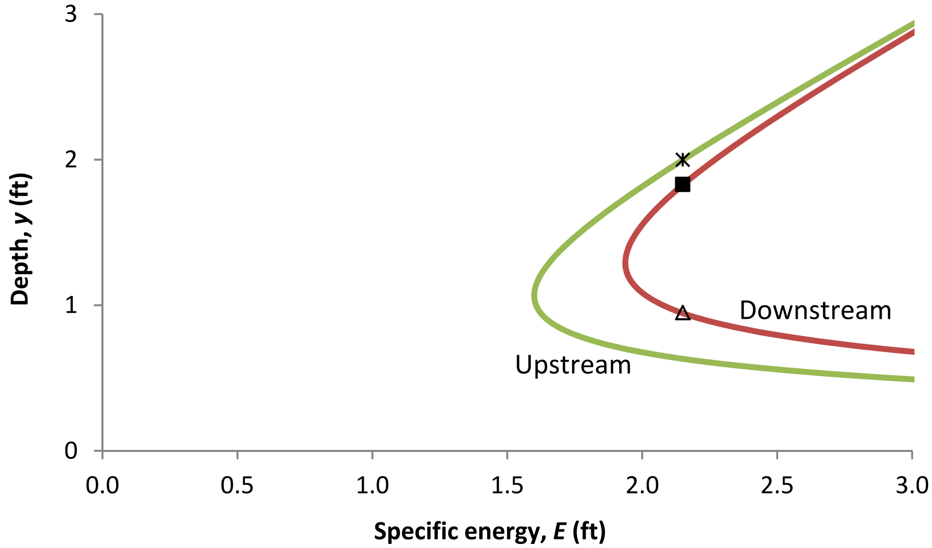
$$q_2 = \frac{Q}{b_2} = \frac{25\ \text{ft}^3/\text{s}}{3\ \text{ft}} = 8.33\frac{\text{ft}^2}{\text{s}}$$

Because the flow is frictionless in the channel was horizontal, the specific energy remains the same as in the upstream section. Therefore:

$$E_2 = 2.15\ \text{ft} = y_2 + \frac{q_2^2}{2g}y_2^{-2} = y_2 + \frac{\left(8.33\ \text{ft}^2/\text{s}\right)^2}{2\left(32.2\ \text{ft/s}^2\right)}y_2^{-2}$$

The two positive roots of this equation are y_2 equal to 1.83 ft or 0.95 ft., so either of those depths meets the energy criterion. (The Froude number at the upstream point is <1, so the flow at that point is sub-critical. As we will see, for the given system geometry, the downstream point also has to be sub-critical, so the correct depth is 1.83 ft. However, since we have not covered that material yet, either depth is an acceptable answer.)

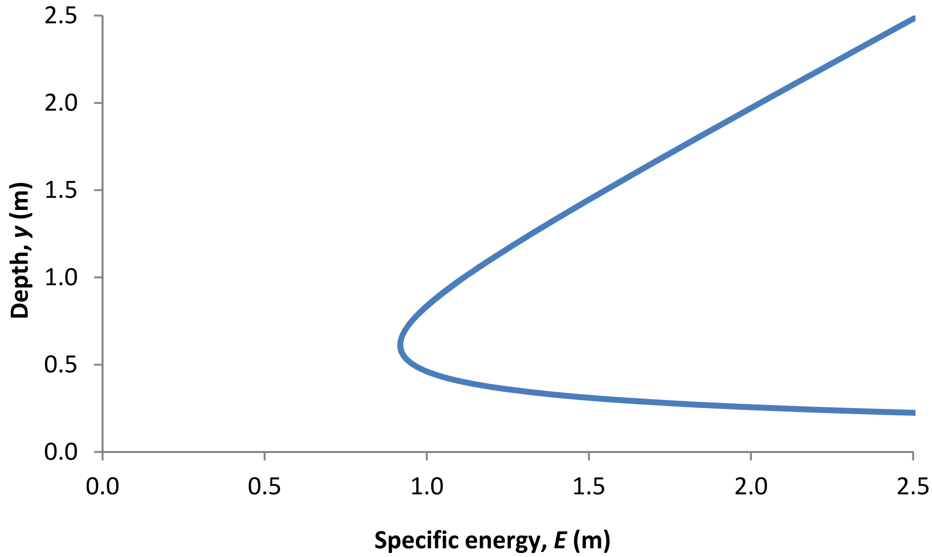
(b) Because the flow rate per unit width, q, changes when the channel narrows, the upstream and downstream points fall on different specific energy curves, even though they have the same value of specific energy. The two curves and the transition undergone by the water are shown in the following diagram.



5. As in the previous problems, the specific energy diagram can be computed based simply on the value of q, which is given as 1.5 m^2/s, by using the following equation:

$$E = y + \frac{q^2}{2g} y^{-2}$$

The diagram is plotted below. At section (1), the depth is 0.5 m, and the corresponding value of E is 0.959 m. If the water loses 0.03 m of head, its specific energy will be 0.929 m, and the depth could be either 0.55 m or 0.68 m. if the water lost 0.06 m of head, its specific energy would be only 0.899 m. This value is less than the critical specific energy, so it would not be possible for the water to continue to flow at the current rate under the circumstances; instead, the water would back up, and the flow rate would decline.



2.5
2.0
1.5
1.0
0.5
0.0
Depth, y (m)
0.0
0.5
1.0
1.5
2.0
2.5
Specific energy, E (m)