CEE 345 Part 2, Assignment #2 Solutions

1. Determination of the flow rates in the various pipes requires that we start with a guesses for those flow rates that meet the continuity criterion. We then use the Hazen-Williams equation to compute the headloss in each pipe, determine the apparent headloss around each loop, and follow the algorithm given in class to improve the guessed values of Q until Δh around each loop is zero. In these iterations, n is the exponent on Q in the H-W equation (1.85), and K is the value of the coefficient in the expression $h_L = KQ^n$. K is a composite parameter that is constant for a given pipe but differs from one pipe to the next. Its calculation depends on a numerical constant and the known values of the pipe diameter, length, and H-W friction coefficient ($C_{H\text{-}W}$, which is more properly thought of as a conductivity coefficient). My results after several iterations, once the corrections to Q have become small, are shown below.

	Given	Given	Coef.	Assumed		HW eqn		
Pipe	D (ft)	length (ft)	C_HW	Q (ft3/s)		h_L (ft)	h_L/Q	ΔQ
ab	1.5	1000	90	3.7320		1.820	0.488	
be	1.33	800	100	0.2816		0.018	0.063	
ei	1.5	800	100	-4.0395		-1.387	0.343	
ih	2	1000	90	-6.2680		-1.170	0.187	
ha	2	1600	90	3.7320		0.717	0.192	
					SUM	**-0.002**	**1.273**	**0.0008**
bc	1.5	500	90	-0.7304		-0.045	0.061	
cf	1.33	800	100	2.2696		0.847	0.373	
fe	1	500	100	-1.3211		-0.790	0.598	
eb	1.33	800	100	-0.2816		-0.018	0.063	
					SUM	**-0.005**	**1.096**	**0.0025**
cd	1.5	500	90	1.1808		0.108	0.092	
dg	1.33	800	90	-0.8192		-0.156	0.191	
gf	1	500	100	1.4093		0.890	0.632	
fc	1.33	800	100	-2.2696		-0.847	0.373	
					SUM	**-0.005**	**1.287**	**0.0021**
ef	1	500	100	1.3211		0.790	0.598	
fg	1	500	100	-1.4093		-0.890	0.632	
gj	1.5	800	90	-2.2285		-0.561	0.252	
ji	2	1000	90	-0.9605		-0.036	0.038	
ie	1.5	800	100	2.7715		0.691	0.249	
					SUM	**-0.007**	**1.768**	**0.0020**

The headloss between points *a* and *j* can be computed by adding the headlosses in any group of pipes connecting those points. Adding the headlosses in pipes *ah*, *hi*, and *ij*, we find:

$$h_{L,aj} = h_{L,ah} + h_{L,hi} + h_{L,ij} = (-0.717 + 1.170 + 0.036)\text{ ft} = 0.489\text{ ft}$$

2. (a) The energy equation written between the surface of the reservoir and the outlet is:

$$\cancel{\frac{p_2}{\gamma}} + z_2 + \frac{V_2^2}{2g} = \cancel{\frac{p_1}{\gamma}} + z_1 + \cancel{\frac{V_1^2}{2g}} + h_{pump} - \sum h_L$$

$$h_{pump} = z_2 - z_1 + \frac{V_2^2}{2g} + \sum h_L = z_2 - z_1 + \frac{V_2^2}{2g} + f\frac{l}{D}\frac{V_2^2}{2g}$$

where the final equality on the right applies because minor losses are negligible, so the headloss can be computed using the D-W equation. The outlet is 3 m below the reservoir surface, and V_2, l, D, and V are given, so the only unknown on the right side of the equation is the friction factor. The friction factor can then be computed from the Moody diagram or an equivalent equation, based on the Reynolds number and the fact that the pipe is smooth:

$$\text{Re} = \frac{DV_2}{\nu} = \frac{(0.05\text{ m})(3\text{ m/s})}{6.85\text{x}10^{-7}\text{ m}^2\text{/s}} = 2.28\text{x}10^5$$

$$f = 0.0152$$

Substituting all the known values into the equation for the head that must be added by the pump, we find:

$$h_{pump} = -3\text{ m} + \frac{(3\text{ m/s})^2}{2(9.81\text{ m/s}^2)} + (0.0152)\frac{200\text{ m}}{0.05\text{ m}}\frac{(3\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 25.3\text{ m}$$

The fluid power and shaft power can then be determined as:

$$P_f = \gamma Q h_{pump} = \left(9.731\frac{\text{kN}}{\text{m}^3}\right)\left[(3\text{ m/s})\left(\frac{\pi(0.05\text{ m})^2}{4}\right)\right](25.3\text{ m}) = 1.45\frac{\text{kN-m}}{\text{s}} = 1.45\text{kW}$$

Because the pump is only 70% efficient, the power that must be provided to the shaft is:

$$P_{shaft} = \frac{P_f}{\eta} = \frac{1.45\text{ kW}}{0.70} = 2.07\text{ kW}$$

(b) The NPSH_A is computed as:

$$\text{NPSH}_\text{A} = \frac{p_S}{\gamma} + \frac{V_S^2}{2g} - \frac{p_{vapor}}{\gamma}$$

Defining the elevation of the suction as the datum elevation, the energy equation written between the reservoir surface and the pump suction (using absolute pressures) is:

$$\frac{p_{\text{atm}}}{\gamma} + \cancel{\frac{V_1^2}{2g}} + z_1 - \cancel{h_L} = \frac{p_S}{\gamma} + \frac{V_S^2}{2g} + \cancel{z_S}$$

Substituting this expression into that for NPSH_A, we obtain:

$$\text{NPSH}_\text{A} = \frac{p_{\text{atm}}}{\gamma} + z_1 - \frac{p_{vapor}}{\gamma}$$

The vapor pressure and specific weight of water at 40°C are given in the tables at the back of the book as 7.376 kN/m^2 and 9.731 kN/m^3, respectively, and normal atmospheric pressure is 101 kPa (101 kN/m^2). Substituting these values, we find:

$$\text{NPSH}_\text{A} = \frac{101\ \text{kN/m}^2}{9.731\ \text{kN/m}^3} + 3\ \text{m} - \frac{7.376\ \text{kN/m}^2}{9.731\ \text{kN/m}^3} = 12.6\ \text{m}$$

3. The energy equation written between the surfaces of the two reservoirs is:

$$\cancel{\frac{p_2}{\gamma}} + z_2 + \cancel{\frac{V_2^2}{2g}} = \cancel{\frac{p_1}{\gamma}} + z_1 + \cancel{\frac{V_1^2}{2g}} + h_{pump} - \sum h_L$$

The downstream reservoir surface is 30 ft higher than the upstream reservoir surface, so:

$$h_{pump} = 30\ \text{ft} + \sum h_L$$

The headlosses are all multiples of the velocity head, with the multiplier being fl/D for the major (pipe friction) headloss, and the minor loss coefficient for the various fittings and valves. The flow velocity can be expressed as a function of Q by:

$$V = \frac{Q}{A} = \frac{Q}{\pi D^2/4} = \frac{Q}{\pi(0.667\ \text{ft})^2/4} = \left(2.86\ \text{ft}^{-2}\right)Q$$

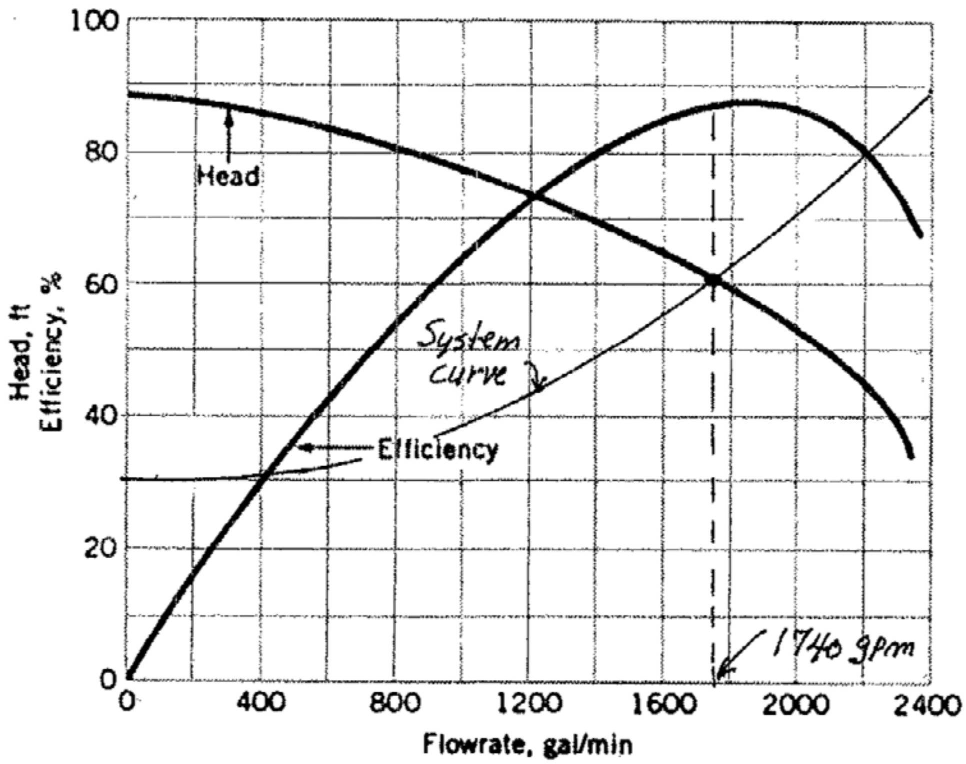
Inserting all this information into the expression for h_{pump}, we obtain:

$$h_{pump} = 30\ \text{ft} + \left(f\frac{l}{D} + 4\left[K_{elbow}\right] + K_{check} + K_{\substack{open\\globe}}\right)\frac{V^2}{2g}$$

$$= 30\ \text{ft} + \left(\left[0.02\right]\frac{100\ \text{ft}}{0.667\ \text{ft}} + 4\left[0.3\right] + 10 + 2\right)\frac{\left[\left(2.86\ \text{ft}^{-2}\right)Q\right]^2}{2\left(32.2\ \text{ft/s}^2\right)}$$

$$= 30\ \text{ft} + \left(2.06\frac{\text{s}^2}{\text{ft}^5}\right)Q^2$$

This equation can be solved to find h_L corresponding to various values of Q. After converting the Q values to gpm units, the results can be plotted to show the system curve on the same graph as the pump curve (shown below). The intersection of the two curves establishes the operating point, which is at Q = 1740 gpm; h = 61.0 ft.



100
80
60
40
20
0
Head, ft
Efficiency, %
Head
System curve
Efficiency
1740 gpm
0
400
800
1200
1600
2000
2400
Flowrate, gal/min

4. When the system is operating steadily, the flow rate through the pump (and therefore the total flow through the two pipes) will be such that the head added by the pump exactly equals the head required to push the water through the system. The relationship between the flow through the pump and the head that the pump adds to the fluid is given by the pump performance curve, which can be plotted from the given data. The relationship between the flow rate and the head required to generate that flow rate, on the other hand, is given by the system curve, which we have to derive based on the system geometry.

   For this particular system geometry, the inlet to the two pipes is the same, so their heads are the same at that point. Their outlet locations are different, so they do not have the same headloss, but we do know the conditions at each outlet, so we can find the flow through each pipe as a function of the head added by the pump, using the energy equation independently for each pipe. Thus, assuming that minor headlosses are negligible, we can write the energy equation between reservoirs A and B as:

$$\cancel{\frac{p_B}{\gamma}} + z_B + \cancel{\frac{V_B^2}{2g}} = \cancel{\frac{p_A}{\gamma}} + z_A + \cancel{\frac{V_A^2}{2g}} + h_{added} - h_{L,1}$$

$$h_{added} = z_B - z_A + f\frac{l_1}{D_1}\frac{V_1^2}{2g}$$

$$= \left[(120-110)\text{ ft}\right] + (0.02)\left(\frac{10{,}000\text{ ft}}{2.5\text{ ft}}\right)\frac{\left[\frac{Q_1}{\pi(2.5\text{ ft})^2/4}\right]^2}{2\left[32.2\text{ ft/s}^2\right]}$$

$$= 10\text{ ft} + \left(0.253\frac{\text{s}^2}{\text{ft}^5}\right)Q_1^2$$

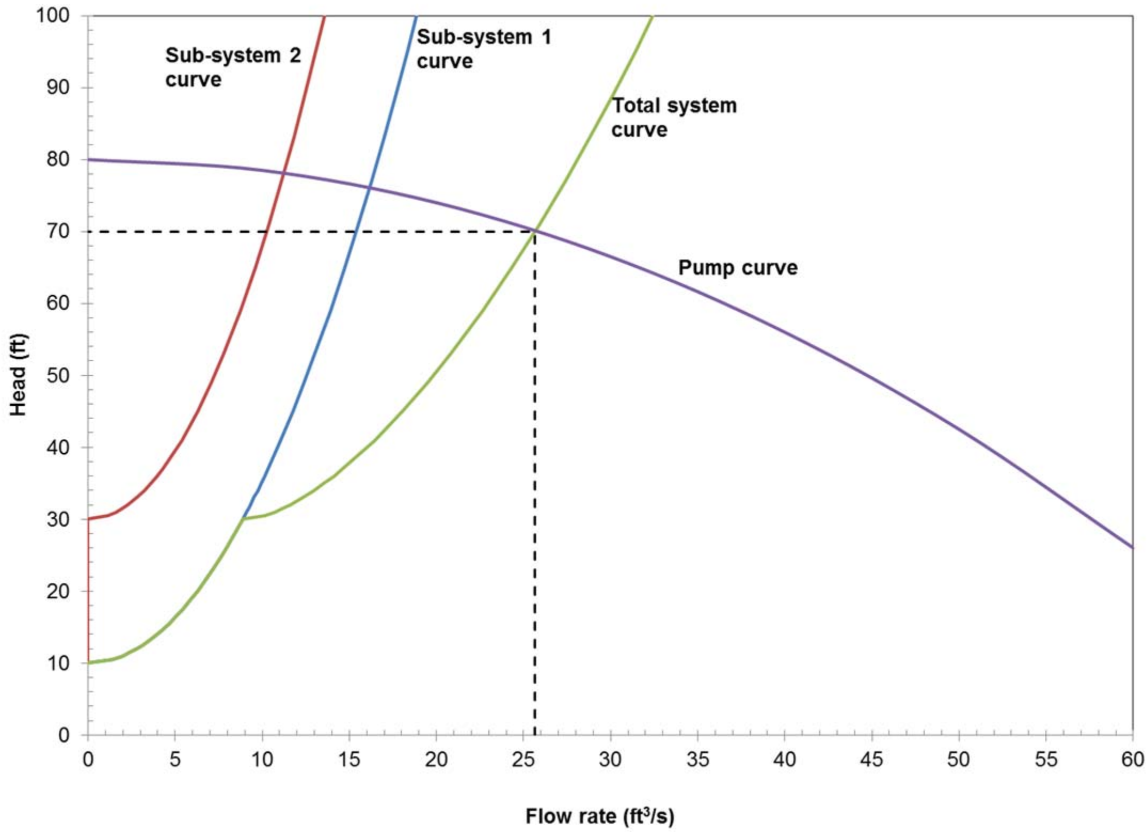
   Analogous calculations for the flow in pipe 2 yield:

$$h_{added} = z_C - z_A + f\frac{l_2}{D_2}\frac{V_2^2}{2g}$$

$$= 30\text{ ft} + \left(0.380\frac{\text{s}^2}{\text{ft}^5}\right)Q_2^2$$

   For any operating scenario, h_{added} must be the same for both pipes. Therefore, we can pick an arbitrary value of h_{added}, solve the two equations for Q_1 and Q_2, and add the two flow rates to obtain the total flow, Q_{tot}. We can then repeat that process for various values of h_{added} to develop a curve of h_{added} vs. Q_{tot}, i.e., the system curve. (Note that, for h_{added} values <10 ft and <30 ft, there is not enough head being added to generate any flow through pipes 1 and 2, respectively, so we assign $Q = 0$ for those scenarios.)

   An alternative to numerical calculation of the system curve is to just plot the two 'sub-system curves' (i.e., h_{added} vs. Q_1 and h_{added} vs. Q_2), and add them graphically. To do that, first plot the two curves. Then, because the addition has to be under conditions where the same value of h_{added} applies to both, we choose a value of h_{added}, read Q_1 and Q_2 from the graph, compute Q_{tot} as $Q_1 + Q_2$, and plot a point for the total system curve at (h_{added}, Q_{tot}). Repeating the process for several points yields the desired curve. The curve generated using either approach is shown along with the pump curve in the following plot.



The operating point is at a flow rate of 25.7 cfs, with 70 ft of head added by the pump.