Applying the RTT to Energy; Headloss

Up to now, we have considered three "types" of energy in fluid systems: mechanical potential energy, gravitational potential energy, and kinetic energy. If we restrict our consideration to isothermal systems with ideal fluids, these are the only forms among which energy can transfer, so they are the only forms that we need to keep track of; fluids do have other forms of energy, but since the amount of energy in those forms is not changing, it has no effect on the fluid's behavior.

If we consider fluids whose temperature can change in the system of interest, then we must also take *internal energy* into account. Internal energy is the energy that is stored in the form of chemical bonds within molecules, chemical interactions among different molecules, and the kinetic energy of the molecules. To a good approximation, the amount of this kind of energy stored per unit mass of fluid depends solely on the temperature, increasing as the temperature increases. We designate internal energy as *u*, and internal energy per unit mass as $\hat{u}$ (the Munson text uses an inverted caret above the *u*, but I can't do that with my software).

In general, fluid energy can be normalized to the fluid mass, weight, or volume. When normalized to mass, the term corresponds to what we have called i_{en} in prior discussions of the Reynolds Transport Theorem (RTT). Expressions for each type of energy normalized in each of these ways are shown in Table 1.

Table 1. Expressions for various types of energy in fluids, normalized in different ways

"Type" of Energy	Primary Parameter	Energy/mass (i_{en})	Energy/wt $(i_{en}g)$	Energy/volume $(i_{en}\rho)$
Mechanical (potential)	p	p/ρ	p/g	p
Gravitational (potential)	z	zg	z	$z\gamma$
Kinetic	V	$V^2/2$	$V^2/2g$	$\rho V^2/2$
Internal	T	$\hat{u}$	$\hat{u}g$	$\rho\hat{u}$

When applied to analyze energy, the RTT contains not only the intensive parameter i_{en}, but also the extensive parameter $\dot{E}_{en}$, which represents the net energy input to the CV per unit time from non-advective processes. Note that i_{en} and $\dot{E}_{en}$ are scalars, because energy is a scalar (as opposed to momentum, which is a vector and therefore appears as $\vec{\boldsymbol{i}}_{mom}$ and $\dot{\boldsymbol{E}}_{mom}$ in the RTT applied to it). $\dot{E}_{en}$ is a *rate* of energy exchange (between the CV and outside), but it is often useful to express that value in terms of the amount of energy exchange per unit of fluid mass, weight, or volume in the system. To accomplish that, we can use the following relationships:

$$\dot{E}_{en} = \left(\frac{\text{Energy}}{\text{mass}}\right)\dot{m} = \left(\frac{\text{Energy}}{\text{weight}}\right)\dot{w} = \left(\frac{\text{Energy}}{\text{weight}}\right)\dot{m}\,g \qquad (1)$$

and

$$\dot{m} = Q\rho\,; \qquad \dot{w} = \dot{m}\,g = Q\gamma \qquad (2)$$

We can now turn our attention to the use of the RTT for energy. The relevant equation is:

$$\frac{\partial}{\partial t}\int_{\text{CV}} i_{en}\rho\, d\cancel{V} = -\int_{\text{CV}} i_{en}\rho \vec{\mathbf{V}} \bullet \vec{\mathbf{n}} dA + \dot{E}_{en} \qquad (3)$$

The terms in the integrals in Equation 3 have been adequately discussed in prior notes; the key difference between the RTT applied to energy versus mass or momentum is in the final term on the right ($\dot{E}_{en}$), so we will focus on that.

In general, the non-advective processes by which energy can be added to or removed from the fluid in a CV include shaft work (pumping energy in or removing it via a turbine) and heat transfer across the CS. It is common to express the amount of energy added to a fluid by a pump or removed from a fluid via a turbine in terms of the gain or less of energy per unit weight of the fluid. As indicated in Table 1, such a term has units of length, and as we have learned previously, energy normalized to the weight of a fluid is commonly referred to as a "head." Thus, the energy added to fluid per unit weight by the action of a pump is called the "pump head," h_P. Similarly, the energy removed from fluid per unit weight by the action of a turbine pump is represented as h_T.

Also, whenever any real fluid flows, viscous forces lead to friction as parcels of fluid flow past one another. This process can convert mechanical, gravitational, or kinetic energy into internal energy (heat). Because it is usually not practical to convert the heat back into the other forms of energy, and therefore not practical to convert it to energy that can be recovered via a turbine or similar machine, the conversion of mechanical, gravitational, or kinetic energy to heat is considered a loss of *available* or *useful* energy from the fluid. When normalized to fluid weight, amount of available energy lost has the units of *head* and hence it is referred to as headloss, h_L. Note that head loss refers to conversion of available (mechanical, gravitational, or kinetic) energy to unavailable (internal) energy, so that the energy is not really 'lost' and the principle of conservation of energy is still satisfied.

In most systems that we will investigate, an assumption is made that the fluid temperature remains constant as the fluid passes through the CV (the system is *isothermal*). For that to occur, the energy that is converted from an available form to internal energy (as heat) must subsequently be transferred out of the CV, e.g., via some radiative process. In such cases, headloss is generated by friction and 'passes through' internal energy on its way out of the CV. However, regardless of whether that energy remains in the CV as internal energy or is subsequently transferred out of the CV, it is considered lost in terms of usefulness.

Based on the preceding discussion and Equations 1 and 2, we can represent the value of $\dot{E}_{en}$ associated with the most common types of energy transfer in systems of interest as follows:

$$\dot{E}_{en} = -\dot{m}\,g\left(h_P - h_S - h_L\right) = -Q\gamma\left(h_P - h_S - h_L\right) \tag{4}$$

Often, it is convenient to separate the portions of the headloss associated with pumps and turbines from that associated with other processes in the system (typically, flow through pipes or channels). When this is done, the second equality in Equation 4 can be written as:

$$\begin{aligned}\dot{E}_{en} &= -Q\gamma\left(h_P - h_S - h_{L,P} - h_{L,S} - h_{L,other}\right) \\ &= -Q\gamma\left[\left(h_P - h_{L,P}\right) - \left(h_S + h_{L,S}\right) - h_{L,other}\right] && \text{(5a)} \\ &= -Q\gamma h_{L,tot} && \text{(5b)}\end{aligned}$$

We can substitute Equation 5 for $\dot{E}_{en}$ in Equation 3. Also, as with other forms of the RTT, we will frequently consider systems that are at steady state, in which case the term on the left side of the RTT (the term representing the rate of accumulation of energy in the CV) is zero.

The integral term on the right side of the equation represents the net advective transport of energy out of the CV (or, when the minus sign is included, into the CV) and, in general, must consider all four forms of energy listed in Table 1. However, we are often interested not primarily in the change in total energy in the system, but rather just in the change in available energy. We can designate that portion of the total energy, normalized to the fluid weight, as $i_{en,av}$, and the net rate at which available energy enters the CV by non-advective processes as $\dot{E}_{en,av}$. However, the processes we considered above (shaft work associated with pumps and turbines and friction) all do affect the available head, so $\dot{E}_{en,av}$ is identical to $\dot{E}_{en}$. Thus:

$$i_{en,av} = \frac{p}{\rho} + zg + \frac{V^2}{2} \tag{6}$$

and $$\dot{E}_{en,av} = \dot{E}_{en} \tag{7}$$

We can therefore write the RTT applied to *available* energy as follows:

$$\int_{CV}\left(\frac{p}{\rho} + zg + \frac{V^2}{2}\right)\rho\vec{\mathbf{V}}\bullet\vec{\mathbf{n}}dA = \int_{CV}\left(\frac{p}{\rho} + zg + \frac{V^2}{2}\right)\rho\vec{\mathbf{V}}\bullet\vec{\mathbf{n}}dA - Q\gamma h_{L,tot} \tag{8}$$

or, at steady state:

$$0 = \int_{\mathrm{CV}} \left(\frac{p}{\rho} + zg + \frac{V^2}{2} \right) \rho \vec{\mathbf{V}} \bullet \vec{\mathbf{n}} dA - Q\gamma h_{L,tot} \qquad (9)$$

$$= \int_{\mathrm{CV}} \left(p + z\gamma + \frac{\rho V^2}{2} \right) \vec{\mathbf{V}} \bullet \vec{\mathbf{n}} dA - Q\gamma h_{L,tot} \qquad (10)$$

If the CV has a limited number of inlets and outlets, we can convert the integration around the control surface to a summation that includes only those inlets and outlets. In that case, if the system is at steady state, we can write the RTT on available energy as:

$$0 = \sum_{inlets} \left[i_{en,av} \rho VA \cos \theta_{VA} \right] - \sum_{outlets} \left[i_{en,av} \rho VA \cos \theta_{VA} \right] - Q\gamma h_{L,tot} \qquad (11)$$

$$0 = \sum_{inlets} \left[\left(p + z\gamma + \frac{\rho V^2}{2} \right) VA \cos \theta_{VA} \right] - \sum_{outlets} \left[\left(p + z\gamma + \frac{\rho V^2}{2} \right) VA \cos \theta_{VA} \right] - Q\gamma h_{L,tot} \qquad (12)$$

A line that shows the total available head of the fluid (the available energy per unit weight) in a system is sometimes shown on a system diagram and labeled as the *energy line* (*EL*). Previously, we considered the energy line to be horizontal, because we did not consider the possibility of headloss. We now see that the energy line declines in the direction of fluid flow, because as fluid flows, there is always a gradual loss of available energy due to friction. In addition to its gradual decline in the direction of flow in the absence of pumps or turbines, the energy line has discontinuities at locations where a spike of energy is added to the fluid (by a pump) or removed (by a turbine). As before, the hydraulic grade line (*HGL*) is defined as a line showing the piezometric head (the sum of the pressure and elevation heads). The HGL is therefore always lower than the EL by an amount equal to the velocity head ($V^2/2g$).