CEE 342 Aut 2004, Exam #1

Work alone. Answer all questions. Total pts: 90. **Always make your thought process clear; if it is not, you will not receive partial credit for incomplete or partially incorrect answers.** Some data that might be useful is provided below; the values that depend on temperature are all for 20°C, which is also the assumed temperature for all the problems.

		Area	Volume	Centroid	I_c
γ_{water} = 62.4 lb/ft^3 = 9800 N/m^3					
$\mu_{\text{water}} = 1.002 \times 10^{-3}$ N-s/m^2	Rectangle	bh		$h/2$	$bh^3/12$
s.g.$_{\text{gasoline}}$ = 0.68	Triangle	$bh/2$		$h/3$	$bh^3/36$
s.g.$_{\text{CCl4}}$ = 1.59	Circle	$\pi d^2/4$		$d/2$	$\pi d^4/64$
p_{atm} = 101.5 kPa = 14.7 psi	Sphere	πd^2	$\pi d^3/6$	$d/2$	
$p_{vapor,Hg} = 1.7 \times 10^{-4}$ kPa					
$p_{vapor,CCl4}$ = 13.1 kPa					

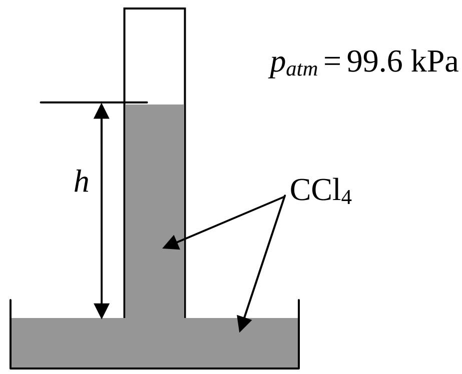
1. (12) A 1 m x 1 m x 1 mm flat plate is submerged in seawater, with the plane of the plate vertical. Designate one of the 1 m x 1 m sides as side A. The plate is then tilted to a 45° angle with the vertical, keeping the center of the plate at the same depth and causing side A to face downward. How does the rotation affect each of the following parameters? Indicate whether you think the parameter will increase, decrease, or not change, and explain your reasoning in 1-2 sentences.

a) The total horizontal force on side A.

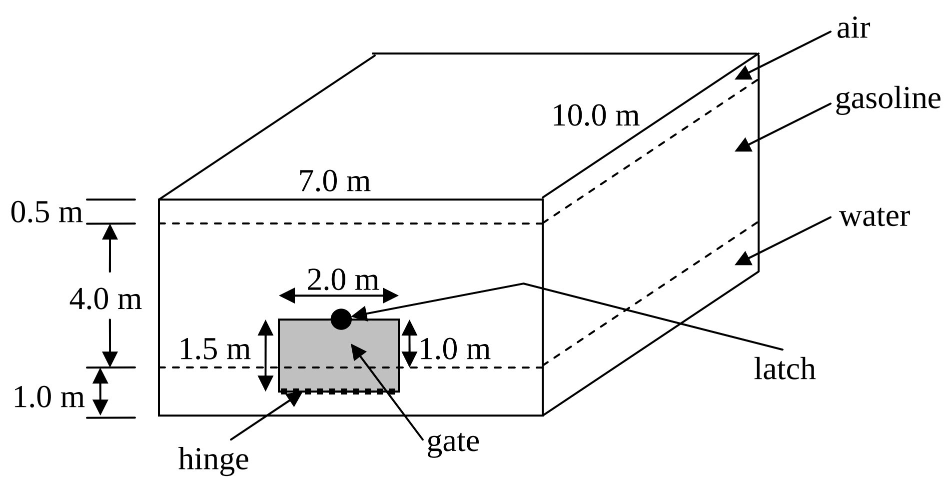
b) The total vertical force on side A.

c) The depth of the centroid of pressure on side A.

2. (20) A barometer modeled after a mercury barometer, but using carbon tetrachloride (CCl_4) as the liquid, is shown schematically below. The local atmospheric pressure is 99.6 kPa. What are the gage and absolute pressures at the gas/liquid interface inside the tube, in kPa? What is the vertical distance h from that interface to the top of the pool of liquid CCl_4?



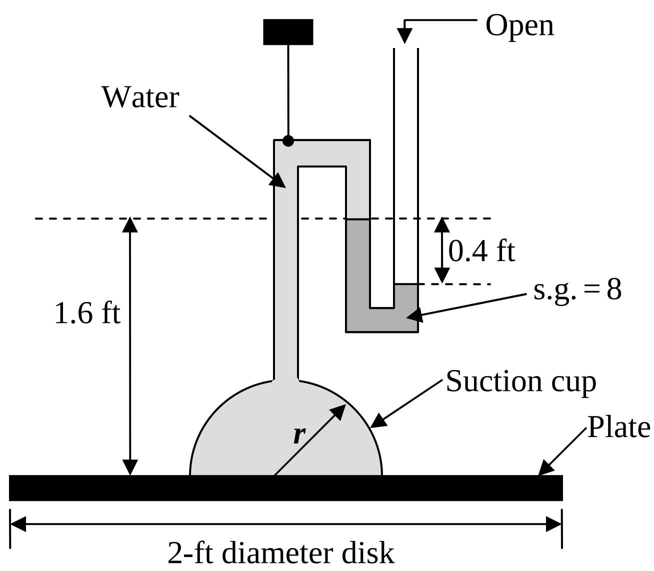
3. The storage tank shown below contains air at a pressure of 2 atm absolute overlying a layer of gasoline and a layer of water. A rectangular gate is present in one of the (vertical) walls.



(20) (a) Draw a pressure prism diagram to describe the force exerted on the gate by the fluids in the tank, indicating the length of each segment of the prism(s). Write out the equation you would use to determine the *net* force on the gate. You do not need to do any calculations or solve for the force.

(8) (b) The gate is hinged at the bottom. If you solved the equation in part *a* and applied a force inward of that magnitude at the top of the gate, would the gate swing outward, swing inward, or remain closed? Explain briefly.

4. (20) A half-sphere suction cup with a radius r of 0.5 ft is used to support a plate with weight W, as shown in the diagram below. Determine the maximum weight of the plate.



CEE 342 Aut 2004, Solutions to Midterm #1

1. a) The horizontal force exerted by a liquid on a surface is $A_{vert}\,\gamma h_c$, where A_{vert} is the area of the surface projected onto a vertical plane, and h_c is the depth of the centroid of area. In the current scenario, γ and h_c remain constant when the plate is rotated, but A_{vert} decreases, so the horizontal force decreases.

   b) The vertical force exerted by a liquid on a surface equals the weight of the liquid that would be contained in a volume defined by the surface and the projection of the surface onto a (horizontal) plane at the top of the liquid layer. In the initial position, the projected area of side A on a horizontal plane is zero, so there is no vertical force on side A. When the plate is tilted, the projected area and the force become finite. Therefore, the force on side A increases.

   c) The centroid of pressure is two thirds of the way down the plate when the plate is vertical. In the current scenario, the plate is 1 m long, so the center of pressure is 2/3 m along the plate, from the top to the bottom. Thus, if the top of the plate is at depth d, the center of pressure is at $d + 2/3$ m. Since this orientation maximizes the amount of area that is below the centerline of the plate, it maximizes the depth of the center of pressure. In the limit, if the plate were rotated about its central axis until it was horizontal, the whole area of the plate would be at depth $d + ½$ m, and the center of pressure would be at that depth also. When the plate is rotated only half this amount, the center of pressure does not rise to $d + ½$ m, but it does rise above the original depth of $d + 2/3$ m.

2. a) The absolute pressure inside the tube is the vapor pressure of CCl_4, which is given as 13.1 kPa. The gage pressure is the difference between the pressures outside and inside the tube, i.e., $p_{gage} = (13.1 - 99.6)\text{ kPa} = -86.5$ kPa.

   b) The pressure at the elevation of the top of the pool of CCl_4 is zero gage. The difference in pressure between the top of the column of CCl_4 and the pool is therefore 86.5 kPa, and the height of the column is:

$$\Delta h = \frac{\Delta p}{\gamma_{\mathrm{CCl_4}}} = \frac{\Delta p}{\left(s.g._{\mathrm{CCl_4}}\right)\gamma_{\mathrm{H_2O}}} = \frac{86.5\text{ kPa}}{(1.59)\left(9.81\text{ kN/m}^3\right)} = 5.55\text{ m}$$

3. a) The force exerted on the gate by the fluids in the tank can be divided into four components: the force exerted by the overlying air, the force exerted by the gasoline above the top of the gate, the force exerted by the gasoline at depths that overlap the gate, and the force generated by the water at depths that overlap the gate. The first two of these components contribute a constant force to the whole area of the gate, whereas the latter two contribute increasing amounts of force as depth increases.

A pressure prism describing the forces on the gate would have the height and width of the gate for two of its dimensions, and the total pressure (the sum of the pressures from the four components described above) at each depth as the third dimension. The pressure contributed by the gas phase is 2 atm, or 203 kPa. The pressure contributed by the gasoline above the gate is:

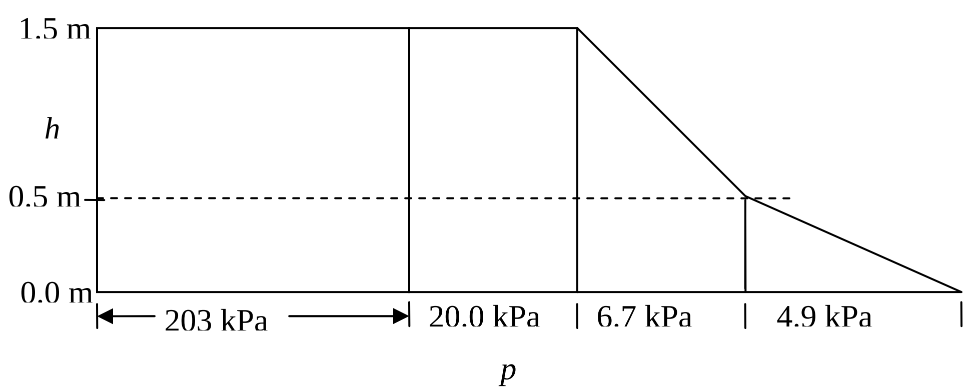
$$p_{\substack{gasoline,\\ above}} = \left(s.g._{gasoline}\right)\gamma_{\mathrm{H_2O}}\left(3\ \mathrm{m}\right) = \left(0.68\right)\left(9.81\ \mathrm{kN/m^3}\right)\left(3\ \mathrm{m}\right) = 20.0\ \mathrm{kPa}$$

These two pressures are constant over the full height of the gate, so they appear as rectangles in the prism. The pressure contributed by the gasoline that is at the same level as the gate increases from zero at the top of the gate to a value that is one-third of the value computed above, since the layer of gasoline in contact with the gate is 1 m thick, whereas the layer above the gate is 3 m thick. The maximum pressure exerted by this layer is therefore 20.0/3, or 6.7 kPa.

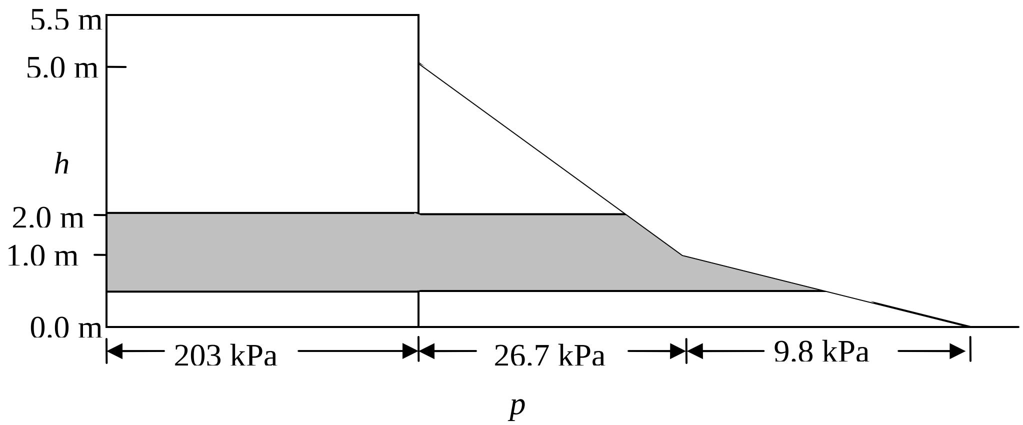
Finally, the pressure contributed by the water increases from zero at the water/gasoline interface to a value at the bottom of the gate given by:

$$p_{water} = \gamma_{\mathrm{H_2O}} h_{\mathrm{H_2O}} = \left(9.81\ \mathrm{kN/m^3}\right)\left(0.5\ \mathrm{m}\right) = 4.9\ \mathrm{kPa}$$

A side view of the prism is drawn below (not to scale); the prism would have a constant width (into the plane of the paper) equal to the width of the gate, i.e., 2.0 m.



Note that this same prism can be seen as a sub-section of the pressure prism on the whole wall of the tank, as shown below. Although the scales are different in the two figures, it should be clear that the shape of the grey area in the figure below is identical to the area shown in the expanded figure above.



The force of all the fluids on the gate can be computed as the volume of the prism, i.e.:

$$F_{tot} = \sum Areas = (203 \text{ kPa} + 20.0 \text{ kPa})(1.5 \text{ m})(2.0 \text{ m}) + \frac{1}{2}(6.7 \text{ kPa})(1.0 \text{ m})(2.0 \text{ m}) + (6.7 \text{ kPa})(0.5 \text{ m})(2.0 \text{ m}) + \frac{1}{2}(4.9 \text{ kPa})(0.5 \text{ m})(2.0 \text{ m})$$

The air pressure on the outside of the gate exerts an opposing force of (101.5 kPa)(1.5 m)(2.0 m), so the net force on the gate is:

$$F_{tot} = \sum Areas = (101.5 \text{ kPa} + 26.7 \text{ kPa})(1.5 \text{ m})(2.0 \text{ m}) + \frac{1}{2}(6.7 \text{ kPa})(1.0 \text{ m})(2.0 \text{ m}) + (6.7 \text{ kPa})(0.5 \text{ m})(2.0 \text{ m}) + \frac{1}{2}(4.9 \text{ kPa})(0.5 \text{ m})(2.0 \text{ m})$$

b) The center of pressure exerted by the fluids in the tank on the gate is somewhere below the centerline of the gate. We don't know exactly where, since two different fluids are in contact with the gate, but we know that it is below the centerline. By exerting an equal force inward at a higher point, we will generate a net moment inward, so the gate will swing inward.

4. The pressure at the top of the liquid in the open arm of the manometer is atmospheric, so if we work in gage pressures, that pressure is zero. The same pressure is present at the same elevation in the arm immediately to the left. The pressure at the interface between the liquid with s.g. of 8 and the water is given by:

$$p_2 = p_1 - \gamma \Delta h = 0 - (8)\left(62.4 \text{ lbs/ft}^3\right)(0.4 \text{ ft}) = -199.7 \frac{\text{lbs}}{\text{ft}^2}$$

The pressure computed above is at a point that is 1.6 ft above the plate, and the fluid filling the tube and suction cup in that space is water, so the gage pressure at the surface of the plate is:

$$p_{plate} = p_2 + \gamma \Delta h = -199.7\,\frac{\text{lbs}}{\text{ft}^2} + \left(62.4\ \text{lbs/ft}^3\right)\left(1.6\ \text{ft}\right) = -99.8\,\frac{\text{lbs}}{\text{ft}^2}$$

There is a net force on the plate only over the area where the pressure on the top surface is different from that on the bottom surface. This area equals the area of a circle with radius 0.5 ft. The net force is the product of this area and the difference between the pressures above and below the area. The gage pressure underneath the plate is zero, so:

$$F_{up} = A\left(p_{below} - p_{above}\right) p = \left(\pi\left(0.5\ \text{ft}\right)^2\right)\left(0 - \left(-99.8\,\frac{\text{lbs}}{\text{ft}^2}\right)\right) = 78.4\ \text{lbs}$$

The upward force balances the weight of the plate, so W_{plate} is 78.4 lbs.