CEE 342 Aut 2003, Exam #1

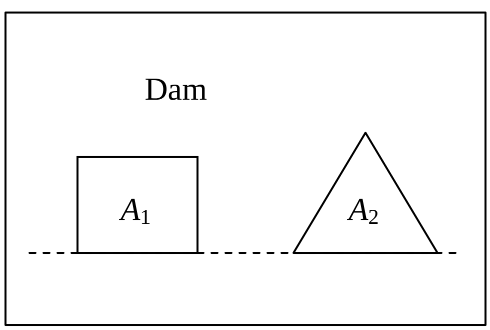
Work alone. Answer all questions. **Always make your thought process clear; if it is not, you will not receive partial credit for incomplete or partially incorrect answers.** Some data that might be useful is provided below.

		Area	Volume	Centroid	I_c
$\gamma_{water} = 62.4$ lb/ft^3 = 9800 N/m^3	Rectangle	bh		$h/2$	$bh^3/12$
s.g.$_{seawater}$ = 1.03	Triangle	$bh/2$		$h/3$	$bh^3/36$
s.g.$_{Hg}$ = 13.6	Circle	$\pi d^2/4$		$d/2$	$\pi d^4/64$
P_{atm} = 101.5 kPa = 14.7 psi	Sphere	πd^2	$\pi d^3/6$	$d/2$	

1. (10) What are the absolute and gage pressures at the bottom of a 1.5-m column of mercury, if the local barometric pressure is 795 mm of Hg? How would these values change if the barometric pressure decreased by 5 mm of Hg? You may give your answers in units of mm of Hg.

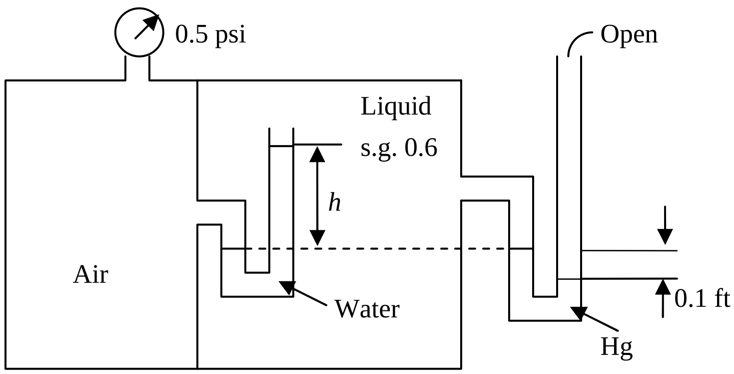
2. (10) Explain why we normally consider the pressure of a gas in a tank to be independent of height, but we do not make the same assumption for liquids. Approximately two sentences should be adequate.

3. (10) The bottoms of a square plate and a plate with the shape of an equilateral triangle are at the same depth on the face of a dam. The two plates have equal total surface area (see diagram below). Do you expect the forces on the two plates to be equal, or will one plate experience a larger force? Explain your reasoning in 1-2 sentences.



$A_1 = A_2$

4. (20) Compartments A and B of the tank shown in the figure below are closed and are filled, respectively, with air and a liquid with specific gravity of 0.6. Determine the value of h if the barometric pressure is 14.7 psia and the pressure gage reads 0.5 psi. The effect of the weight of the air is negligible.



5. (20) You wish to raise a tightly-sealed trunk from the bottom of a lake that is 15 m deep. The mass of the trunk is 1200 kg, and its dimensions (l x w x h) are 1.5 x 0.8 x 0.8 m. You decide to attach an inflatable buoy to the trunk, and fill the buoy with air until it starts to rise. Assuming that the buoy is spherical and that its center is 2 m above the bottom of the lake, what must its diameter be to begin bringing the trunk to the surface? Will the trunk-and-buoy combination be more, less, or equally buoyant at the surface, compared to their buoyancy at depth? You may assume that the weight of the buoy is negligible compared to that of the trunk.

6. (30) A plastic tube that is 3.0 m long and has a 1-m diameter is sealed on both ends by square plates that are 1.2 m on each side. The absolute pressure inside the tube is 50 kPa. The tube is lowered into seawater until the middle of the top plate is at a depth of 40 m, and it is held at that depth at a 45^{o} angle.

(a) Identify the direction and magnitude of the force exerted by the water on the top plate (you need not determine the line of action). Consider the force of all the water that contacts the plate.

(b) What is the net force exerted by the top plate on the tube, i.e., the difference between the force of the water and the force of the air. Be careful about absolute vs. gage pressures!

(c) If the bottom plate is removed, but the top plate remains in place, do you think that any changes will occur in the system? If so, describe them in a few sentences; if not, explain why not.

(d) What will be the direction and magnitude of the net force (the difference between the force of the water and the force of the air) on the top plate, after the bottom plate is removed, and the system has adjusted fully to that modification?

CEE 342 Aut 2003; Exam #1 Solutions

1. The gage pressure equals the pressure generated by the Hg column. Since we are given the height of the column in mm of Hg and we want to express pressure in the same units, the gage pressure is simply 1.5 m of Hg, or 1500 mm of Hg. The absolute pressure is the sum of the gage pressure and atmospheric (barometric) pressure, or 2295 mm of Hg.

If the barometric pressure decreased by 5 mm of Hg, the gage pressure would remain the same as before (1500 mm of Hg), but the absolute pressure would decrease by 5 mm, to 2290 mm of Hg.

2. The pressure in a static fluid varies with elevation z in a way that causes the sum $p+\gamma z$ to be constant everywhere in the fluid. Gases have such low values of γ that this sum is essential just equal to p, so the pressure is the same regardless of the value of z. By contrast, γ is much larger for liquids, so as z increases, p decreases.

3. The force on a submerged plate is the product of the force at its centroid of area and the total area of the plate:

$$F = p_c A_{tot} = \gamma\left(z-h_c\right)A_{tot}$$

where z is the depth at the bottom of the plate, and h_c is measured upward from that point to the centroid, i.e., $z-h$ is the depth of the centroid. The centroid of the triangle is deeper in the water than that of the square. Since the values of γ and A_{tot} are the same for the two plates, the force on the triangle is greater.

4. Knowing that the pressure at the air/Hg interface is 14.7 psi, and noting that, for any two locations in a continuous phase separated by a vertical distance Δz, $\Delta p = \gamma\Delta z$, we can work our way around the system to the air chamber as follows:

$$14.7\text{ psi} - \gamma_{\text{Hg}}\left(0.1\text{ ft}\right) - \gamma_{\text{liquid}}\left(h\right) + \gamma_{\text{water}}\left(h\right) = p_{\text{air, closed container}} = \left(14.7+0.5\right)\text{ psi}$$

Subtracting 14.7 psi from both sides, expressing γ_{Hg} and γ_{liquid} in terms of the specific gravities and γ_{water}, and carrying out some algebra, we find:

$$0.5\text{ psi} = -13.6\gamma_{\text{water}}\left(0.1\text{ ft}\right) - 0.6\gamma_{\text{water}}\left(h\right) + \gamma_{\text{water}}\left(h\right)$$

$$h = \frac{\left(0.5\text{ psi}\right)\dfrac{144\text{ in}^2}{\text{ft}^2} + 13.6\left(62.4\dfrac{\text{lb}}{\text{ft}^3}\right)\left(0.1\text{ ft}\right)}{0.4\left(62.4\dfrac{\text{lb}}{\text{ft}^3}\right)} = 6.28\text{ ft}$$

5. *Note: On this problem, many people seemed to assume that the trunk was sitting on the bottom of the lake with no water under it. In truth, the sand and other sediments supporting objects on a lake bed touch the surface over a negligible area (it's like a sheet*

of plywood sitting on a bed of marbles), so that virtually all the bottom area of the object is in contact with water. My solution below is based on that view, so that there is a buoyant force on the trunk. However, if you solved the problem ignoring that buoyant force, I didn't take off any points.

(a) For the combination of the trunk and buoy to rise, the weight of the water that they displace must be greater than their combined weight. The weight of the buoy is negligible, and that of the trunk is $m_{trunk}g$, so the total weight to be raised is:

$$W = m_{trunk}\,g = 1200\text{ kg}\left(9.8\text{ N/kg}\right) = 11,760\text{ N}$$

The volume of water that has this weight is:

$$V_{water} = \frac{W}{\gamma_{water}} = \frac{11,760\text{ N}}{9800\text{ N/m}^3} = 1.20\text{ m}^3$$

The volume of the trunk is $(1.5 \times 0.8 \times 0.8)\text{m}^3$, or 0.96 m^3, so it displaces that volume of water. The buoy must therefore displace an additional 0.24 m^3, i.e., $V_{buoy} = 0.24\text{ m}^3$. The buoy must therefore be inflated to a diameter of:

$$d_{buoy} = \sqrt[3]{\frac{6}{\pi}V_{buoy}} = \sqrt[3]{\frac{6}{\pi}\left(0.24\text{ m}^3\right)} = 0.77\text{ m}$$

Note that neither the depth of the lake nor that of the buoy have any bearing on this result; the buoy would have to be this size to lift the trunk, regardless of where either the trunk or the buoy is located in the water.

(b) As the trunk and buoy rise, the buoy will become larger (because the water pressure will decline), so the combined buoy-plus-trunk combination will become more buoyant.

6. (a) The force of the water on the plate is perpendicular to its surface. Over the areas of the plate that protrude beyond the tube, the pressures on the two sides of the plate are of equal magnitude, but in opposite directions, so the resultant of those forces is zero. However, in the area where the plate covers the tube, the water pressure is exerted on only one side, leading to a net force toward the tube. The area of this region equals the area of the tube.

The force of a fluid on a planar object equals the product of the pressure at the centroid of the area and the total area. The centroid of the circular area of interest is in seawater at a depth of 40 m, so the pressure at that point is $40\gamma_{sw}$. The net force of the water on the plate is therefore:

$$F_{sw} = p_c A_{tot} = \gamma_{sw} z_c A_{tot}$$

$$= \left[(1.03)\left(9800 \text{ N/m}^3\right)\right](40 \text{ m})\left(\pi \frac{(1.0 \text{ m})^2}{4}\right) = 317{,}000 \text{ N} = 317 \text{ kN}$$

As noted, this force is directed at a 45° angle, perpendicular to the plate and toward the tube.

(b) The force computed in part *a* is attributable to the seawater alone. The total force on the exterior of the tube is the sum of the forces due to the water and the overlying atmosphere, at a pressure of 100 kPa. The air in the tube exerts a force in the opposite direction, equal to the product of the pressure inside the tube and the area. This pressure was 50 kPa when the tube was at the surface, and since the tube was sealed before it was lowered, the pressure stays at that value. The net force of the plate on the tube is therefore:

$$F_{net} = F_{sw} + F_{atm} - F_{air}$$

$$= 317 \text{ kN} + \left[(100-50) \text{ kPa}\right]\left(1.0 \frac{\text{kN/m}^2}{\text{kPa}}\right)\left(\pi \frac{(1.0 \text{ m})^2}{4}\right) = 356 \text{ kN}$$

(c) When the plate is removed, the pressure of the seawater and overlying atmosphere, which was previously exerted on the plate, is instead exerted on the air in the tube. That pressure is somewhat greater than 40 γ_{sw}, since the bottom of the tube is lower than the top; the exact value is $(40 + 3*\sin 45^{o})\ \gamma_{sw}$, or 447 kN/m^2, i.e., 447 kPa. The counteracting pressure of the air is only 50 kPa, so the water will enter the tube. As it does so, the level of the water/air interface rises slightly, so the pressure on the seawater side of the interface decreases slightly. At the same time, the air gets compressed, so its pressure increases. The process continues until the air is so compressed that its pressure equals that of the seawater, and the whole system is in equilibrium.

(d) Once the process described in part *c* is complete, the air pressure will be essentially the same as that of the seawater, so there will be no net force on the upper plate.