Key concepts: Conservation of mass, momentum, energy

<u>Fluid</u>: a material that deforms continuously and permanently under the application of a shearing stress, no matter how small.

Fluids are either gases or liquids. (Under very specialized conditions, a phase of intermediate properties can be stable, but we won't consider that possibility.) In liquids, the molecules are relatively closely spaced, allowing the magnitude of their (attractive, electrically-based) interaction energy to be of the same magnitude as their kinetic energy. As a result, they exist as a loose collection of clusters. In gases, the molecules are much more widely separated, so the kinetic energy (at a given temperature, identical to that in the liquid) is far greater than the interaction energy (much less than in the liquid), and molecule do not form clusters. In a liquid, the molecules themselves typically occupy a few percent of the total space available; in a gas, they occupy a few thousandths of a percent. Nevertheless, for our purposes, all fluids are considered to be continua (no voids or holes). The absence of significant intermolecular attraction allows gases to fill whatever volume is available to them, whereas the presence of such attraction in liquids prevents them from doing so. The attractive forces in liquid water are unusually strong, compared to other liquids.

Properties of Fluids:

Density is mass/volume: $\rho = \frac{m}{V}$. The density of liquid water is ~1.0 kg/m³; that of air at 20°C is ~1.2x10⁻³ kg/m³.

Specific weight is weight/volume: $\gamma = \frac{mg}{V} = \frac{W}{V} = \rho g$

Specific gravity is density normalized to the density of water: $s.g. = \frac{\gamma_i}{\gamma_w}$

Specific volume is volume/mass: $\overline{\Psi} = \frac{\Psi}{m} = \frac{1}{\rho}$

Bulk modulus or **modulus of elasticity** is the pressure change per fractional change in volume or density: $E_v = -\frac{dp}{dV} = \frac{dp}{d\rho/\rho}$. The

larger the bulk modulus, the larger the pressure change required for a given fractional reduction in volume; *i.e.*, increasing bulk modulus corresponds to decreasing compressibility. E_v of water is ~2x10⁹ Pa, or ~20,000 atm; *i.e.*, it compresses by ~1% when subjected to a pressure of ~200 atm.

An *ideal gas* is a gas whose molecules behave according to theories developed for hard spheres that have kinetic energy and that interact strictly by perfectly elastic collisions (no other attraction or repulsion). All gases behave nearly ideally under conditions relevant for this class. The *equation of state* for an ideal gas is:

$$\frac{n_{mol}}{\Psi} = \rho_{mol} = \frac{p}{R_0 T}$$
$$\frac{n_{mol} (MW)}{\Psi} = \frac{M}{\Psi} = \rho = \frac{p}{R_0 T} (MW) = \frac{p}{RT}$$

where n_{mol} is the number of moles of gas in a volume V_{-} , ρ_{mol} is the *molar density* (moles per unit volume) of the gas, MW is the *molecular weight* of the gaseous species, and $R = R_0$ /MW. (Note: R_0 is the Rydberg constant, more often called the universal gas constant.)

Pressure is the force exerted per unit of surface, resulting from the impact of molecules colliding with the surface. The surface can be real or imaginary. Pressure can also be thought of in a 3-D context, as the mechanical potential energy that an object has per unit

volume of the object. Consider an analogy: Any object on the earth has the same *gravitational* potential energy <u>per unit mass</u>, equal to *gh*. Thus, the gravitational potential energy of any object can be computed as *mgh*, regardless of the size, shape, or other properties of the object. (To carry out the calculation, one has to define a datum where h=0, but that can be done arbitrarily.)

Similarly, objects have *mechanical* potential energy <u>per unit</u> <u>volume</u> equal to p. That is, the mechanical potential energy of an object with volume \bigvee can be computed as $\bigvee p$, regardless of its mass, shape, or other properties. Again, a datum must be defined, in this case to establish where p=0. The most common datum levels are the pressure in an absolute vacuum and the local atmospheric pressure; when the former datum is used, the pressure is called the *absolute pressure*, and when the latter is used, the pressure is called the *gage pressure*.

Mechanical, gravitational, and other forms of potential energy can be interconverted, and these conversions are central features in many fluid processes of interest.



At any solid boundary, any fluid has zero velocity. This is known as the *no-slip condition*. The recognition of the no-slip condition in 1904 by Prandtl ushered in the modern era of fluid dynamics. If a fluid is flowing past a solid surface, the fluid velocity must increase from zero to a finite value as one moves away from the surface, *i.e.*, a velocity gradient must exist near the solid/liquid interface. Far from solid boundaries, fluid flows with very little velocity gradient and hence moves as a solid mass; such fluid is said to be *inviscid* and to behave as an *ideal fluid*.

(Absolute) Viscosity is shear stress per unit velocity gradient: $\mu = \frac{\tau}{du/dy}$. The velocity gradient can also be interpreted as the rate of shear strain $\left(\frac{du}{dy} = \frac{d(dx/dt)}{dy} = \frac{d^2x}{dy dt} = \frac{d(dx/dy)}{dt}\right)$. A large viscosity means that a large shear stress is required to generate a relatively small velocity gradient.

Kinematic Viscosity is the ratio of absolute viscosity to density: $v = \frac{\mu}{\rho}$.

Imagine three experiments in which a fluid is place between two parallel plates. In one case, the upper plate is moved in the x direction at a velocity U, and no pressure gradient is applied in the x direction. In the second case, a pressure gradient is applied, but the plates are stationary. And in the third case, the plate is moved and a pressure gradient is applied. Empirically, the results might be as follows:



At any point in these systems, we can define a variable μ as the ratio of the (local) shear stress to the (local) velocity gradient: $\mu = \frac{\tau}{du/dy}$. In (*a*), the shear stress and velocity gradient are the same everywhere in the system, but in (b) and (c) they are not. Thus:

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(a):

$$\frac{F_{x,top}}{A_{top}} = \frac{F_{x,bot}}{A_{bot}} = \tau = \mu \frac{U}{Y} = \mu \frac{du}{dy}$$
(b):

$$\frac{F_{x,top}}{A_{top}} = \frac{F_{x,bot}}{A_{bot}} = \tau = \mu \left(\frac{du}{dy}\right)_{wall}$$
(c):

$$\frac{F_{x,top}}{A_{top}} = \tau_{top} = \mu \left(\frac{du}{dy}\right)_{top wall}$$

$$\frac{F_{x,bot}}{A_{bot}} = \tau_{bot} = \mu \left(\frac{du}{dy}\right)_{bottom wall}$$

$$F_{x,top} \neq F_{x,bot}$$

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If general, μ varies from one fluid to the next, and also for a single fluid as a function of temperature. For many fluids, μ also varies as a function of τ . However, for many others, μ is independent of τ ; such fluids are said to be Newtonian. (Note: the calculation of viscosity as the ratio of the shear stress to the velocity gradient applies only to *laminar* flows. Under conditions where flow is *turbulent*, the equation does not apply; in that case, the product $\mu du/dy$ accounts for a portion of the shear stress, but the total shear stress includes contributions due to the turbulence as well. We will discuss this situation later in the term.)

Viscosity reflects the slowing of one layer of fluid due to interactions with the adjacent layers. In liquids, this slowing is caused primarily by the attraction of clusters in adjacent layers for one another. Those interactions slow down the molecules in the faster moving layer and speed up the molecules in the slower moving layer. An increase in temperature increases the kinetic energy of the molecules, reducing the relative importance of their attraction for one another, so viscosity decreases.

In gases, attractive interactions among molecules are negligible; in this case the main effect of an increase in temperature and kinetic energy is to increase the frequency with which molecules from one layer of the fluid move into the adjacent layer. Assuming that the density of the gas is constant, the movement of *x* molecules from layer A to layer B must be accompanied by movement of the same number of molecules in the opposite direction. This exchange slows down the faster moving layer and accelerates the slower moving layer; *i.e.*, it has the same effect as the interactions among clusters in adjacent layers of liquid. However, the frequency of molecular exchanges across layers increases with increasing temperature, so gases become more viscous when the temperature is increased.



Vapor pressure is the pressure exerted by the gas-phase molecules of a liquid, when the gas and liquid reach equilibrium. Vapor pressure increases with temperature.

Many different species can co-exist in a gas phase, and each exerts a pressure in accord with the ideal gas law. If a liquid and gas are in contact with one another, molecules of the liquid will evaporate until the pressure exerted by those molecules in the gas equals the <u>vapor pressure</u> of the liquid at that temperature. If, for some reason, the pressure exerted by those molecules becomes greater than the vapor pressure, they will condense to form more liquid, thereby lowering the exerted pressure until it equals the vapor pressure. These changes occur regardless of what other gaseous species are present, and how much pressure they exert.

In addition to evaporation and condensation, if the <u>total</u> pressure on the liquid is less than the vapor pressure, the liquid will boil. Boiling can occur even if no gas phase is initially present. When a pressure drop in a pipe or pump causes boiling, it is called *cavitation*. If the pressure subsequently increases, the bubbles generated by cavitation collapse.



 $p_i < p_v, p_{tot} > p_v$

Evaporation, no boiling $p_i > p_v, p_{tot} > p_v$

Condensation, no boiling $p_i < p_v, p_{tot} < p_v$

Evaporation and boiling

 $p_{tot} < p_v$, no gas phase

No evaporation, Boiling (cavitation)

Surface tension

Absent other forces, all freely suspended liquid droplets tend to become spherical. This observation can be interpreted in terms of either energy or force; we will consider an energy analysis here, and show the force analysis after developing some other principles.

Energy balance analysis. Molecules in a liquid phase are attracted to one another, which is to say that when a molecule moves from a place far from others to a place where it is surrounded by others, it loses potential energy. This process is energetically favorable and therefore occurs spontaneously; an analogy in a more familiar system is that of a ball rolling downhill, which also occurs spontaneously and which we ascribe to the loss of (gravitational) potential energy. In each case, the process can be reversed, but only by putting energy back into the system.

Returning to the liquid droplet, we note that molecules at the surface are in contact with fewer other molecules than those in the interior, so the surface molecules have higher energy and are less stable. The system spontaneously minimizes its total potential energy by minimizing the number of molecules at the surface (*i.e.*, by minimizing the surface area), which leads to the spherical shape.

Now imagine that a droplet that is initially spherical becomes distorted. This process requires an energy input, because it forces some interior (lower energy) molecules to move to the (higher energy) surface. The energy required per unit increase in surface area is called the *surface tension*, σ . Liquids with large surface tension strongly resist distortion, which is to say that a relatively large input of energy is required to distort them and increase the surface area by even a small amount. This leads to the image of an elastic "skin" covering the liquid surface: the larger the surface tension, the "tougher" the skin.

The same idea applies to liquid surfaces that are flat, e.g., water in a bowl. If certain objects are gently placed on the liquid surface, they might float, even though they would sink if immersed in the liquid. The explanation is that, when initially placed on the surface, these objects sink very slightly, but continue to float. As they sink, their gravitational potential energy declines by $mg \Delta z$. However, simultaneously, they distort the surface slightly. As a result, the surface area of the liquid increases, increasing the liquid's potential energy (by $\sigma \Delta A$). If, for an incremental lowering of the object by an amount dz, the gravitational energy declines more than the surface-tension-based potential energy increases, that change will be favored. If such a change is favored until the object has moved deep enough into the liquid, the liquid will cover the object, and it will sink. However, if at some point in that process the product mg dz equals σdA (where dA is the increment of new area generated when the object descends distance dz), then the object will be stable at that location.

At a much larger scale, this process is similar to what happens when a person stands in the middle of a trampoline. As he or she descends, the gravitational energy of the system declines, and the energy stored in the stretched-out trampoline surface increases. This process continues until the loss of gravitational potential energy by sinking farther would generate an equal increase in "stretching" potential energy; at that point, neither sinking nor rising is energetically favored, and the system remains stable.

