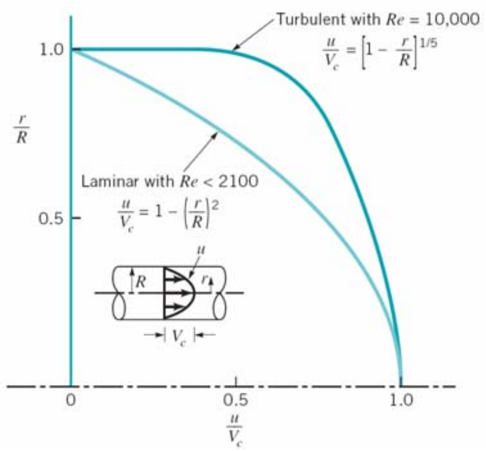
CEE 342 Aut 2005 HW#9 Solutions

8.26.



The average velocity for any flow pattern is given by $V = Q/A$. If the velocity is not uniform, then Q, and hence V, must be determined by an integration across the cross-section. For a circular cross-section, for example, the integration is as follows:

$$V = \frac{Q}{A} = \frac{\int v dA}{\pi R^2} = \frac{\int_{r=0}^{R} v(2\pi r)dr}{\pi R^2}$$

(a) For laminar flow, we have already carried out the integration in class, and it is also presented in the text. The result is that $V = V_{max}/2$, and that the radial profile of the velocity is $V(r) = V_{max}\left(1 - \left(\frac{r}{R}\right)^2\right)$. Therefore, the question comes down to determining the value of r at which $V(r)$ equals $V_{max}/2$. Substituting this equality into the equation for the velocity profile, we obtain:

$$\frac{V_{max}}{2} = V_{max}\left(1 - \left(\frac{r}{R}\right)^2\right)$$

$$\frac{1}{2} = 1 - \left(\frac{r}{R}\right)^2$$

$$\frac{r}{R} = \sqrt{\frac{1}{2}} = 0.707$$

Thus, the pitot tube should be placed at a radius 70.7% of the distance from the center to the pipe wall.

(b) For turbulent flow with a velocity profile that corresponds to $V_{avg} = V_{\max}\left(1-\frac{r}{R}\right)^{1/n}$, the average velocity is given by:

$$V = \frac{\int_{r=0}^{R} v\left(2\pi r\right)dr}{\pi R^2} = \frac{\int_{r=0}^{R}\left[V_{\max}\left(1-\frac{r}{R}\right)^{1/n}\right]2\pi r dr}{\pi R^2} = 2V_{\max}\int_{r=0}^{R}\left(1-\frac{r}{R}\right)^{1/n}\frac{r}{R}\frac{dr}{R}$$

This expression can be integrated analytically by defining a new variable y as $1-r/R$. Then, $r/R = 1-y$, and $dr/R = -y$. Also, r values of 0 and R at the limits of integration correspond to y values of 1 and 0, respectively. Making these substitutions:

$$V = 2V_{\max}\int_{y=1}^{0} y^{1/n}\left(1-y\right)dy = 2V_{\max}\int_{y=1}^{0}\left(y^{1/n}-y^{(n+1)/n}\right)\left(-dy\right) = 2V_{\max}\int_{y=1}^{0}\left(y^{(n+1)/n}-y^{1/n}\right)dy$$

$$= 2V_{\max}\left(\frac{n}{2n+1}y^{(2n+1)/n}-\frac{n}{n+1}y^{(n+1)/n}\right)_{y=1}^{0} = 2V_{\max}\left(\left[0-\frac{n}{2n+1}\right]-\left[0-\frac{n}{n+1}\right]\right)$$

$$= 2V_{\max}\frac{n^2}{\left(2n+1\right)\left(n+1\right)}$$

In the current case, with $n = 5$, we have:

$$V = 2V_{\max}\frac{5^2}{\left(2*5+1\right)\left(5+1\right)} = \frac{50}{66}V_{\max}$$

Thus, the average velocity if 50/66 of $V_{\max}$. Based on the velocity profile, this average velocity occurs at a value of r/R given by:

$$\frac{50}{66}V_{\max} = V_{\max}\left(1-\frac{r}{R}\right)^{1/5}$$

$$\frac{r}{R} = 1-\left(\frac{50}{66}\right)^5 = 0.75$$

Thus, the pitot tube should be placed at 75% of the distance from the centerline to the wall.

8.35. We are given information about the length, diameter, and roughness of a pipe, and we need to find the upstream pressure when the water velocity in the pipe is 6 ft/s, if the pipe discharges as a free jet. Applying the energy equation, we have:

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{l}{D}\frac{V^2}{2g}$$

where V is the velocity in the pipe. For this problem, $z_1 = z_2$, $p_2 = 0$, and $V_1 = V_2 = V = 6$ ft/s. Thus:

$$p_1 = f\frac{l}{D}\frac{V^2}{2g}\gamma = f\frac{l}{D}\frac{\rho V^2}{2}$$

The relative roughness and Reynolds number for the flow in the pipe are:

$$\frac{\varepsilon}{D}=\frac{0.0009\text{ ft}}{(0.5/12)\text{ft}}=0.0216$$

$$\text{Re}=\frac{DV}{\nu}=\frac{(0.5/12\text{ ft})(6\text{ ft/s})}{1.21\text{x}10^{-5}\text{ ft}^2\text{/s}}=20662$$

With the given relative roughness and Reynolds number, we can solve iteratively for f using the Colebrook equation, yielding $f = 0.052$. Then, plugging values into the equation for p_1, we find:

$$p_1 = f\frac{l}{D}\frac{\rho V^2}{2}=0.052\frac{70\text{ ft}}{(0.5/12\text{ ft})}\frac{(1.94\text{ slug/ft}^3)(6\text{ ft/s})^2}{2}\frac{1\text{ lb}}{\text{slug-ft/s}^2}$$

$$=3051\text{ psf }=21.2\text{ psi}$$

8.47. Writing the energy equation between points upstream and downstream of the contraction, we have:

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+K_L\frac{V_2^2}{2g}$$

Note that the loss coefficient for the contraction is based on the velocity head *after* the contraction. The pipe is horizontal, so $z_1 = z_2$. Also, the flow rate is given, so we can determine V_1 and V_2:

$$V_1=\frac{Q}{A_1}=\frac{0.04\text{ m}^3\text{/s}}{\frac{\pi}{4}(0.12\ )^2}=3.54\frac{\text{m}}{\text{s}}$$

$$V_2=V_1\frac{A_1}{A_2}=\left(3.54\frac{\text{m}}{\text{s}}\right)(4)=14.2\frac{\text{m}}{\text{s}}$$

From Figure 8.26, for $A_2/A_1 = 0.25$, we see that $K_L = 0.4$. Therefore, multiplying through the energy equation by γ and inserting values, we find:

$$p_1 + \frac{\rho V_1^2}{2} + \cancel{z_1\gamma} = p_2 + \frac{\rho V_2^2}{2} + \cancel{z_2\gamma} + K_L \frac{\rho V_2^2}{2}$$

$$p_1 - p_2 = \frac{\rho}{2}\left[\left(V_2^2 - V_1^2\right) + K_L V_2^2\right] = \frac{\rho}{2}\left[(1+K_L)V_2^2 - V_1^2\right]$$

$$= \frac{1000 \text{ kg/m}^3}{2}\left[(1+0.4)\left(14.2\frac{\text{m}}{\text{s}}\right)^2 - \left(3.54\frac{\text{m}}{\text{s}}\right)^2\right] = 1.35\text{x}10^5 \frac{\text{kg}}{\text{m-s}^2} = 135 \text{ kPa}$$

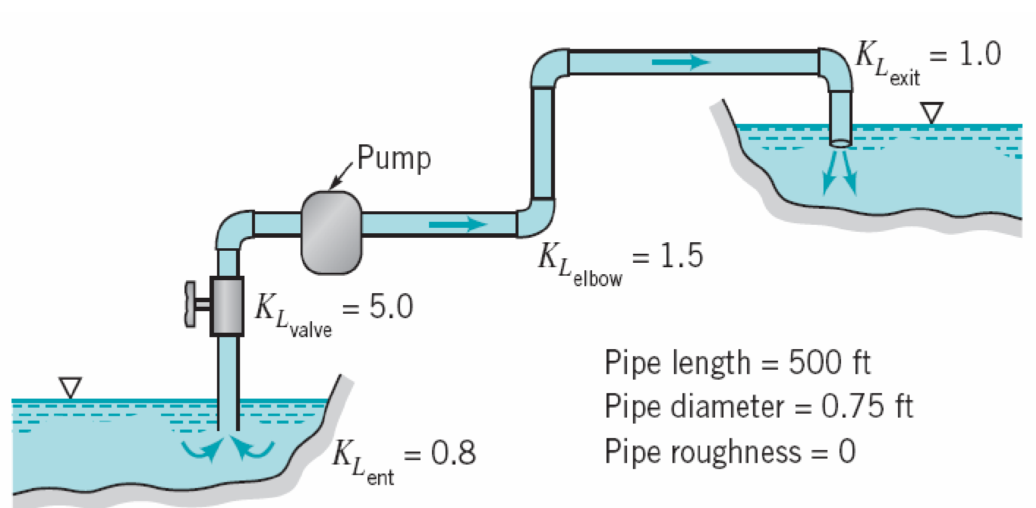
The portion of the pressure loss due to friction is $K_L \frac{\rho V_2^2}{2}$, which is:

$$K_L \frac{\rho V_2^2}{2} = 0.40 \frac{\left(1000 \text{ kg/m}^3\right)\left(14.2 \text{ m/s}\right)^2}{2} = 40,330 \ \frac{\text{kg}}{\text{m-s}^2} = 40.3 \text{ kPa}$$

Correspondingly, the portion due to the change in velocity is:

$$\Delta p_{vel} = \Delta p_{tot} - \Delta p_{geometry} = (135 - 40.3)\text{ kPa} = 94.7 \text{ kPa}$$

8.75.



Applying the energy equation between the surfaces of the two reservoirs, we have:

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1+h_{pump}=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_L$$

The pressures and velocities at both locations are zero, the elevation difference is 200 ft, and the delivers 250 ft of head to the water. The head loss due to friction includes the loss for flow through the pipe and minor losses associated with the entry into the pipe, the valve, the four 90^o elbows, and the exit into the upper reservoir. Thus:

$$z_1-z_2+h_{pump}=\left(f\frac{l}{D}+\sum K_L\right)\frac{V^2}{2g}$$

$$-200\text{ ft}+250\text{ ft}=\left(f\frac{500\text{ ft}}{0.75\text{ ft}}+\left[0.8+4(1.5)+5.0+1\right]\right)\frac{V^2}{2\left(32.2\text{ ft/s}^2\right)}$$

$$\frac{(50\text{ ft})2\left(32.2\text{ ft/s}^2\right)}{f\frac{500\text{ ft}}{0.75\text{ ft}}+12.8}=V^2$$

$$V=\sqrt{\frac{3220\text{ ft}^2\text{/s}^2}{667f+12.8}}$$

Because the pipe is smooth, the Colebrook equation can be simplified as follows:

$$\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{2.51}{\text{Re}\sqrt{f}}\right)$$

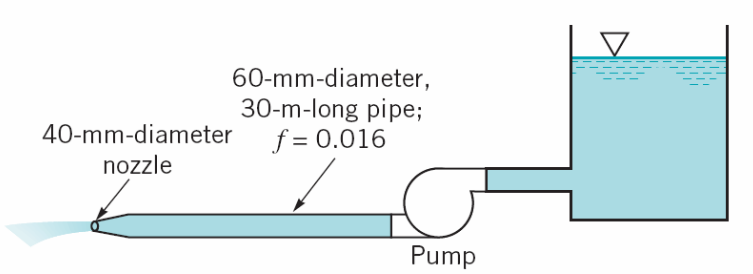
$$\text{Re}=\frac{DV\rho}{\mu}=\frac{(0.75\text{ ft})V\left(1.94\frac{\text{slug}}{\text{ft}^3}\right)}{2.34\text{x}10^{-5}\text{ lb-s/ft}^2}\left(1\frac{\text{lb}}{\text{slug-ft/s}^2}\right)=\left(62180\frac{\text{s}}{\text{ft}}\right)V$$

We now have three equations relating V, f, and Re to one another. Solving these equations simultaneously (by T & E), we find $V = 12.4$ ft/s, and $f = 0.012$. The power added to the water by the pump is:

$$P=Q\gamma h_{pump}=\left(12.4\frac{\text{ft}}{\text{s}}\right)\left(\frac{\pi}{4}\left[0.75\text{ ft}\right]^2\right)\left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(250\text{ ft})=8.55\text{x}10^4\frac{\text{ft-lb}}{\text{s}}$$

$$=\left(8.55\text{x}10^4\frac{\text{ft-lb}}{\text{s}}\right)\left(\frac{1\text{ hp}}{550\text{ ft-lb/s}}\right)=155\text{ hp}$$

8.89.



The energy equation written between the top of the reservoir and the outlet for the conditions with the pump in place is:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_{pump} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

For the specified conditions, $p_1 = p_2 = 0$, and $V_1 = 0$. Thus:

$$z_1 - z_2 + h_{pump} = \frac{V_2^2}{2g} + h_L$$

$$h + h_{pump} = \frac{V_2^2}{2g} + f\frac{l}{D}\frac{V^2}{2g}$$

We can find h_{pump}, V_2 and V (the velocity in the pipe) from the given flow rate:

$$h_{pump} = \frac{P}{Q\gamma} = \frac{25000 \text{ W}}{\left(0.04 \text{ m}^3\text{/s}\right)\left(9800 \text{ N/m}^3\right)}\left(1\frac{\text{N-m/s}}{\text{W}}\right) = 63.8 \text{ m}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.04 \text{ m}^3\text{/s}}{\frac{\pi}{4}\left(0.040 \text{ m}\right)^2} = 31.8\frac{\text{m}}{\text{s}}$$

$$V = \frac{Q}{A} = \frac{0.04 \text{ m}^3\text{/s}}{\frac{\pi}{4}\left(0.060 \text{ m}\right)^2} = 14.1\frac{\text{m}}{\text{s}}$$

Thus:

$$h = \frac{V_2^2}{2g} + f\,\frac{l}{D}\frac{V^2}{2g} - h_{pump}$$

$$= \frac{(31.8\text{ m/s})^2}{2(9.8\text{ m/s}^2)} + 0.016\left(\frac{30\text{ m}}{0.060\text{ m}}\right)\frac{(14.1\text{ m/s})^2}{2(9.8\text{ m/s}^2)} - 63.8\text{ m} = 69.0\text{ m}$$

Without the pump, we can write the same energy equation, but with $h_{pump} = 0$. In addition, we can relate V and V_2 by continuity. Thus:

$$V = V_2\,\frac{A_2}{A} = V_2\,\frac{(0.040\text{ m})^2}{(0.060\text{ m})^2} = 0.444V_2$$

$$h = \frac{V_2^2}{2g} + f\,\frac{l}{D}\frac{V^2}{2g} = \frac{V_2^2}{2g} + f\,\frac{l}{D}\frac{(0.444V_2)^2}{2g} = \frac{V_2^2}{2g}\left(1 + (0.444)^2\,f\,\frac{l}{D}\right)$$

$$V_2 = \sqrt{\frac{2gh}{1 + (0.444)^2\,f\,\frac{l}{D}}} = \sqrt{\frac{2(9.8\text{ m/s}^2)(69.0\text{ m})}{1 + (0.444)^2(0.016)\frac{30\text{ m}}{0.06\text{ m}}}} = 22.9\,\frac{\text{m}}{\text{s}}$$

$$Q = V_2 A_2 = \left(22.9\,\frac{\text{m}}{\text{s}}\right)\left(\frac{\pi}{4}[0.04\text{ m}]^2\right) = 0.0289\,\frac{\text{m}^3}{\text{s}}$$