CEE 342 Aut 2005 HW#8 Solutions

8.7. For laminar flow in a pipe or system with a similar flow geometry, flow is laminar at Reynolds numbers <2100. At 10^{o}C, the kinematic viscosity (ν) of water is $1.31 \text{ x } 10^{-6}$ m^2/s (from Table B.2, p.761). The velocity of the fluid is:

$$V = \frac{Q}{A} = \frac{\left(4 \text{ cm}^3\text{/s}\right)}{\pi\left(0.4 \text{ cm}\right)^2 / 4} = 31.8\frac{\text{cm}}{\text{s}}$$

The Reynolds number is therefore:

$$\text{Re} = \frac{DV}{\nu} = \frac{\left(0.4 \text{ cm}\right)\left(31.8 \text{ cm/s}\right)}{\left(1.31\text{x}10^{-6} \text{ m}^2\text{/s}\right)\left(100 \text{ cm/m}\right)^2} = 971$$

The flow is laminar. For laminar flow, the ratio of the entrance length to the diameter is given by:

$$\frac{l_e}{D} = 0.06\,\text{Re} = 0.06\left(971\right) = 58.3$$

$$l_e = 58.3D = 58.3\left(0.4 \text{ cm}\right) = 23.3 \text{ cm}$$

Since the straw length (25 cm) is longer than l_e, the flow is fully developed.

8.15. The maximum pressure is the pressure that causes the Reynolds number to equal 2100. Thus:

$$\text{Re} = 2100 = \frac{DV\rho}{\mu}$$

$$V = \frac{2100\mu}{D\rho} = \frac{2100\left(0.30 \text{ N-s/m}^2\right)}{\left(0.1 \text{ m}\right)\left(10^3 \text{ kg/m}^3\right)\left(1 \ \frac{\text{N}}{\text{kg-m/s}^2}\right)} = 6.3\frac{\text{m}}{\text{s}}$$

For laminar flow, the headloss and pressure change due to friction are given by the Hagen-Poiseuille equation. If, in addition to friction, pressure changes because of a change in elevation, the overall pressure change is given by Equation 8.11 or 8.12. Using Equation 8.11, we find:

$$V = \frac{\left(\Delta p - \gamma l \sin\theta\right)D^2}{32\mu l}$$

$$\Delta p = \frac{32\mu l V}{D^2} + \gamma l \sin\theta = \frac{32\left(0.30\text{ N-s/m}^2\right)\left(10\text{ m}\right)\left(6.3\text{ m/s}\right)}{\left(0.10\text{ m}\right)^2} + \left(9810\frac{\text{N}}{\text{m}^3}\right)\left(10\text{ m}\right)\sin\left(-90^{\text{o}}\right)$$

$$= -37,620\text{ Pa} = -37.6\text{ kPa}$$

8.19. The velocity profile for laminar flow in a pipe is given by:

$$V = V_c\left(1 - \left[\frac{r}{R}\right]^2\right)$$

where V_c is the centerline velocity, r is the distance measured from the centerline, and R is the pipe radius. In the current case, $R = 0.05$ cm, $r = (0.050 - 0.012)$ cm, or 0.038 cm, and $V = 0.8$ m/s. Also, the average velocity for laminar flow in a pipe is one-half of the maximum (centerline) velocity. The centerline velocity and flow are therefore:

$$V_c = \frac{V}{1 - \left(r/R\right)^2} = \frac{0.8\text{ m/s}}{1 - \left(0.038/0.05\right)^2} = 1.89\text{ m/s}$$

$$Q = V_{avg}A = 0.5V_cA = \left(0.5\right)\left(1.89\text{ m/s}\right)\left(\pi\left(0.05\text{ m}\right)^2\right) = 7.42\text{x}10^{-3}\,\frac{\text{m}^3}{\text{s}}$$

8.21. The Darcy-Weisbach equation applies for either laminar or turbulent flow. Thus, for either case of interest:

$$h_L = f\,\frac{l}{D}\frac{V^2}{2g}$$

Since l, D, V and g are the same for the two cases, the ratio of the headloss in laminar to turbulent flow is the ratio of the corresponding friction factors. For laminar flow, $f = 64/\text{Re}$; for $\text{Re} = 6000$, $f = 0.0107$. For turbulent flow with a smooth pipe, we can find the friction factor from the Moody diagram, with the result that, for $\varepsilon/D = 0$ and $\text{Re} = 6000$, $f = 0.035$. Therefore, if the flow were laminar, the headloss would be reduced by:

$$\frac{0.035 - 0.0107}{0.035} = 0.694 = 69.4\%$$