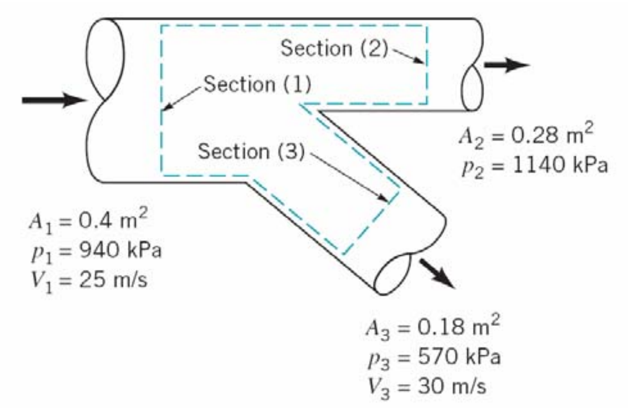
CEE 342 Aut 2005 HW#7 Solutions

5.105. An appropriate CV for analyzing the system is shown below. We are asked to find the power loss as water flows through the *y* connection, which lies in a horizontal plane.



The velocity at location 2 can be found based on the continuity equation:

$$Q = V_1A_1 = V_2A_2 + V_3A_3$$

$$V_2 = \frac{V_1A_1 - V_3A_3}{A_2} = \frac{\left(25\,\frac{\text{m}}{\text{s}}\right)\left(0.4\text{ m}^2\right) - \left(30\,\frac{\text{m}}{\text{s}}\right)\left(0.18\text{ m}^2\right)}{\left(0.28\text{ m}^2\right)} = 16.4\,\frac{\text{m}}{\text{s}}$$

We can apply the energy equation to the CV as a version of the RTT, substituting the volumetric flow rate Q for the product $AV\cos\theta$, based on the fact that the flow is perpendicular to the area at each inlet and outlet:

$$0 = \sum_{inlets}\left(i_{en}\rho VA\cos\theta\right) - \sum_{outlets}\left(i_{en}\rho VA\cos\theta\right) + \dot{E}_{en} = \sum_{inlets}\left(i_{en}\rho Q\right) - \sum_{outlets}\left(i_{en}\rho Q\right) + \dot{E}_{en}$$

The energy terms in the RTT can be expressed in various units. In the following equation, the terms are expressed as the product $i_{en}\rho$, which has units of energy unit volume of fluid. Also, note that $\dot{E}_{en}$ is the non-advective rate of energy input (i.e., the power input) into the CV. The only non-advective energy transfer in this system is due to friction, so the value we are seeking (the rate at which the fluid loses power due to friction, $-\Delta P$) is $-\dot{E}_{en}$. Thus:

$$-\Delta P = -\dot{E}_{en} = \left(p_1 + z_1\gamma + \rho\frac{V_1^2}{2}\right)Q_1 - \left(p_2 + z_2\gamma + \rho\frac{V_2^2}{2}\right)Q_2 - \left(p_3 + z_3\gamma + \rho\frac{V_3^2}{2}\right)Q_3$$

Because the connection is in a horizontal plane, $z_1 = z_2 = z_3$, so the fluid neither gains nor loses gravitational energy, and the terms containing z all cancel. Writing each Q as VA, we have:

$$-\Delta P = \left(940\ \text{kPa} + \left(10^3\,\frac{\text{kg}}{\text{m}^3}\right)\frac{\left(25\,\frac{\text{m}}{\text{s}}\right)^2}{2}\right)\left(25\,\frac{\text{m}}{\text{s}}\right)\left(0.4\text{m}^2\right)$$

$$-\left(1140\ \text{kPa} + \left(10^3\,\frac{\text{kg}}{\text{m}^3}\right)\frac{\left(16.4\,\frac{\text{m}}{\text{s}}\right)^2}{2}\right)\left(16.4\,\frac{\text{m}}{\text{s}}\right)\left(0.28\text{m}^2\right) - \left(570\text{kPa} + \left(10^3\,\frac{\text{kg}}{\text{m}^3}\right)\frac{\left(30\,\frac{\text{m}}{\text{s}}\right)^2}{2}\right)\left(30\,\frac{\text{m}}{\text{s}}\right)\left(0.18\text{m}^2\right)$$

$$-\Delta P = 1165\text{x}10^3\,\frac{\text{kg-m}^2/\text{s}^2}{\text{s}} = 1.16\text{x}10^6\,\frac{\text{J}}{\text{s}} = 1.16\ \text{MW}$$

7.7. We are trying to find a set of pi terms to relate the dependent variable (c, the speed of the waves), to h, ρ, σ, and λ. The dimensions of these parameters, using M, L, and T as the base set of dimensions, are:

$$c[=]\frac{\text{L}}{\text{T}}; \qquad h[=]\text{L} \qquad \rho[=]\frac{\text{M}}{\text{L}^3} \qquad \sigma[=]\frac{\text{M}}{\text{T}^2} \qquad \lambda[=]\text{L}$$

Since we are told to use h, ρ, and σ as repeating variables, the dimensionless Π terms we are trying to identify will contain those three variables and either h (for Π_1) and λ (for Π_2). The requirements that these Π terms be dimensionless leads to the following relationships:

$$\Pi_1 = ch^{a_1}\rho^{b_1}\sigma^{c_1}[=]\left(\frac{\text{L}}{\text{T}}\right)\left(\text{L}\right)^{a_1}\left(\frac{\text{M}}{\text{L}^3}\right)^{b_1}\left(\frac{\text{M}}{\text{T}^2}\right)^{c_1}$$

For M: $0 = b_1 + c_1$

For L: $0 = 1 + a_1 - 3b_1$

For T: $0 = -1 - 2c_1$

We conclude that $c_1 = -½$, $b_1 = ½$, $a_1 = ½$, so $\Pi_1 = c\sqrt{\frac{h\rho}{\sigma}}$. Following the same procedure for Π_2:

$$\Pi_2 = \lambda h^{a_2} \rho^{b_2} \sigma^{c_2} [=] (\mathrm{L})(\mathrm{L})^{a_2} \left(\frac{\mathrm{M}}{\mathrm{L}^3}\right)^{b_2} \left(\frac{\mathrm{M}}{\mathrm{T}^2}\right)^{c_2}$$

For M: $0 = b_2 + c_2$

For L: $0 = 1 + a_2 - 3b_2$

For T: $0 = 0 - 2c_2$

Thus: $c_2 = 0$, $b_2 = 0$, $a_2 = -1$, and $\Pi_2 = \dfrac{\lambda}{h}$. The conclusion is that $c\sqrt{\dfrac{h\rho}{\sigma}} = fcn\left(\dfrac{\lambda}{h}\right)$.

7.33. For dynamic similarity, the force ratios must be identical in the two systems. For flow in a pipe, the relevant forces are the inertial force driving the flow and the viscous force resisting it, so the relevant dimensionless ratio is the Reynolds number. Equating the Reynolds number in the model and prototype, we have:

$$\frac{V_m d_m}{\nu_m} = \frac{V_p d_p}{\nu_p}$$

$$d = \frac{V_p \nu_m}{V_m \nu_p} d_p = \frac{\left(0.30\,\frac{\mathrm{m}}{\mathrm{s}}\right)\left(1.46\mathrm{x}10^{-5}\,\frac{\mathrm{m}^2}{\mathrm{s}}\right)}{\left(2\,\frac{\mathrm{m}}{\mathrm{s}}\right)\left(6.03\mathrm{x}10^{-7}\,\frac{\mathrm{m}^2}{\mathrm{s}}\right)}\left(0.03\ \mathrm{m}\right) = 0.109\ \mathrm{m} = 109\ \mathrm{mm}$$

7.43. (a) The dependent variable (F) is assumed to depend on Q, ρ, and l, and we suspect that the gravitational constant g might also appear in the relationship. The dimensions of these terms, using F, L, and T as the base set of dimensions, are:

$$F [=] \mathrm{F}; \qquad Q [=] \frac{\mathrm{L}^3}{\mathrm{T}} \qquad \rho [=] \frac{\mathrm{FT}^2}{\mathrm{L}^4} \qquad g [=] \frac{\mathrm{L}}{\mathrm{T}^2} \qquad l [=] \mathrm{L}$$

The variables ρ, g, and l contain all the relevant dimensions and they are independent of one another, so we can choose them as the repeating variables. The analysis is carried out as follows:

$$\Pi_2 = F\rho^{a_2} g^{b_2} l^{c_2} \,[=]\, (\mathrm{F})\left(\frac{\mathrm{FT}^2}{\mathrm{L}^4}\right)^{a_2}\left(\frac{\mathrm{L}}{\mathrm{T}^2}\right)^{b_2}(\mathrm{L})^{c_2}$$

$$\left.\begin{array}{ll} \text{F:} & 0 = 1 + a_1 \\ \text{L:} & 0 = -4a_1 + b_1 + c_1 \\ \text{T:} & 0 = 2a_1 - 2b_1 \end{array}\right\} \quad a_1 = -1; \quad b_1 = -1; \quad c_1 = -3$$

$$\Pi_1 = \frac{F}{\rho g l^3}$$

$$\Pi_2 = Q\rho^{a_2} g^{b_2} l^{c_2} \,[=]\left(\frac{\mathrm{L}^3}{\mathrm{T}}\right)\left(\frac{\mathrm{FT}^2}{\mathrm{L}^4}\right)^{a_2}\left(\frac{\mathrm{L}}{\mathrm{T}^2}\right)^{b_2}(\mathrm{L})^{c_2}$$

$$\left.\begin{array}{ll} \text{F:} & 0 = a_2 \\ \text{L:} & 0 = 3 - 4a_2 + b_2 + c_2 \\ \text{T:} & 0 = -1 + 2a_2 - 2b_2 \end{array}\right\} a_2 = 0; \quad b_2 = -\frac{1}{2}; \; c_2 = -\frac{5}{2}$$

$$\Pi_2 = \frac{Q}{\sqrt{gl^5}}$$

(b) From part *a* , we conclude that the dimensionless group containing the dependent parameter (Π_1) depends only on the group found for Π_2. Thus, for the systems to be similar, they must have identical values of Π_2. Since g is inherently the same in the two systems, they will be similar if the following equality is achieved:

$$\frac{Q_m}{\sqrt{gl_m^5}} = \frac{Q_p}{\sqrt{gl_p^5}}$$

$$Q_m = \frac{\sqrt{gl_m^5}}{\sqrt{gl_p^5}} Q_p = \left(\frac{l_m}{l_p}\right)^{5/2} Q_p = \left(\frac{1}{20}\right)^{5/2}\left(1000\frac{\mathrm{ft}^3}{\mathrm{s}}\right) = 0.559\frac{\mathrm{ft}^3}{\mathrm{s}}$$

(c) If the model is set up so that it has the same value of Π_2 as the prototype, the two systems are guaranteed to also have the same value of Π_1. Thus, since g and ρ will be the same in the two systems:

$$\frac{F_m}{\rho g l_m^3} = \frac{F_p}{\rho g l_p^3}$$

$$F_p = \frac{\rho g l_p^3}{\rho g l_m^3} F_m = \left(\frac{l_p}{l_m}\right)^3 F_p = (20)^3 (20 \text{ lb}) = 1.6\text{x}10^5 \text{ lb}$$