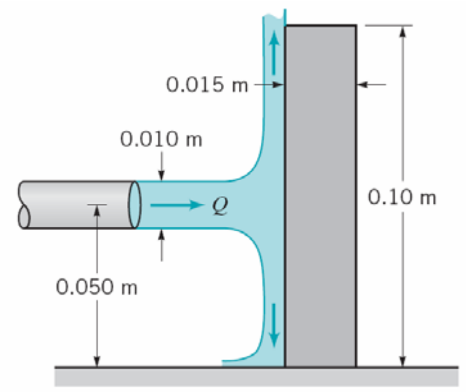
CEE 342 Aut 2005 HW#6 Solutions

5.29. The schematic of the scenario of interest is reproduced below.



When the block is about to tip, the moment about the toe at the lower right of the block is zero. The block weighs 6 N, and this force acts downward at a horizontal distance 7.5 mm to the left of the toe, exerting a counter-clockwise moment. The force of the water is horizontal and is applied at a location 50 mm above the toe, exerting a clockwise moment. Therefore, the required force from the water is given by:

$$(6\text{ N})(0.0075\text{ m}) + F_x(0.050\text{ m}) = 0$$

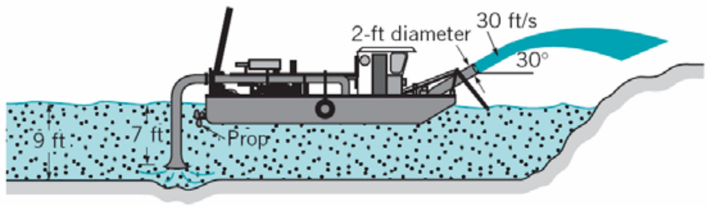
$$F_x = -\frac{(6\text{ N})(0.0075\text{ m})}{0.050\text{ m}} = -0.90\text{ N}$$

The horizontal component of the velocity decreases from V_1 just after the water exits the pipe to zero at the block. The pressure on the water is atmospheric (zero gage) everywhere in this CV, so force of the water can be computed from the impulse-momentum equation, as follows.

$$F_x = \rho Q\left(\cancel{V_{x,2}} - V_{x,1}\right) = \left(1000\frac{\text{kg}}{\text{m}^3}\right)Q\left(-\frac{Q}{\frac{\pi}{4}(0.01\text{ m})^2}\right) = \left(-1.27\text{x}10^7\frac{\text{kg}}{\text{m}^5}\right)Q^2$$

$$Q = \sqrt{\frac{-0.90\text{ N}}{-1.27\text{x}10^7\frac{\text{kg}}{\text{m}^5}\left(1\frac{\text{N}}{\text{kg-m/s}^2}\right)}} = 2.66\text{x}10^{-4}\frac{\text{m}^3}{\text{s}}$$

5.41. The schematic is reproduced below. We define the CV to include the whole barge, so that the fluid entering the CV has zero horizontal momentum, but the fluid leaving the CV has finite momentum in the horizontal direction.

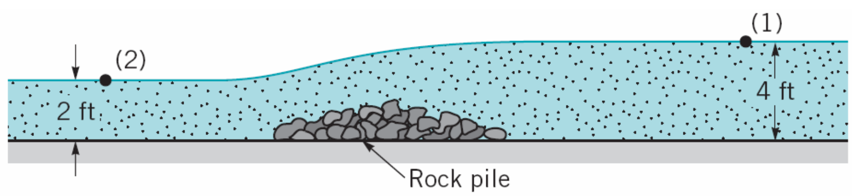


According to the impulse-momentum equation, the horizontal force applied to the water/sand mixture in the x direction as it passes through the CV is:

$$F_x = \rho Q\left(V_{x,2} - \cancel{V_{x,1}}\right) = \left[(1.2)\left(1.94\frac{\text{slug}}{\text{ft}^3}\right)\right]\left[\left(\pi\frac{(2\text{ ft})^2}{4}\right)\left(30\frac{\text{ft}}{\text{s}}\right)\right]\left(\left(30\frac{\text{ft}}{\text{s}}\right)\cos 30^\text{o} - 0\right)$$

$$= 5700\frac{\text{slug-ft}}{\text{s}^2}\left(1\frac{\text{lb}}{\text{slug-ft/s}^2}\right) = 5700\text{ lb}$$

5.87. The system diagram is shown below.



At the location where the depth of the river is 4 ft (location 1), its velocity is:

$$V_1 = \frac{Q}{A_1} = \frac{2400\text{ ft}^3/\text{s}}{(100\text{ ft})(4\text{ ft})} = 6\frac{\text{ft}}{\text{s}}$$

The velocity at location 2 must be twice that at location 1, or 12 ft/s. The water must lose energy due to friction (i.e., headloss must be positive) in the direction of flow. We will assume tentatively that the flow is from left to right (location 2 to location 1). In that case, the energy equation is:

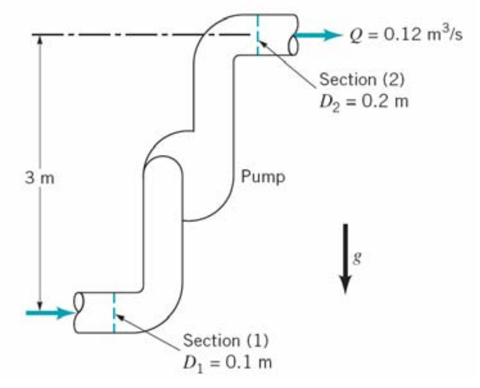
$$\left(\frac{p}{\gamma} + z + \frac{V^2}{2g}\right)_2 - \left(\frac{p}{\gamma} + z + \frac{V^2}{2g}\right)_1 = h_L$$

$$\left(0\text{ ft} + 4\text{ ft} + \frac{(12\text{ ft/s})^2}{2\left(32.2\text{ ft/s}^2\right)}\right) - \left(0\text{ ft} + 2\text{ ft} + \frac{(6\text{ ft/s})^2}{2\left(32.2\text{ ft/s}^2\right)}\right) = h_L$$

$$-0.323\text{ ft} = h_L$$

Since the computed headloss is negative in the direction of flow, the assumption of the direction of flow must have been incorrect. The correct calculation of the headloss just involves reversing the calculation, so that the magnitude of h_L computed above is correct, but the direction is wrong. That is, the flow is from right to left, and the headloss attributable to the rocks is 0.323 ft.

5.111. This question asks us to determine the pressure drop between points 1 and 2 in the gasoline pumping system shown below, if the head loss is given as $h_L g = 0.3V_1^2/2$ and the pump delivers 20 kW of power.



The energy equation between points 1 and 2 can be written as follows, with the terms inside the parentheses each expressed as energy per unit volume of fluid:

$$0 = \left(p + z\gamma + \frac{\rho V^2}{2} \right)_1 Q - \left(p + z\gamma + \frac{\rho V^2}{2} \right)_2 Q + \dot{W}_p - h_L \gamma Q$$

$$p_2 - p_1 = \left[(z_1 - z_2)\gamma + \frac{\rho}{2}\left(V_1^2 - V_2^2\right) \right] + \frac{\dot{W}_p}{Q} - h_L g \rho$$

Everything on the right-hand side is known, other than the velocities at points 1 and 2. These velocities can be computed as:

$$V_1 = \frac{Q}{\frac{\pi}{4} D_1^2} = \frac{0.12 \text{ m}^3/\text{s}}{\frac{\pi}{4}(0.10 \text{ m})^2} = 15.28 \frac{\text{m}}{\text{s}}$$

$$V_2 = V_1 \left(\frac{D_1}{D_2} \right)^2 = \left(15.28 \frac{\text{m}}{\text{s}} \right) \left(\frac{0.10 \text{ m}}{0.20 \text{ m}} \right)^2 = 3.82 \frac{\text{m}}{\text{s}}$$

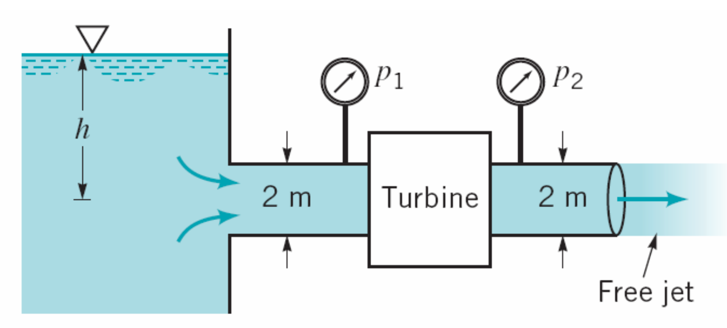
Therefore:

$$p_2 - p_1 = \left\{(-3\text{ m})\left(0.68*9800\frac{\text{N}}{\text{m}^3}\right) + \frac{0.68\left(1000\text{ kg/m}^3\right)}{2}\left[\left(15.28\frac{\text{m}}{\text{s}}\right)^2 - \left(3.82\frac{\text{m}}{\text{s}}\right)^2\right]\right\}$$

$$+\frac{20,000\text{ N-m/s}}{0.12\frac{\text{m}^3}{\text{s}}} - 0.3\frac{\left(15.28\frac{\text{m}}{\text{s}}\right)^2}{2}\left(0.68*1000\ \frac{\text{kg}}{\text{m}^3}\right)$$

$$= 197,281\text{ Pa}$$

5.119. The schematic of the system is shown below. We need to calculate the pressure difference across the turbine, assuming negligible head loss, if the flow rate is 20 m^3/s and the turbine develops 2500 kW. We also need to determine the elevation *h*, if the head loss for the entire system is 2.5 m.



(a) Choosing the CV to include just the turbine, and noting that the elevation and the velocity do not change from the inlet to the outlet of this CV, the energy equation is:

$$0 = \left(p + z\gamma + \frac{\rho V^2}{2g}\right)_1 Q - \left(p + z\gamma + \frac{\rho V^2}{2g}\right)_2 Q - \dot{W}_T$$

$$p_1 - p_2 = \left[\cancel{(z_2 - z_1)}\gamma + \frac{\rho}{2}\cancel{\left(V_2^2 - V_1^2\right)}\right] + \frac{\dot{W}_T}{Q}$$

$$= \frac{2500\text{ kN-m/s}}{20\text{ m}^3\text{/s}} = 125\frac{\text{kN}}{\text{m}^2}$$

(b) Now choosing the fluid in the entire diagram as the CV, and designating the top of the reservoir as location 3 and the beginning of the free jet as location 4, we can write:

$$0 = \left(\frac{p}{\gamma} + z + \frac{V^2}{2g}\right)_3 Q\gamma - \left(\frac{p}{\gamma} + z + \frac{V^2}{2g}\right)_4 Q\gamma - \dot{W}_T - h_L Q\gamma$$

$$0=\left(\frac{p_3-p_4}{\gamma}+z_3-z_4+\frac{V_3^2-V_4^2}{2g}\right)Q\gamma-\dot{W}_T-h_LQ\gamma$$

We assume that the velocity at the top of the reservoir is zero. We can find the velocity in the jet as:

$$V_4=\frac{Q}{A}=\frac{20\text{ m}^3/\text{s}}{\frac{\pi}{4}\left(2\text{ m}\right)^2}=6.37\,\frac{\text{m}}{\text{s}}$$

The pressure is zero at both points 3 and 4, so:

$$0=\left(\cancel{\frac{p_3-p_4}{\gamma}}+h+\frac{0-V_4^2}{2g}\right)Q\gamma-\dot{W}_T-h_LQ\gamma$$

$$h=\frac{\dot{W}_T}{Q\gamma}+h_L+\frac{V_4^2}{2g}$$

$$=\frac{2500\text{ kN-m/s}}{\left(20\text{ m}^3/\text{s}\right)\left(9800\text{ N/m}^3\right)}+2.5\text{ m}+\frac{\left(6.37\text{m/s}\right)^2}{2\left(9.81\text{ m/s}^2\right)}=17.3\text{ m}$$