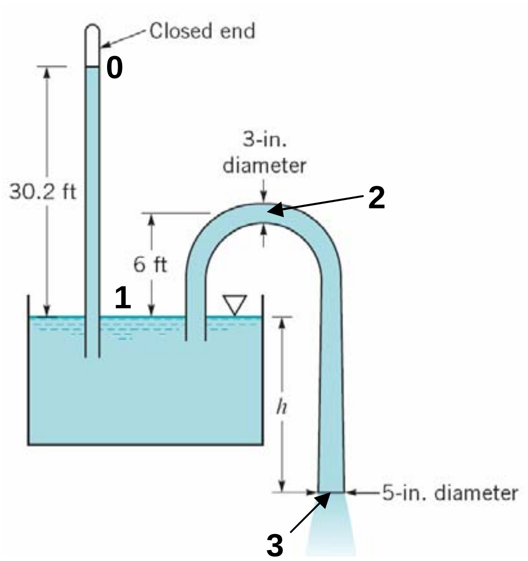
CEE 342 Aut 2005 HW#4 Solutions



3.39. Cavitation will occur at point 2 when the pressure at that point drops below the vapor pressure of water. That pressure is present at point 0, where the velocity is zero, so if we write the Bernoulli equation between points 0 and 2 we find:

$$\cancel{\frac{p_0}{\gamma}} + \cancel{\frac{v_0^2}{2g}} + z_0 = \cancel{\frac{p_2}{\gamma}} + \frac{v_2^2}{2g} + z_2$$

$$v_2 = \sqrt{2g\left(z_0 - z_2\right)} = \sqrt{2\left(32.2\ \text{ft/s}^2\right)\left(24.2\ \text{ft}\right)} = 39.5\ \text{ft/s}$$

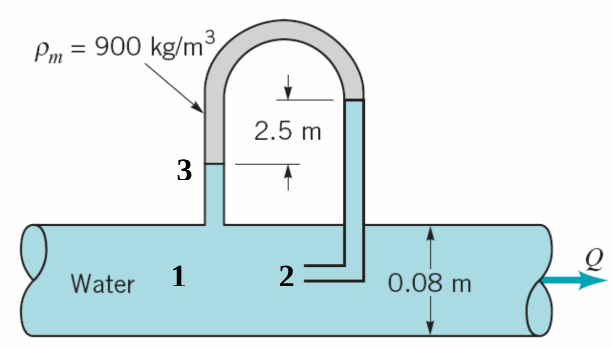
The velocity at point 3 can be related to that at point 2 via continuity:

$$v_3 = v_2 \frac{A_2}{A_3} = \left(39.5\ \text{ft/s}\right)\frac{\left(\pi/4\right)\left(3\ \text{in.}\right)^2}{\left(\pi/4\right)\left(5\ \text{in.}\right)^2} = 14.2\ \text{ft/s}$$

Using that information and writing the Bernoulli equation between points 1 and 3, we find:

$$\cancel{\frac{p_1}{\gamma}} + \cancel{\frac{v_1^2}{2g}} + z_1 = \cancel{\frac{p_3}{\gamma}} + \frac{v_3^2}{2g} + z_3$$

$$h = z_1 - z_3 = \frac{v_3^2}{2g} = \frac{v_2^2}{2g} = \frac{\left(14.2\ \text{ft/s}\right)^2}{2\left(32.2\ \text{ft/s}^2\right)} = 3.14\ \text{ft}$$



3.47. We can choose points 1 and 2 to be at the same elevation as the opening in the tube, with point 1 sufficiently upstream that the fluid is unaffected by the tube. Also, define l as the vertical distance between point 3 and either point 1 or point 2. Write the Bernoulli equation between points 1 and 2 we find:

$$\frac{p_1}{\gamma_w} + \frac{v_1^2}{2g} + \cancel{z_1} = \frac{p_2}{\gamma_w} + \cancel{\frac{v_2^2}{2g}} + \cancel{z_2}$$

$$v_1 = \sqrt{2g\frac{p_2 - p_1}{\gamma_w}}$$

Also, because the pressure distribution is hydrostatic in a pipe with steady flow, we can relate the pressures at points 1 and 2 by moving around the manometer as follows:

$$p_1 - l\gamma_w - (2.5 \text{ m})\gamma_m + (2.5 \text{ m} + l)\gamma_w = p_2$$

$$(2.5 \text{ m})(\gamma_w - \gamma_m) = p_2 - p_1$$

where γ_m is the specific weight of the manometer fluid. Substituting this result into the previous expression for v_1 we find:

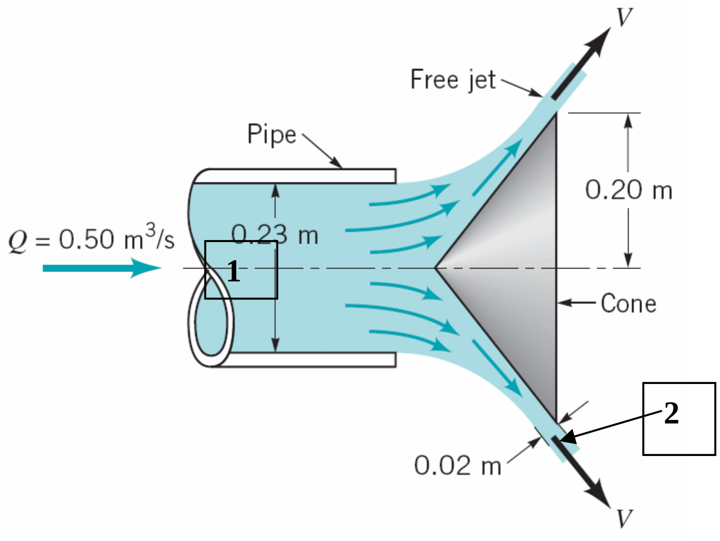
$$v_1 = \sqrt{2g\frac{(2.5 \text{ m})(\gamma_w - \gamma_m)}{\gamma_w}} = \sqrt{2g(2.5 \text{ m})\left(1 - \frac{\gamma_m}{\gamma_w}\right)}$$

The ratio of the specific weights is the same as the ratio of densities, so γ_m / γ_w is 0.9, and v_1 is:

$$v_1 = \sqrt{2\left(9.81\frac{\text{m}}{\text{s}^2}\right)(2.5 \text{ m})(1 - 0.9)} = 2.21\frac{\text{m}}{\text{s}}$$

The flow rate is therefore:

$$Q = v_1 A_1 = \left(2.21\frac{\text{m}}{\text{s}}\right)\left(\frac{\pi}{4}[0.08 \text{ m}]^2\right) = 0.0111\frac{\text{m}^3}{\text{s}}$$



3.87. If we tentatively assume the air is incompressible (i.e., that it has constant density, an assumption we will check later), we can write Bernoulli's equation between points 1 and 2 (as shown in the diagram), as follows:

$$\frac{p_1}{\gamma}+\frac{v_1^2}{2g}+z_1=\cancel{\frac{p_2}{\gamma}}+\frac{v_2^2}{2g}+z_2$$

$$p_1=\gamma\left(\frac{v_2^2-v_1^2}{2g}+\left(z_2-z_1\right)\right)$$

In the system of interest, "point 2" is actually an annular space around the cone. The fluid in this space is all at atmospheric pressure and has uniform velocity, but different values of z. Fortunately, for gases, changes in the elevation head are almost invariably negligible compared to those in the velocity and/or pressure head, so we can ignore the $z_2 - z_1$ term in the preceding equation, yielding:

$$p_1=\gamma\left(\frac{v_2^2-v_1^2}{2g}\right)$$

We can solve for v_1 based on the given volumetric flow rate and pipe diameter:

$$v_1=\frac{Q}{A_1}=\frac{0.50\ \text{m}^3/\text{s}}{(\pi/4)(0.23\ \text{m})^2}=12.0\frac{\text{m}}{\text{s}}$$

The cross-sectional area of flow at point 2 (the annular space around the cone) is $2\pi r\,(\Delta r)$ so, from continuity:

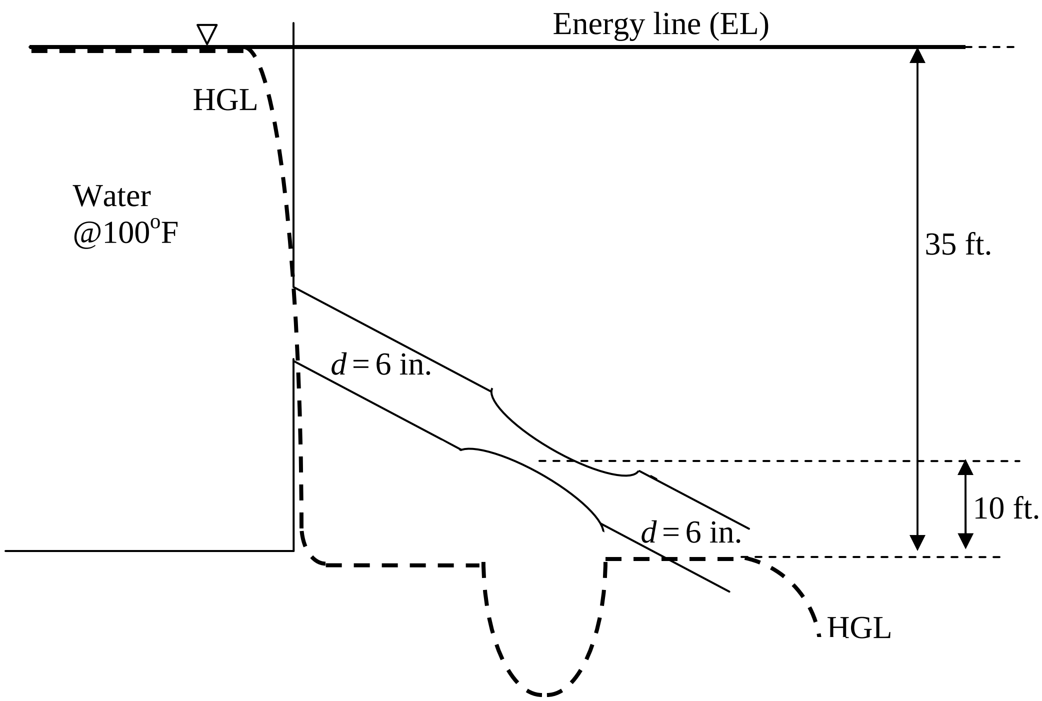
$$v_2 = \frac{Q}{A_2} = \frac{0.50 \text{ m}^3/\text{s}}{2\pi\left(0.20 \text{ m}\right)\left(0.02 \text{ m}\right)} = 19.9\frac{\text{m}}{\text{s}}$$

Substituting these values into the expression derived above for p_1:

$$p_1 = \left(11.81\frac{\text{N}}{\text{m}^3}\right)\frac{\left[19.9 \text{ m/s}\right]^2 - \left[12.0 \text{ m/s}\right]^2}{2\left(9.81\frac{\text{m}}{\text{s}^2}\right)} = 152\frac{\text{N}}{\text{m}^2} = 157 \text{ Pa}$$

The computed value is a gage pressure (based on the fact that we defined atmospheric pressure at point 2 to be zero). The absolute pressure of the atmosphere is ~10^5 Pa, so the pressure changes by less than 0.2% between points 1 and 2; this validates our assumption that the density of the air is approximately constant, so that it can be treated as incompressible and Bernoulli's equation holds. Note also that the term $\frac{v_2^2 - v_1^2}{2g}$ equals approximately 12.8 m, so that ignoring Δz (which ranges from +0.2 to –0.2 m, and has a mean value of zero) seems justified.

The figure from the problem statement is shown again below, with the EL and HGL added. Since we are assuming that frictional losses are negligible, the EL is horizontal at the level of the water surface in the reservoir. The HGL is at that same level at locations far from the outlet pipe, but as we approach the pipe, some piezometric head is converted into velocity head, and the elevation of the HGL drops. In the 6-in pipe, the HGL reaches a level that is $v_{6"}^2/2g$ below the EL. Note that this level must be the level of the outlet, since the total head is constant throughout the system, and the velocity everywhere in the 6" pipe is the same as at the pipe outlet. The HGL remains at that level until the pipe begins to constrict. At that point, it drops further, reaching a minimum at the throat of the constriction, and then reverses direction and returns to the pre-constriction level when the pipe diameter returns to 6 in. It then remains horizontal until the water is discharged. At the throat of the constriction, the absolute pressure equals the vapor pressure of the water.



To quantify the description given above and calculate the critical diameter, we note that the density and vapor pressure of water at 100°F are given in tables at the back of the text as 62.0 lb/ft^3 and 0.95 psia, respectively. The pressure head of the atmosphere is:

$$\frac{p_{atm}}{\gamma} = \frac{\left(14.0\ \text{lb/ft}^2\right)\left(144\ \text{in}^2/\text{ft}^2\right)}{62.0\ \text{lb/ft}^3} = 32.5\ \text{ft}$$

The velocity at the pipe outlet can be computed using the Bernoulli equation between the top of the reservoir (point 1) and the outlet (point 2):

$$\cancel{\frac{p_1}{\gamma}} + z_1 + \cancel{\frac{v_1^2}{2g}} = \cancel{\frac{p_2}{\gamma}} + z_2 + \frac{v_2^2}{2g}$$

$$v_2 = \sqrt{2g\left(z_1 - z_2\right)} = \sqrt{2\left(32.2 \text{ ft/s}^2\right)\left(35 \text{ ft}\right)} = 47.5 \text{ ft/s}$$

Knowing the velocity and pressure at the outlet, we can write the Bernoulli equation again, this time with point 2 being the outlet and point 1 being the throat of the constriction. Specifying that the pressure at the constriction is the vapor pressure of water, we can determine what the velocity must be at that point:

$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$

$$v_1 = \sqrt{v_2^2 + 2g\left(\frac{p_2 - p_1}{\gamma} + z_2 - z_1\right)}$$

$$= \sqrt{\left(47.5 \text{ ft/s}\right)^2 + 2\left(32.2 \text{ ft/s}^2\right)\left(\frac{\left[\left(14.0 - 0.95\right)\text{lb/in}^2\right]\left(144 \text{ in}^2\text{/ft}^2\right)}{62.0 \text{ lb/ft}^3} + \left(-10 \text{ ft}\right)\right)}$$

$$= 59.7 \text{ ft/s}$$

Finally, knowing *v* at the throat, we can compute the diameter of the throat by applying the continuity equation between the throat (point 1) and the outlet (point 2):

$$A_1 v_1 = A_2 v_2$$

$$\frac{v_2}{v_1} = \frac{A_1}{A_2} = \frac{d_1^2}{d_2^2}$$

$$d_1 = \sqrt{\frac{v_2 d_2^2}{v_1}} = \sqrt{\frac{\left(47.5 \text{ ft/s}\right)\left(6 \text{ in}\right)^2}{\left(59.7 \text{ ft/s}\right)}} = 5.35 \text{ in}$$

4.7. The components of the velocity vector at any location (x, y) can be expressed as $u = x$ and $v = x(x-1)(y+1)$. The slope of the streamline (i.e., the value of dy/dx along the streamline) at any location is v/u, so:

$$\frac{dy}{dx} = \frac{v}{u} = \frac{x(x-1)(y+1)}{x} = (x-1)(y+1)$$

Rearranging and integrating yields:

$$\int_{y_1}^{y_2} \frac{dy}{y+1} = \int_{x_1}^{x_2} (x-1)\,dx$$

$$\ln\frac{y_2+1}{y_1+1} = \left(\frac{x_2^2}{2} - x_2\right) - \left(\frac{x_1^2}{2} - x_1\right)$$

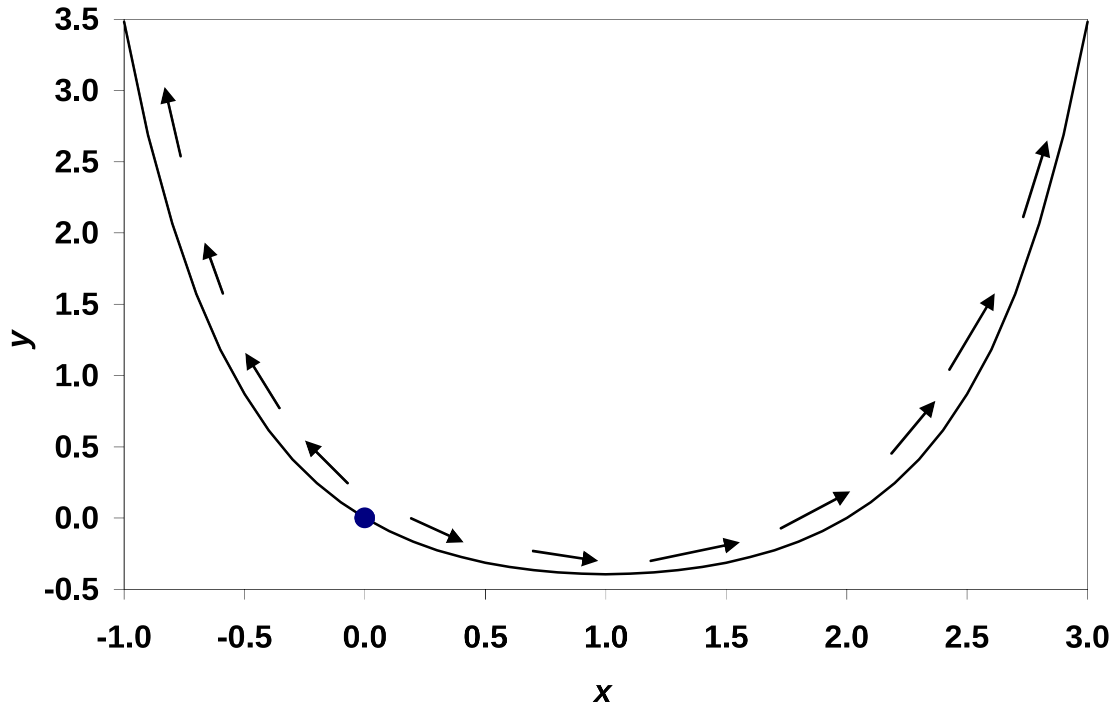
Since we are interested in the streamline that passes through the origin, we know that the point $x_1 = y_1 = 0$ is on the line. Using that information, the equation giving all other points on the line is:

$$\ln(y_2+1) = \frac{x_2^2}{2} - x_2$$

$$y = \exp\left(\frac{x^2}{2} - x\right) - 1$$

where the subscript has been dropped since the streamline includes any point (x, y) that satisfies the equation. The streamline is plotted below; it is symmetric around the value $(1.0, -0.393)$, where the velocity (given by the slope) is strictly in the $+x$ direction (i.e., it has a positive value of u and a v value of zero). The velocity is zero at $(0, 0)$ and away from that point everywhere else, so a source of the fluid must be present at the origin. The arrows drawn on the figure indicate the direction of flow but have not been drawn to scale to indicate the magnitude of the velocity.

Because neither u nor v depends on time, the flow is steady, so the streamline is identical to the streakline (and the pathline) that passes through the origin.



4.17. (a) The local acceleration is, by definition, $\partial V / \partial t$. In this system, the only component of the velocity is in the x direction (i.e., $V = u$), so the local acceleration is $\partial u / \partial t$. Thus, at points 1 and 2, the local acceleration is:

$$\left.\frac{\partial u}{\partial t}\right|_1 = \frac{\partial\left(4t \text{ ft/s}\right)}{\partial\left(t \text{ s}\right)} = 4\frac{\text{ft}}{\text{s}^2} \qquad \left.\frac{\partial u}{\partial t}\right|_2 = \frac{\partial\left(2t \text{ ft/s}\right)}{\partial\left(t \text{ s}\right)} = 2\frac{\text{ft}}{\text{s}^2}$$

(b) For a fluid that has velocity only in the x direction, the convective acceleration is $u\dfrac{\partial u}{\partial x}$.

Between points 1 and 2, u is positive and $\dfrac{\partial u}{\partial x}$ can be approximated as:

$$\frac{\partial u}{\partial x} \approx \frac{\delta u}{\delta x} = \frac{u_2 - u_1}{\delta x}$$

Since u decreases in the $+x$ direction, $\dfrac{\partial u}{\partial x}$ is negative. As noted, u is positive, so the convective acceleration ($u\dfrac{\partial u}{\partial x}$) must be negative.