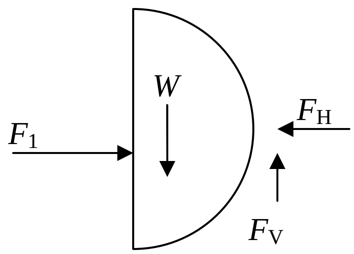
CEE 342 Aut 2005 HW#3 Solutions

2-78. A free body diagram of the fluid in the bulge is shown below. The vertical forces on the fluid include gravity (accounting for a downward force of *W*) and the net upward force of the wall. Thus, the upward force exerted by the wall on the water equals the weight of the water. For a 1-ft length of the bulge, this weight is:



$$F_V = \gamma_{H_2O} \forall = \left(62.4 \frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{\pi (3\text{ ft})^2}{4}\right)(1\text{ ft}) = 882\text{ lb}$$

Note that the upward force exerted by the lower half of the bulge is greater than F_V, but part of this force is counteracted by a downward force exerted by the upper half, so that the net force exerted by the wall is 882 lb upward. The force exerted by the water is the opposite of the that computed, or 882 lb downward.

The horizontal force exerted by the water equals the product of the pressure at the centroid and the projected area of the bulge:

$$F_1 = \gamma h_c A = \left(62.4 \frac{\text{lb}}{\text{ft}^3}\right)(6\text{ ft} + 3\text{ ft})(6\text{ ft x } 3\text{ ft}) = 337\text{ lb}$$

This force is to the right.

2-83. The weight of the two blocks must be countered by the buoyant force, which equals the weight of water displaced by the blocks. The submerged volume of the wooden block is one-half of the total volume of that block, so:

$$V_{submerged} = \frac{1}{2} LHW = \frac{1}{2}(1.3\text{m}*0.7\text{m}*0.7\text{m}) = 0.318\text{m}^3$$

$$F_{B,wood} = \gamma_{water} V_{submerged} = \left(9.80 \frac{\text{kN}}{\text{m}^3}\right)\left(0.318 \text{ m}^3\right) = 3.12 \text{ kN}$$

The net downward force exerted by the concrete block must be the difference between the buoyant force due to the wooden block and the weight of the wooden block:

$$F_{net,concrete} = -F_{net,wood} = -(2.4 \text{ kN} - 3.12 \text{ kN}) = 0.72 \text{ kN}$$

This force also equals the difference between the weight of the concrete block and the buoyant force on it:

$$0.72 \text{ kN} = F_{g,concrete} - F_{B,concrete} = V_{concrete}\left(\gamma_{concrete} - \gamma_{water}\right)$$

$$V_{concrete} = \frac{0.72 \text{ kN}}{\gamma_{concrete} - \gamma_{water}} = \frac{0.72 \text{ kN}}{(23.6 - 9.80)\frac{\text{kN}}{\text{m}^3}} = 0.052 \text{ m}^3$$

3.27. (a) We can apply the Bernoulli equation between a point far upstream of the test section (1) and another at the same horizontal location as the manometer (2), both along the centerline of the wind tunnel:

$$p_1 + \rho\frac{v_1^2}{2} + \gamma z_1 = p_2 + \rho\frac{v_2^2}{2} + \gamma z_2$$

Defining p as the gage pressure, we can assign $p_1 = 0$, $v_1 = 0$, and $z_1 = z_2$. Then, since $v_2 =$ 60 mph = 88 ft/s:

$$0 + 0 + \cancel{\gamma z_1} = p_2 + \left(2.373\frac{\text{slugs}}{\text{ft}^3}\right)\frac{(88 \text{ ft/s})^2}{2}\left(\frac{1 \text{ lb}}{\text{slug}\frac{\text{ft}}{\text{s}^2}}\right) + \cancel{\gamma z_2}$$

$$p_2 = -\left(2.373\text{x}10^{-3}\frac{\text{slugs}}{\text{ft}^3}\right)\frac{(88 \text{ ft/s})^2}{2} = -9.19\frac{\text{lb}}{\text{ft}^2}$$

Because air is a gas, we can assume that the pressure changes negligibly with z, so p_2 is also the pressure at the interface with the water in the manometer. Then, computing pressure changes through the manometer, we find:

$$p_2 + \gamma_{water} h - \gamma_{oil}\left(\frac{1}{12}\text{ ft}\right) = 0\frac{\text{lb}}{\text{ft}^2}$$

$$-9.19\frac{\text{lb}}{\text{ft}^2} + \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)h - 0.9\left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{1}{12}\text{ ft}\right) = 0\frac{\text{lb}}{\text{ft}^2}$$

$$h = \frac{0.9\left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{1}{12}\text{ ft}\right) + 9.19\frac{\text{lb}}{\text{ft}^2}}{62.4\frac{\text{lb}}{\text{ft}^3}} = 0.222\text{ ft} = 2.67\text{ in}$$

(b) The pressure difference between the test section and the stagnation point in the front of the vehicle is the pressure generated by converting the kinetic energy in the test section into mechanical energy. (The change in gravitational potential energy is negligible, since the fluid is a gas.) Thus:

$$\Delta p = \rho\frac{v^2}{2} = \left(2.373\text{x}10^{-3}\frac{\text{slugs}}{\text{ft}^3}\right)\frac{(88\text{ ft/s})^2}{2}\left(\frac{1\text{ lb}}{\text{slug}\frac{\text{ft}}{\text{s}^2}}\right) = 9.19\frac{\text{lb}}{\text{ft}^2}$$

3.45. (a) The static pressure at the constriction along the midline of the pipe (point 1) can be computed from the manometer reading, as follows:

$$p_1 = 0 - h_{\text{Hg}}\gamma_{\text{Hg}} - h_{\text{H}_2\text{O}}\gamma_{\text{H}_2\text{O}} = 0 - \left(\frac{2}{12}\text{ft}\right)\left(13.6*62.4\frac{\text{lb}}{\text{ft}^3}\right) - \left(\frac{5}{12}\text{ft}\right)\left(62.4\frac{\text{lb}}{\text{ft}^3}\right) = -167\frac{\text{lb}}{\text{ft}^2}$$

Also, the velocity at the constriction can be related to that in the larger portion of the pipe by the continuity equation:

$$v_1 A_1 = v_2 A_2$$

$$v_2 = v_1\frac{A_1}{A_2} = v_1\frac{\pi(1.0\text{ in})^2/4}{\pi(2.0\text{ in})^2/4} = \frac{v_1}{4}$$

Writing the Bernoulli equation between point 1 and the point along the centerline of the pipe where the water exits (point 3), and noting that $p_3 = 0$ and $z_1 = z_3$, we find:

$$p_1 + \rho \frac{v_1^2}{2} + \cancel{\gamma z_1} = 0 + \rho \frac{v_3^2}{2} + \cancel{\gamma z_3}$$

Noting that $v_2 = v_3$, and substituting the relationship found from the continuity equation:

$$p_1 + \rho \frac{(4v_3)^2}{2} = \rho \frac{v_3^2}{2}$$

$$p_1 = -7.5 \rho v_3^2$$

$$v_3 = \sqrt{\frac{-p_1}{7.5\rho}} = \sqrt{\frac{\left(167 \text{ lb/ft}^2\right)\left[\left(1 \text{ slug-ft/s}^2\right)/\text{lb}\right]}{7.5\left(1.94 \text{ slug/ft}^3\right)}} = 3.39 \frac{\text{ft}}{\text{s}}$$

$$Q_3 = v_3 A_3 = \left(3.39 \frac{\text{ft}}{\text{s}}\right)\left(\frac{\pi}{4}\left[\frac{2}{12}\text{ ft}\right]^2\right) = 0.074 \frac{\text{ft}^3}{\text{s}}$$

(b) Applying the Bernoulli equation between the end of the pipe at the centerline (point 3) and the stagnation point in the Pitot tube (point 4):

$$p_3 + \rho \frac{v_3^2}{2} + \gamma z_3 = p_4 + \rho \frac{v_4^2}{2} + \gamma z_4$$

Because $z_3 = z_4$, $p_3 = 0$, and $v_4 = 0$:

$$\rho \frac{v_3^2}{2} = p_4$$

$$p_4 = \left(1.94 \text{ slug/ft}^3\right) \frac{\left(3.39 \frac{\text{ft}}{\text{s}}\right)^2}{2} \left(\frac{1 \text{ lb}}{\text{slug-ft/s}^2}\right) = 11.1 \frac{\text{lb}}{\text{ft}^2}$$

$$h = \frac{p_4}{\gamma} = \frac{11.1 \frac{\text{lb}}{\text{ft}^2}}{62.4 \frac{\text{lb}}{\text{ft}^3}} = 0.179 \text{ ft} = 2.14 \text{ in.}$$

3.83. (a) Because the fluid velocity at the top of the reservoir is negligible and the pressures at the top of the reservoir and the outlets of both pipes are zero, the velocity exiting each pipe is developed strictly by conversion of gravitational potential energy to kinetic energy. According to the Bernoulli equation, for such a situation:

$$\frac{v_2^2 - \cancel{v_1^2}}{2g} = z_1 - z_2$$

$$v_2 = \sqrt{2g\left(z_1 - z_2\right)}$$

Designating the top of the reservoir as point 0, the outlet from the 0.03-m pipe as point 2, and that from the 0.02-m pipe as point 3:

$$v_2 = \sqrt{2\left(9.81\text{ m/s}^2\right)\left(3\text{ m}\right)} = 7.67\frac{\text{m}}{\text{s}}$$

$$v_3 = \sqrt{2\left(9.81\text{ m/s}^2\right)\left(7\text{ m}\right)} = 11.7\frac{\text{m}}{\text{s}}$$

The corresponding flow rates are determined by multiplying by the respective cross-sectional areas of the pipes:

$$Q_2 = v_2 A_2 = \left(7.67\frac{\text{m}}{\text{s}}\right)\left(\frac{\pi}{4}\left[0.03\text{ m}\right]^2\right) = 0.0054\frac{\text{m}^3}{\text{s}}$$

$$Q_3 = v_3 A_3 = \left(11.7\frac{\text{m}}{\text{s}}\right)\left(\frac{\pi}{4}\left[0.02\text{ m}\right]^2\right) = 0.0037\frac{\text{m}^3}{\text{s}}$$

The total flow is the sum of Q_2 and Q_3:

$$Q_1 = Q_2 + Q_3 = 0.0054\frac{\text{m}^3}{\text{s}} + 0.0037\frac{\text{m}^3}{\text{s}} = 0.0091\frac{\text{m}^3}{\text{s}}$$

The pressure at point 1 can then be found by applying the Bernoulli equation between that point and the top of the reservoir:

$$\frac{p_0}{\gamma} + \frac{v_0^2}{2g} + z_0 = \frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1$$

$$0 + 0 + z_0 = \frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1$$

$$p_1 = \left(z_0 - z_1 - \frac{v_1^2}{2g}\right)\gamma$$

The velocity at point 1 is:

$$v_1 = \frac{Q_1}{A_1} = \frac{0.0091\frac{\text{m}^3}{\text{s}}}{\frac{\pi}{4}[0.05\text{ m}]^2} = 4.63\frac{\text{m}}{\text{s}}$$

$$p_1 = \left(z_0 - z_1 - \frac{v_1^2}{2g}\right)\gamma = \left(7\text{ m} - \frac{(4.63\text{ m/s})^2}{2(9.81\text{ m/s}^2)}\right)\left(9.80\frac{\text{kN}}{\text{m}^3}\right) = 57.9\frac{\text{kN}}{\text{m}^2} = 57.9\text{ kPa}$$

3.95. Applying the Bernoulli equation between points at atmospheric pressure and where the water surface is flat on either side of the gate, we have:

$$\cancel{\frac{p_0}{\gamma}} + \frac{v_0^2}{2g} + z_0 = \cancel{\frac{p_1}{\gamma}} + \frac{v_1^2}{2g} + z_1$$

$$z_0 - z_1 = \frac{v_1^2 - v_0^2}{2g}$$

$$5\text{ ft} = \frac{v_1^2 - v_0^2}{2(32.2\text{ ft/s}^2)}$$

In addition, the continuity equation tells us that the velocity toward the gate at a point far to the left must be related to that as a point far to the right by:

$$v_1 = v_0\frac{A_0}{A_1} = v_0\frac{6\text{ ft*8 ft}}{1\text{ ft*8 ft}} = 6v_0$$

Substituting this result into the result of the Bernoulli equation, and then utilizing $Q = vA$, we have:

$$(5\text{ ft}) = \frac{(6v_0)^2 - v_0^2}{2(32.2\text{ ft/s}^2)}$$

$$(5\text{ ft})(2)(32.2\text{ ft/s}^2) = (6v_0)^2 - v_0^2$$

$$v_0 = \sqrt{\frac{(5\text{ ft})(2)(32.2\text{ ft/s}^2)}{35}} = 3.03\frac{\text{ft}}{\text{s}}$$

$$Q = v_0 A_0 = \left(3.03\frac{\text{ft}}{\text{s}}\right)(6\text{ ft*8 ft}) = 145\frac{\text{ft}^3}{\text{s}}$$