**CEE 342 Aut 2005. Homework #2 Solutions**

**2.9.** A free-body diagram of the lower hemisphere is shown below. This set of boundaries for the free body is chosen because it isolates the force on the bolts, which is the parameter we are trying to solve for. (Note: assuming that the bolts have to support all the mercury in the sphere might seem reasonable, but unless a free body diagram can be drawn that confirms that assumption based on physical principles, the assumption cannot be relied upon.) Define the relevant variables as follows:

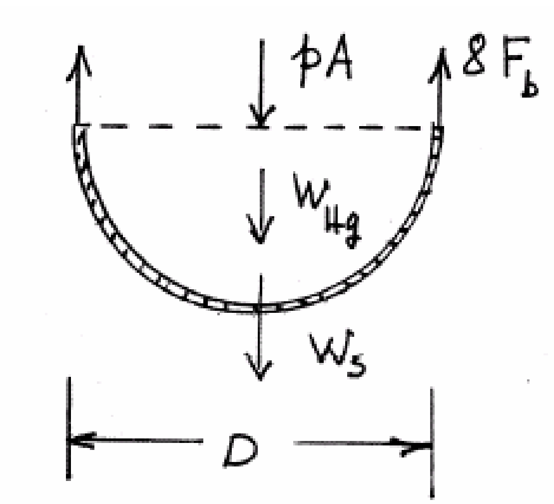
$F_b$ = force on a single bolt

$p$ = the pressure inside the sphere anywhere in the plane at mid-depth

$A$ = the area inside the sphere at mid-depth

$W_{Hg}$ = weight of mercury in the lower hemisphere

$W_s$ = weight of material forming the lower hemisphere




For the sphere to remain stationary, the sum of the vertical forces on the free body must equal zero. The downward forces on the body include (1) the pressure-based force on the upper surface and (2) the weight of mercury and the material forming the lower hemisphere of the container. The only upward force is that on the eight bolts. Therefore, the force balance can be written and solved as follows:

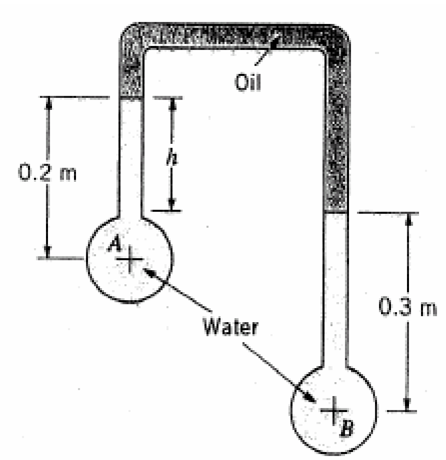
$$pA + W_{\mathrm{Hg}} + W_s = 8F_b$$

$$F_b = \frac{1}{8}\left(pA + W_{\mathrm{Hg}} + W_s\right) = \frac{1}{8}\left(\left[\gamma_{\mathrm{Hg}}\frac{D}{2}\right]\left[\pi\left(\frac{D}{2}\right)^2\right] + \gamma_{\mathrm{Hg}}\left[\frac{1}{2} * \frac{\pi D^3}{6}\right] + W_s\right)$$

$$= \frac{1}{8}\left(\left[847\frac{\mathrm{lb}}{\mathrm{ft}^3}\right]\left[1.5\ \mathrm{ft}\right]\left[\pi\left[1.5\ \mathrm{ft}\right]^2\right] + \left[847\frac{\mathrm{lb}}{\mathrm{ft}^3}\right]\left[\frac{1}{2}\frac{\pi\left(3.0\ \mathrm{ft}\right)^3}{6}\right] + 150\ \mathrm{lb}\right)$$

$$= 1890\ \mathrm{lb}$$

**2.29.** (As noted, the figure in the text [and copied below] is wrong; the heights of water above points *A* and *B* are 1.2 m and 1.3 m, respectively, not 0.2 m and 0.3 m.) The pressure differential between points *A* and *B* is given as $p_B - p_A = 5 \text{ kPa} = 5 \text{ x } 10^3 \text{ N/m}^2$. The height *h* can be determined in a variety of ways, all of which involve either starting at one endpoint and computing the pressure changes as we move to the other endpoint, or else starting at the two endpoints and computing the pressure differences as we move through the system from both endpoints until a meeting point is reached somewhere in the middle. Using the former approach, and noting that the pressure is identical in the two arms of the manometer at an elevation corresponding to the oil/water interface in the left arm (because the space from that location to the same elevation in the right arm is filled with a single fluid), we can write:




$$p_A - \gamma_w (0.2 \text{ m}) + \gamma_{oil} h + \gamma_w (0.3 \text{ m}) = p_B$$

$$h = \frac{(p_B - p_A) + \gamma_w (0.2 \text{ m}) - \gamma_w (0.3 \text{ m})}{\gamma_{oil}} = \frac{(p_B - p_A) - \gamma_w (0.1 \text{ m})}{\gamma_{oil}}$$

$$h = \frac{5\text{x}10^3 \text{ N/m}^2 - (9.80\text{x}10^3 \text{ N/m}^3)(0.1 \text{ m})}{8.95\text{x}10^3 \text{ N/m}^3} = 0.45 \text{ m}$$

**2.35.** (a) The specific weight of the liquid, $\gamma_{liq}$, is:

$$\gamma_{liq} = \rho_{liq} g = \left(800 \frac{\text{kg}}{\text{m}^3}\right)\left(9.81 \frac{\text{m}}{\text{s}^2}\right) = 7850 \frac{\text{N}}{\text{m}^3}$$

For convenience, we designate the surface of the liquid in the tank as location A and the level of the gage as location B. The absolute pressure at B is:

$$p_{B,abs} = p_{A,abs} + \gamma_{liq} h_{A\text{-}B} = 120 \text{ kPa} + \left(7850 \frac{\text{N}}{\text{m}^3}\right)(1.0 \text{ m})\left(\frac{1 \text{ kPa}}{1000 \text{ N}}\right) = 127.85 \text{ kPa}$$

Atmospheric pressure is 101 kPa, so the gage pressure at B is:

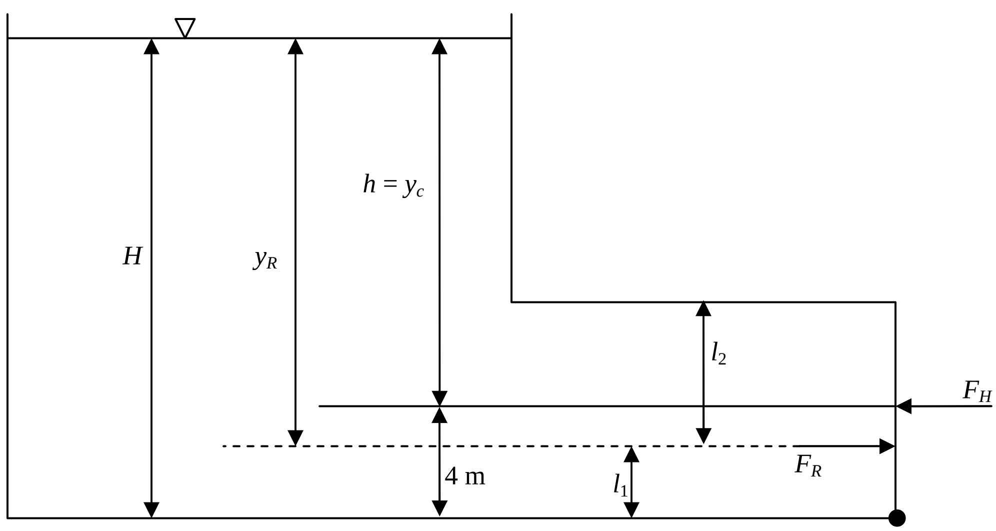
$$p_{\text{B,gage}} = p_{\text{B,abs}} - p_{\text{atm}} = (127.85 - 101)\ \text{kPa} = 26.85\ \text{kPa}$$

(b) The pressure at the bottom of the height designated as *h* is that at the gage, since the two locations are connected by a continuous, uniform fluid. Furthermore, the gage pressure at the Hg/air interface, which we designate as location C, is zero. Therefore, we can write:

$$p_{\text{B,gage}} - \gamma_{\text{Hg}} h = p_{\text{C,gage}} = 0$$

$$h = \frac{p_{\text{B,gage}}}{\gamma_{\text{Hg}}} = \frac{26.85\ \text{kPa}\left(1000 \frac{\text{N/m}^2}{\text{kPa}}\right)}{133\text{x}10^3\ \text{N/m}^3} = 0.20\ \text{m}$$

**2.57.** (a) The maximum water height that will allow the gate to remain closed will be the height that causes the moment around the hinge to be zero, when the maximum force $F_H$ is applied externally. The counter-clockwise moment equals the product $F_{H,max}$(4 m). We must find the water depth that causes the resultant force on the gate from the water, and its point of action, to provide an equal and opposite moment.




The magnitude of the resultant force exerted on the gate by the water equals the product of the pressure at the centroid and the area. Because the gate is rectangular, the centroid is at mid-depth, so the resultant force is:

$$F_R = h\gamma_{\mathrm{w}} A = h\left(9.80\frac{\mathrm{kN}}{\mathrm{m}^3}\right)(3\mathrm{m} \text{ x } 8\mathrm{m}) = \left(235.2\frac{\mathrm{kN}}{\mathrm{m}}\right)h$$

The location of action of this force is:

$$y_R = \frac{I_{\mathrm{xc}}}{y_c A} + y_c = \frac{(1/12)(3\text{ m})(8\text{ m})^3}{h(3\mathrm{m} \text{ x } 8\mathrm{m})} + h = \frac{5.33\text{ m}^2}{h} + h$$

Noting that, by geometry, $l_1$ can be written as $H - y_R = (h+4) - y_R$, we find:

$$l_1 = h + 4 - y_R = h + 4\mathrm{m} - \left[\frac{5.33\text{ m}^2}{h} + h\right] = 4\mathrm{m} - \frac{5.33\text{ m}^2}{h}$$

We can now balance the moments around the hinge.

$$F_R l_1 = F_{H,\max}(4\text{ m})$$

$$\left[\left(235.2\frac{\mathrm{kN}}{\mathrm{m}}\right)h\right]\left(4\mathrm{m} - \frac{5.33\text{ m}^2}{h}\right) = (3500\text{ kN})(4\text{ m})$$

$$(940.8\text{ kN})h - 1254\text{ kN-m} = 14,000\text{ kN-m}$$

$$h = \frac{14,000\text{ kN-m} + 1254\text{ kN-m}}{940.8\text{ kN}} = 16.2\text{ m}$$

(b) If the gate is hinged at the top, the magnitude and location of action of the resultant force due to the water remain the same. However, the moment arm for $F_R$ is longer, extending from the top of the gate to the location of action. This distance is designated $l_2$ in the figure and can be written in terms of other system parameters as follows:

$$l_2 = 8\mathrm{m} - l_1 = 4\mathrm{m} + \frac{5.33\text{ m}^2}{h}$$

In this case, the balancing of moments becomes:

$$F_R l_2 = F_{H,\max}(4\text{ m})$$

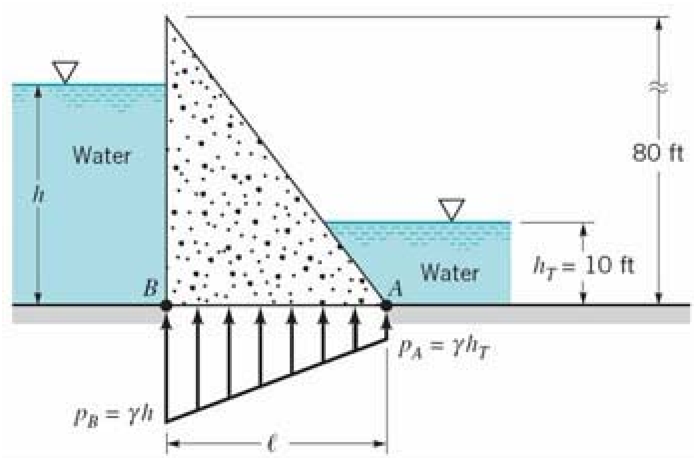
$$\left[\left(235.2\frac{\mathrm{kN}}{\mathrm{m}}\right)h\right]\left(4\mathrm{m} + \frac{5.33\text{ m}^2}{h}\right) = (3500\text{ kN})(4\text{ m})$$

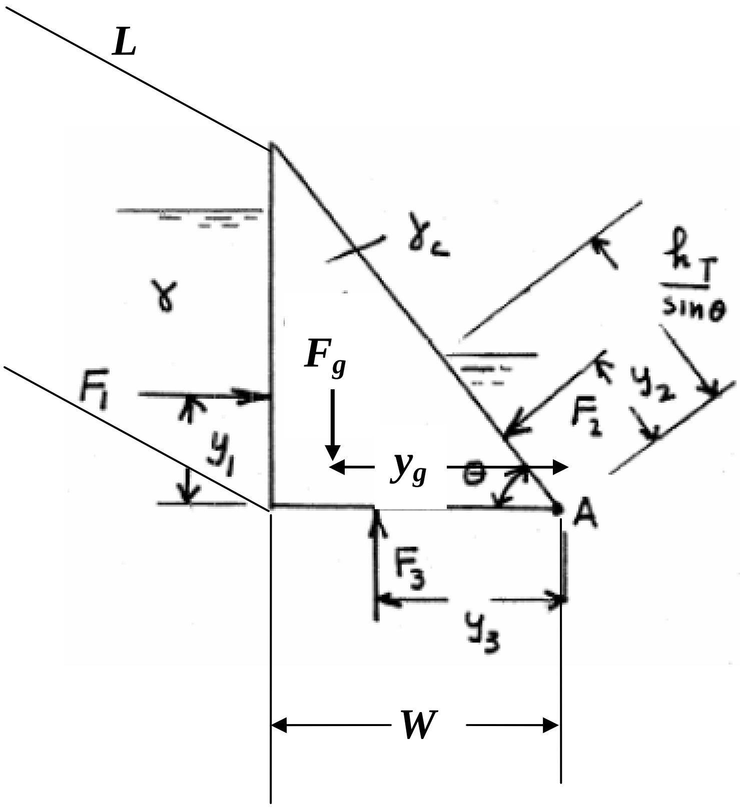
$$(940.8\text{ kN})h + 1254\text{ kN-m} = 14,000\text{ kN-m}$$

$$h = \frac{14,000 \text{ kN-m} - 1254 \text{ kN-m}}{940.8 \text{ kN}} = 13.5 \text{ m}$$

Thus, the maximum height of water is less when the gate is hinged at the top.

**2.69.** The diagram defining the problem and a free-body diagram of the dam are shown below. (Note that the parameter labeled ***ℓ*** in the definition diagram has been relabeled as *W* (width) in the free-body diagram.)






We can write expressions for $F_1$, $F_2$, $F_3$, $y_1$, and $y_2$ directly based on the system geometry and the fact that the three surfaces of interest are all planer. Specifically, we know that the pressure-based force exerted on a plane surface by a uniform fluid has magnitude $\gamma h_c A$, where $h_c$ is the pressure at the centroid, and that the direction of the force is perpendicular to the plane. Thus, the forces shown in the free-body diagram can be represented as:

$$F_1 = \gamma h_{c,1} A_1 = \gamma \left( \frac{h}{2} \right) (hL) = \frac{\gamma h^2 L}{2}$$

$$F_2 = \gamma h_{c,2} A_2 = \gamma \left( \frac{h_\mathrm{T}}{2} \right) \left( \frac{h_\mathrm{T}}{\sin\theta} L \right) = \frac{\gamma h_\mathrm{T}^2 L}{2\sin\theta}$$

$$F_3 = \gamma h_{c,3} A_3 = \gamma \left( \frac{h + h_\mathrm{T}}{2} \right) (LW) = \frac{\gamma LW \left( h + h_\mathrm{T} \right)}{2}$$

$$F_g = \gamma_c V_{dam} = \left( 150 \frac{\mathrm{lb}}{\mathrm{ft}^3} \right) \left( \frac{1}{2} \right) (80 \text{ ft}) LW = \left( 6000 \frac{\mathrm{lb}}{\mathrm{ft}^2} \right) LW$$

If we want to simplify the algebra by eliminating one parameter, we can carry out the calculations per unit length of the dam. However, that is not critical, and so it is not done here (despite the recommendation in the problem statement to do so).

Because $F_1$ and $F_3$ increase linearly from zero to their maximum values, the location of action (*i.e.*, the center of pressure) of each force is two-thirds of the way from the location where $F = 0$ to the location where $F = F_{\max}$. Thus:

$$y_1 = \frac{h}{3} \qquad \text{and} \qquad y_2 = \frac{1}{3}\frac{h_\mathrm{T}}{\sin\theta}$$

The pressure on the bottom of the dam can be expressed as the sum of a constant pressure ($F_{con}$) of $\gamma h_\mathrm{T}$ and a variable pressure ($F_{var}$) that increases linearly from zero at the toe to $\gamma(h - h_\mathrm{T})$ on the high-water side. Treating these two contributions independently, we see that the center of pressure of the constant component is in the middle of the underside at a horizontal distance $W/2$ from either point $A$ or point $B$. Because the variable component of the force increases linearly from zero to its maximum value, its center of pressure is two-thirds of the way from point $A$ to point $B$.

We can add the two components of force applied to the bottom of the dam to obtain the center of pressure on the bottom ($y_3$); the calculation is based on a summation of the moments around point $A$ and the knowledge that the total force is the sum of the two components forces. Thus:

$$F_3 = F_{con} + F_{var} = \gamma h_\mathrm{T} LW + \frac{\gamma\left(h - h_\mathrm{T}\right)LW}{2} = \gamma LW\left(h + \frac{h_\mathrm{T}}{2}\right)$$

$$F_3 y_3 = \left(F_{con} + F_{var}\right)y_3 = F_{con}y_{con} + F_{var}y_{var}$$

$$y_3 = \frac{F_{con}\dfrac{W}{2} + F_{var}\dfrac{2W}{3}}{F_{con} + F_{var}} = \frac{\gamma h_\mathrm{T} LW\dfrac{W}{2} + \gamma\dfrac{\left(h - h_\mathrm{T}\right)}{2}LW\dfrac{2W}{3}}{\gamma h_\mathrm{T} LW + \dfrac{\gamma\left(h - h_\mathrm{T}\right)LW}{2}} = \frac{W}{3}\frac{h_\mathrm{T} + 2h}{h_\mathrm{T} + h}$$

Finally, we note that because the dam is triangular, the location of action of its weight along the horizontal axis is two-thirds of the distance from $A$ to $B$, so:

$$y_g = \frac{2}{3}W$$

For equilibrium, the sum of the moments around point $A$ must be zero, so the summation of moments can be written as:

$$F_1 y_1 - F_2 y_2 + F_3 y_3 - F_g y_g = 0$$

$$\left(\frac{\gamma h^2 L}{2}\right)\left(\frac{h}{3}\right)-\left(\frac{\gamma h_{\mathrm{T}}^2 L}{2\sin\theta}\right)\left(\frac{1}{3}\frac{h_{\mathrm{T}}}{\sin\theta}\right)+\left[\gamma LW\left(\frac{h+h_{\mathrm{T}}}{2}\right)\right]\left(\frac{W}{3}\frac{h_{\mathrm{T}}+2h}{h_{\mathrm{T}}+h}\right)-\left(6000\frac{\mathrm{lb}}{\mathrm{ft}^2}LW\right)\left(\frac{2}{3}W\right)=0$$

All the terms on the left side of the equation contain $L$, so we can divide through by $L$ and eliminate that variable. Then, for a given value of $W$, since $h_{\mathrm{T}}$ is given as 10 ft, the only unknown is $h$. Solving for $h$ for each of the specified values of $W$, we find:

| $W$ (ft) | $H$ (ft) |
|---|---|
| 20 | 48.2 |
| 30 | 61.1 |
| 40 | 71.8 |
| 50 | 81.1 |
| 60 | 89.1 |

Note that, in the end, the calculation of $F_3$ and $y_3$ was unnecessary; we could have just as easily solved the problem treating the two components of force on the dam bottom separately, without determining their resultant.