CEE 342 Aut 2005, HW#1 Solutions

1.9. Both sides of the equation must have the same dimensions, and sometimes people report coefficients (such as the value 0.04 to 0.09 in the given equation) that implicitly have dimensions that are not reported. To test for this possibility, we can compare the dimensions of the terms that are shown explicitly. For this effort, we will use the FLT set of dimensions. The equation of interest is:

$$h = (0.04 \text{ to } 0.09)(D/d)^4 (V^2/2g)$$

The parameter on the left is identified as energy loss per unit weight. In the FLT system, energy has dimensions FL (energy = force x distance), so *h* has dimensions of FL/F, or simply L. *V* is a velocity, with dimensions L/T, and *g* is acceleration, with dimensions L/T^2. Therefore, a dimensional analysis of the equation can be expressed as:

$$\left(\frac{\text{FL}}{\text{F}}\right) \doteq (0.04 \text{ to } 0.09 \text{ dimensions?})\left(\frac{\text{L}}{\text{L}}\right)^4 \left(\frac{(\text{L/T})^2}{\text{L/T}^2}\right)$$

$$\text{L} \doteq (0.04 \text{ to } 0.09 \text{ dimensions?})(\text{L})$$

The result indicates that the coefficient has no dimensions. The equation is therefore a general homogeneous equation that is valid in any system of units.

1.23. The values needed to compute the Froude number are given in BG units. The corresponding values in SI units are as follows:

$$V = \left(10\frac{\text{ft}}{\text{s}}\right)\left(0.3048\frac{\text{m}}{\text{ft}}\right) = 3.05\frac{\text{m}}{\text{s}}$$

$$g = 9.81\frac{\text{m}}{\text{s}^2}$$

$$l = (2 \text{ ft})\left(0.3048\frac{\text{m}}{\text{ft}}\right) = 0.610 \text{ m}$$

The value of the Froude number using the two sets of units is:

BG units: $$\frac{V}{\sqrt{gl}} = \frac{10 \text{ ft/s}}{\sqrt{\left(32.2 \text{ ft/s}^2\right)\left(2 \text{ ft}\right)}} = 1.25$$

SI units: $$\frac{V}{\sqrt{gl}} = \frac{3.05 \text{ m/s}}{\sqrt{\left(9.81 \text{ m/s}^2\right)\left(0.610 \text{ m}\right)}} = 1.25$$

The value is the same regardless of which units are used, as must be the case for a dimensionless number.

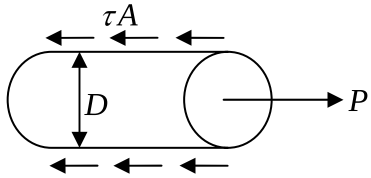
1.54. The shear stress on each surface can be computed as the product of the viscosity and the velocity gradient at that surface. Thus:

Fluid 1: $$\tau_1 = \mu_1 \left(\frac{du}{dy}\right)_{top} = \left(0.4 \frac{\text{N-s}}{\text{m}^2}\right)\left(\frac{3\frac{\text{m}}{\text{s}} - 2\frac{\text{m}}{\text{s}}}{0.02 \text{ m}}\right) = 20 \frac{\text{N}}{\text{m}^2}$$

Fluid 2: $$\tau_2 = \mu_2 \left(\frac{du}{dy}\right)_{bottom} = \left(0.2 \frac{\text{N-s}}{\text{m}^2}\right)\left(\frac{2\frac{\text{m}}{\text{s}}}{0.02 \text{ m}}\right) = 20 \frac{\text{N}}{\text{m}^2}$$

The shear stresses on the two surfaces are equal, so $\tau_{top} / \tau_{bottom} = 1.0$.

1.57. Because the shaft is moving at a steady velocity, the net force on it in the *x* direction must be zero: $\sum F_x = 0$. The force *P* in the +*x* direction is what we are trying to find, and the resisting force in the –*x* direction is due to the shear stress on the surface of the shaft. Equating these two forces, we find:



$$P=\tau A=\left(\mu\frac{\text{velocity of shaft}}{\text{gap width}}\right)(\pi Dl)=\left(\mu\frac{V}{b}\right)(\pi Dl)$$

The kinematic viscosity is given, whereas the dynamic viscosity appears in the preceding equation. We can relate the two by: $\mu=\nu\rho=\nu(\text{s.g.})\left(\rho_{\text{H}_2\text{O @ 4}^\text{o}C}\right)$. Thus:

$$P=\left[\nu(\text{s.g.})\left(\rho_{\text{H}_2\text{O @ 4}^\text{o}C}\right)\right]\left(\frac{V}{b}\right)(\pi Dl)$$

$$=\left[\left(8.0\text{x}10^{-4}\,\frac{\text{m}^2}{\text{s}}\right)(0.91)\left(10^3\,\frac{\text{kg}}{\text{m}^3}\right)\right]\left(\frac{3\text{ m/s}}{0.0003\text{ m}}\right)(\pi)(0.025\text{ m})(0.5\text{ m})$$

$$=286\frac{\text{kg-m}}{\text{s}^2}=286\text{ N}$$

1.94. For the water strider to remain on the surface, the water must exert an upward force on its six "feet" equal to the insect's weight. Around the perimeter of the depression surrounding each foot, the surface tension causes the water to act as if it is a membrane (or a trampoline) that is being stretched. At each location, the force is tangential to the water surface, so it is in a direction pointing upward (out of the water) and outward (radially). The radial forces on opposite sides cancel one another, so the resultant is an upward force equal to $\sigma l\sin\theta$, where σ is the surface tension, l is the circumference of the circle, and θ is the angle of the water surface at the perimeter of the foot ($\theta = 0$ corresponds to a flat water surface, and $\theta = 90^\text{o}$ corresponds to a vertical surface). To get a rough idea of the required value of l, we will assume $\theta = 90^\text{o}$, although a somewhat smaller value would be a reasonable (and better) estimate. Making that assumption, a force balance yields $W=\sigma l_{tot}$, where l_{tot} is the total length of the perimeters of all six feet. We then find, (a) for the bug and (b) for a human:

(a) $l_{tot}=\dfrac{W}{\sigma}=\dfrac{10^{-4}\text{ N}}{7.34\text{x}10^{-2}\text{ N/m}}=1.36\text{x}10^{-3}\text{ m}=1.36\text{ mm}$

This is the total length of contact around the perimeters of all six feet required for the water to exert an upward force equal to the insect's weight. If we assume that each of the six feet has a circular shape (not quite right, since the contact surface is more like a cylinder), the diameter of each foot would have to be:

$$d=\frac{\text{Circumference}}{\pi}=\frac{1.36\text{ mm}/6}{\pi}=0.072\text{ mm}=72\ \mu\text{m}$$

(b) For a human: $l_{tot}=\dfrac{W}{\sigma}=\dfrac{750\text{ N}}{7.34\text{x}10^{-2}\text{ N/m}}=1.02\text{x}10^4\text{ m}\ (>6.3\text{ miles!})$