CEE 342 Aut 2005 HW#10 Solutions

10.14. Writing the energy balance between the two ends of the Pitot tube, we have:

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

where point 1 is the tip of the Pitot tube facing upstream and point 2 is the end of the tube in contact with the atmosphere. Defining the datum as the bottom of the channel, z_2 is given as 4.5 ft. We do not know z_1 or p_1, since the depth of the Pitot tube is not given. However, since the pressure distribution in the channel is hydrostatic, we know that the piezometric head, $\frac{p}{\gamma} + z$, is the same at any depth. That is, wherever the Pitot tube is placed, the sum of p/γ and z is the same. We can therefore equate $\frac{p_1}{\gamma} + z_1$ with $\frac{p_{surf}}{\gamma} + z_{surf}$, where *surf* refers to the water surface. p_{surf} is zero, and since the datum is the channel bottom, z_{surf} is the depth of the water, y. Thus, the energy equation becomes:

$$y + \frac{V_1^2}{2g} = 4.5 \text{ ft}$$

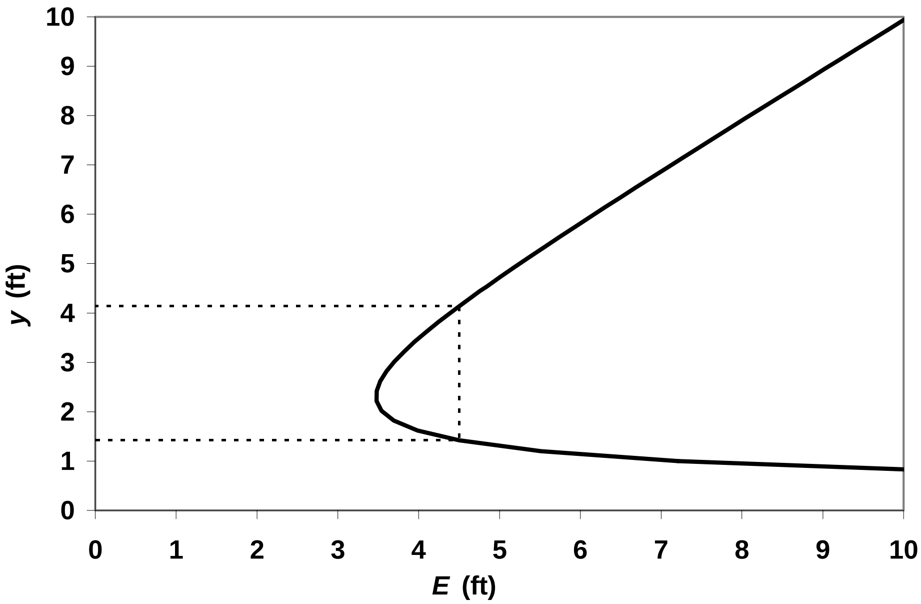
We can express V as q/y, where q is given as 20 cfs/ft, yielding:

$$y + \frac{(q/y)^2}{2g} = 4.5 \text{ ft}$$

$$y + \frac{\left(20 \text{ ft}^2\text{/s}\right)^2}{2\left(32.2 \text{ ft/s}^2\right)} \frac{1}{y^2} = 4.5 \text{ ft}$$

$$\left(6.21 \text{ ft}^3\right)\frac{1}{y^2} = 4.5 \text{ ft} - y$$

By inspection, any positive roots of this equation must fall in the range from $0 < y < 4.5$ ft. (At $y > 4.5$ ft, the left side of the equation is positive and the right side negative, so no solution exists.) The roots in the feasible range can be found by trial and error to be 4.14 ft (sub-critical flow) and 1.42 ft super -critical flow). The specific energy diagram (a plot of y vs. E) is shown below.



10.15. Assuming that friction is negligible, the energy equation can be written as:

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

$$z_{bot,1} + y_1 + \frac{V_1^2}{2g} = z_{bot,2} + y_2 + \frac{V_2^2}{2g}$$

The flow per unit width, q, is (30 ft^3/s) /(5 ft), or 6 ft^2/s. Substituting q/y for V, 2.5 ft for y_1, and 0.2 ft for $z_{bot,2} - z_{bot,1}$, we find:

$$y_1 + \frac{q^2 / y_1^2}{2g} = \left(z_{bot,2} - z_{bot,1}\right) + y_2 + \frac{q^2 / y_2^2}{2g}$$

$$2.5\text{ ft} + \frac{\left(6\text{ ft}^2\text{/s}\right)^2 / \left(2.5\text{ ft}\right)^2}{2\left(32.2\text{ ft/s}^2\right)} = 0.2\text{ ft} + y_2 + \frac{\left(6\text{ ft}^2\text{/s}\right)^2 / y_2^2}{2\left(32.2\text{ ft/s}^2\right)}$$

The equation is satisfied at y_2 values of 0.55 ft and 2.28 ft. The critical depth in this system is:

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{\left[6\text{ ft}^2\text{/s}\right]^2}{32.2\text{ ft/s}^2}\right)^{1/3} = 1.04\text{ ft}$$

Since the geometry of the system does not cause y to ever get that small, the flow remains sub-critical throughout, and $y_2 = 2.28$ ft.

Pipe Network Problem. The flow rates in the various pipes can be determined by an iterative process, using the Hardy-Cross approach. The initial guesses I made are shown below, along with the calculations for the first set of revisions in Q. Friction factors were computed using the equation from Problem 8.37 in Munson. Note that the adjustment in Q_3 in each iteration had to include the computed ΔQ values for both loops, since the pipe is part of both loops. The final values of Q are shown in the last row of the second table.

Pipe	D (m)	L (m)	e/D (--)	A (m^2)	Q (m^3/s)	V (m/s)	Re	Friction factor
1	0.3	1000	0.0033	0.071	0.010	0.14	4.24E+04	0.027
2	0.3	500	0.0033	0.071	–0.010	0.14	4.24E+04	0.027
3	0.2	1000	0.0050	0.031	–0.030	0.95	1.91E+05	0.030
4	0.3	500	0.00333	0.071	–0.060	0.85	2.55E+05	0.027
3	0.2	1000	0.0050	0.031	0.030	0.95	1.91E+05	0.030
5	0.2	500	0.0050	0.031	0.000	0.00	0.00E+00	0.000
6	0.1	1000	0.0100	0.008	–0.020	2.55	2.55E+05	0.038
7	0.7	500	0.0014	0.385	–0.030	0.08	5.46E+04	0.021

Pipe	h_L (m)	h_L/Q (s/m^2)	$\Sigma(h_L)$ (m)	sum(h_L/Q) (s/m^2)	ΔQ (m^3/s)	new Q (m^3/s)	**Final Q (m^3/s)**
1	0.09	9.2				0.0257	**0.0428**
2	–0.05	4.6				0.0057	**0.0228**
3	–7.05	235.2				–0.0234	**–0.0142**
4	–1.65	27.5	–8.66	276.4	0.0157	–0.0443	**–0.0272**
3							
5	7.05	235.2				0.0234	**0.0142**
6	0.00	0.0				0.0091	**0.0170**
7	–125.23	6261.7				–0.0109	**–0.0030**
	0.00	0.2	–118.18	6497.0	0.0091	–0.0209	**–0.0130**

Because frictional losses are assumed to be negligible between the reservoir and the inlet to the network, the pressure head at the entrance is the elevation difference between the reservoir surface and the network, or 60 m. The pressure at each junction point can then be determined from the value at the inlet and the headloss through the various pipes leading to the junction. The velocities in the junctions are not known, but the velocity heads are assumed to be negligible compared to the pressure heads throughout the system, so we can ignore them. The pressure losses computed for the final iteration are shown in the table below. For example, the pressure at the lower right of the system (junction of pipes 5 and 6) can be computed as follows:

$$p_{5\text{-}6} = p_{entrance} - p_1 - p_2 - p_5 = \left(60.00 - 1.662 - 0.234 - 1.126\right)\text{m} = 56.98\text{ m}$$

The gage pressure at all the junctions is also shown in the table.

Pipe	Final Q (m^3/s)	h_L (m)
1	0.0428	1.662
2	0.0228	0.234
3	–0.0142	–1.616
4	–0.0272	–0.344
3	0.0142	1.616
5	0.0170	1.126
6	–0.0030	–2.910
7	–0.0130	–0.001

Junction	P (m)	P (kPa)
4-1	60.00	588.6
1-2	58.34	572.3
2-3-5	58.10	570.0
3-4-7	59.72	585.9
5-6	56.98	559.0
6-7	59.89	587.5